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Confinement and chiral symmetry breaking, one or two critical points?

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We study the QCD phase diagram, in particular we study the critical points of the two main QCD phase transitions, confinement and chiral symmetry breaking. Confinement drives chiral symmetry breaking, and, due to the finite quark mass, at small density both transitions are a crossover, while they are a first or second order phase transition in large density. We study the QCD phase diagram with a quark potential model including both confinement and chiral symmetry. This formalism, in the Coulomb gauge hamiltonian formalism of QCD, is presently the only one able to microscopically include both a quark-antiquark confining potential and a vacuum condensate of quark-antiquark pairs. Our order parameters are the Polyakov loop and the quark mass gap. The confining potential is extracted from the Lattice QCD data of the Bielefeld group. We scan the QCD phase diagram for different quark masses, in order to address how the quark masses affect the critical point location in the phase diagram.

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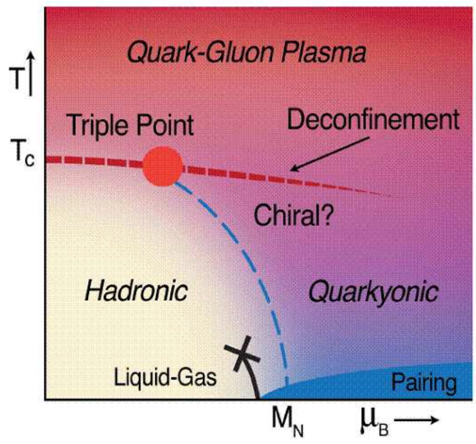

Figure 1: A sketch of the QCD Phase Diagram, according to the collaboration CBM at FAIR [1]. It is usually assumed that the critical point for deconfinement coincides with the critical point for chiral symmetry restoration.

1. Motivation

Our main motivation is to contribute to understand the QCD phase diagram, for finite T and μ, to be studied at LHC, RHIC and FAIR.

Using modern quark models for light quarks we now study chiral symmetry breaking, i. e. quark mass generation, at finite T. In the last Bormio Meeting we used the bottomonium and charmonium as good prototypes to study finite T quark-antiquark potentials. In that case it was sufficient to solve the Schrödinger equation with static lattice QCD potentials, since $m_b, m_c >> \Lambda_{QCD}$ and $m_b, m_c >> Tc$ allowed us to neglect in the quark sector, temperature effects, spontaneous chiral symmetry breaking, relativistic effects and coupled channels. However, to address light quarks at finite temperature T it is also necessary to include temperature, not only in the confining quark-antiquark interaction, but also in the quark mass generation and in the quark propagator.

Thus the present work, not only addresses the QCD phase diagram, but it also constitutes the first step to allow us in the future to,
- compute the spectrum of any hadron at finite T ,
- compute the interaction of any hadron-hadron system at finite T .

Here we address the finite temperature string tension, the quark mass gap for a finite current quark mass and temperature, and the deconfinement and chiral restoration crossovers. We conclude on the separation of the critical point for for chiral symmetry restoration from the critical point for deconfinement.

2. Fits for the finite T string tension from the Lattice QCD energy F_1

At vanishing temperature $T = 0$, the confinement, modelled by a string, is dominant at moderate distances,

$$V(r) \simeq \frac{\pi}{12r} + V_0 + \sigma r \ . \tag{2.1}$$

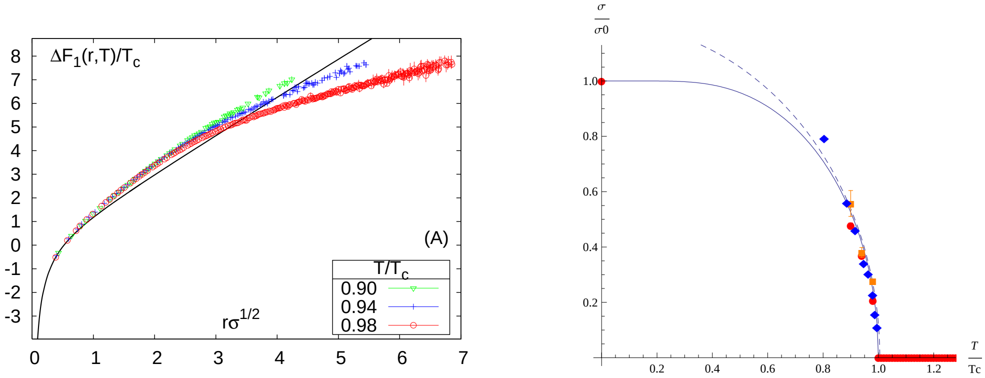

Figure 2: Left: The Bielefeld fee F_1 energy at $T < T_c$. Right: comparing the magnetization critical curve with the string tension σ/σ_0, fitted from the long distance part of F_1, they are quite close.

At short distances we have the Luscher or Nambu-Gotto Coulomb due to the string vibration + the OGE coulomb, however the Coulomb is not important for chiral symmetry breaking. At finite temperature the string tension $\sigma(T)$ should also dominate chiral symmetry breaking, and thus one of our crucial steps here is the fit of the string tension $\sigma(T)$ from the Lattice QCD data of the Bielefeld Lattice QCD group, [2, 3, 4, 5, 6].

The Polyakov loop is a gluonic path, closed in the imaginary time t_4 (proportional to the inverse temperature T^{-1}) direction in QCD discretized in a periodic boundary euclidian Lattice. It measures the free energy F of one or more static quarks,

$$P(0) = N e^{-F_q/T} \ , \ \ P^a(0)\overline{P^a}(r) = N e^{-F_{q\bar{q}}(r)/T} \ . \tag{2.2}$$

If we consider a single solitary quark in the universe, in the confining phase, his string will travel as far as needed to connect the quark to an antiquark, resulting in an infinite energy F. Thus the 1 quark Polyakov loop P is a frequently used order parameter for deconfinement. With the string tension $\sigma(T)$ extracted from the $q\bar{q}$ pair of Polyakov loops we can also estimate the 1 quark Polyakov loop $P(0)$. At finite T, we use as thermodynamic potentials the free energy F_1 and the internal energy U_1, computed in Lattice QCD with the Polyakov loop [2, 3, 4, 5, 6]. They are related to the static potential $V(r) = -fdr$ with $F1(r) = -fdr - SdT$ adequate for isothermic transformations. In Fig. 2 we extract the string tensions $\sigma(T)$ from the free energy $F_1(T)$ computed by the Bielefeld group, and we also include string tensions previously computed by the Bielefeld group [7].

We also find an ansatz for the string tension curve, among the order parameter curves of other physical systems related to confinement, i. e. in ferromagnetic materials, in the Ising model, in superconductors either in the BCS model or in the Ginzburg-Landau model, or in string models, to suggest ansatze for the string tension curve. We find that the order parameter curve that best fits our string tension curve is the spontaneous magnetization of a ferromagnet [8], solution of the algebraic equation,

$$\frac{M}{M_{sat}} = \tanh\left(\frac{T_c}{T}\frac{M}{M_{sat}}\right) \ . \tag{2.3}$$

In Fig. 2 we show the solution of Eq. 2.3 obtained with the fixed point expansion, and compare it with the string tensions computed from lattice QCD data.

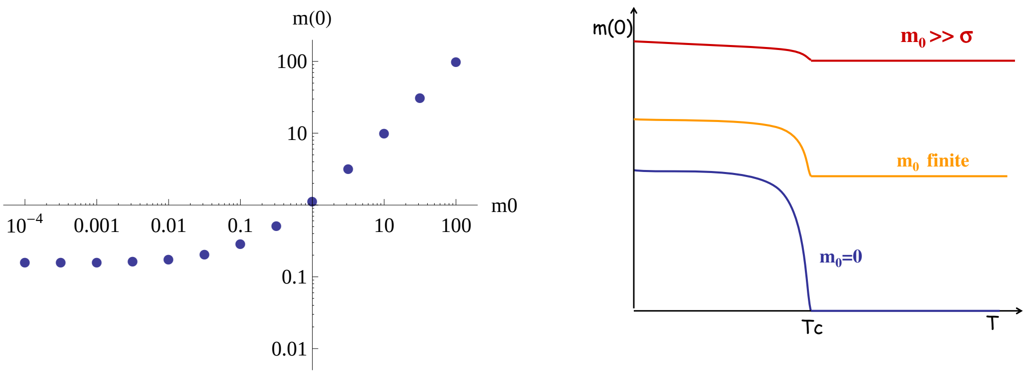

Figure 3: Left: the mass gap $m(0)$ solution of as a function of the quark current mass m_0, in units of $\sigma = 1$. Right: sketch of the effect of m_0 on the crossover versus phase transition of choral restoration at finite T.

3. The mass gap equation with finite T and finite current quark mass m_0.

Now, the critical point occurs when the phase transition changes to a crossover, and the crossover in QCD is produced by the finite current quark mass m0, since it affects the order parameters P or σ, and the mass gap $m(0)$ or the quark condensate $\langle\bar{q}q\rangle$. The mass gap equation at the ladder/rainbow truncation of Coulomb Gauge QCD in equal time reads,

$$
\begin{aligned}
m(p) \;=\; & m_0 + \frac{\sigma}{p^3}\int_0^\infty \frac{dk}{2\pi}\frac{1}{\sqrt{k^2+m(k)^2}}\left\{\left[\frac{pk}{(p-k)^2+\mu^2}-\frac{pk}{(p+k)^2+\mu^2}\right]m(k)p\right. \\
& \left. -\left[\frac{pk}{(p-k)^2+\mu^2}+\frac{pk}{(p+k)^2+\mu^2}+\frac{1}{2}\log\frac{(p-k)^2+\mu^2}{(p+k)^2+\mu^2}\right]m(p)k\right\} . \qquad (3.1)
\end{aligned}
$$

The mass gap equation (3.1) for the running mass $m(p)$ is a non-linear integral equation with a nasty cancellation of Infrared divergences [9, 10, 11]. We devise a new method with a rational ansatz, and with relaxation, to get a maximum precision in the IR where the equation is nearly almost unstable. The solution $m(p)$ is shown in Fig. 3 for a vanishing momentum $p = 0$.

At finite T, one only has to change the string tension to the finite T string tension $\sigma(T)$ [12], and also to replace an integral in ω by a sum in Matsubara Frequencies. Both are equivalent to a reduction in the string tension, $\sigma \to \sigma^*$ and thus all we have to do is to solve the mass gap equation in units of σ^* . The results are depicted in Fig. 3. Thus at vanishing m_0 we have a chiral symmetry phase transition, and at finite m_0 we have a crossover, that gets weaker and weaker when m_0 increases. This is also sketched in Fig. 3.

4. Chiral symmetry and confinement crossovers with a finite current quark mass

In what concerns confinement, the linear confining quark-antiquark potential saturates when the string breaks at the threshold for the creation of a quark-antiquark pair. Thus the free energy $F(0)$ of a single static quark is not infinite, but is the energy of the string saturation, of the order of the mass of a meson i. e. of $2m_0$. For the Polyakov loop we get,

$$
P(0) \simeq N e^{-2m_0/T} . \qquad (4.1)
$$

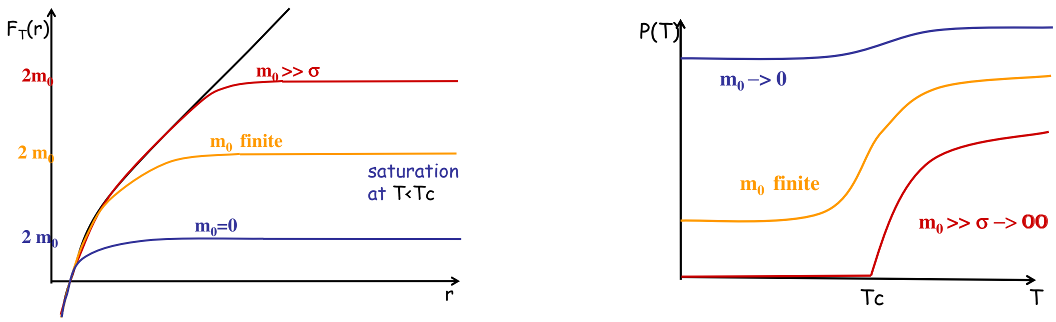

Figure 4: Sketches of the saturation of confinement (left), and of the corresponding crossover in the order parameter P polyakov loop (righ).

Thus at infinite m_0 we have a confining phase transition, while at finite m_0 we have a crossover, that gets weaker and weaker when m_0 decreases. This is sketched in Fig. 4.

Since the finite current quark mass affects in opposite ways the crossover for confinement and the one for chiral symmetry, we conjecture that at finite T and μ there are not only one but two critical points (a point where a crossover separates from a phase transition). Since for the light u and d quarks the current mass m_0 is small, we expect the crossover for chiral symmetry restoration critical to be closer to the $\mu = 0$ vertical axis, and the crossover for deconfinement to go deeper into the finite μ region of the critical curve in the QCD phase diagram depicted in Fig. 1.

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