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The causality concept in quantum field theory and in quantum mechanics.

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We discuss the causality problem in quantum theory. We show that there exists a formulation of quantum theory that, on one hand, preserves the mathematical apparatus of the standard quantum mechanics and, on the other hand, ensures the satisfaction of the causality condition for each individual event including the measurement procedure.

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The causality problem is one of the main problems in the quantum theory. It attracted especially close attention during the construction of the quantum field theory, where the causality (locality) axiom plays a central role [1, 2]. This axiom has different formulations; however, without going into mathematical subtleties, it can be reduced to the following: boson fields must commute at space-like separated points, while fermion fields must anti-commute.

The following argument is often used as a physical justification of this axiom. The results of the measurement in a bounded domain of a Minkowski space (a local measurement) are determined by boson-field values and by bilinear combinations of fermion fields in this domain.

Such locality requirement is purely mathematical in its nature. It can be formulated only in the framework of a particular mathematical formalism, and it is a part of that formalism. In a general discussion of causality it is desirable to proceed from requirements that can be formulated in physics terminology and that can be checked in the experiment directly. That is, such formulation must be fairly obvious.

It is Einstein causality. If two bounded domains $\mathcal{O}_1$ and $\mathcal{O}_2$ of the Minkowski space are space-like separated, then the results of measurements in the domain $\mathcal{O}_1$ do not depend on any manipulations in the $\mathcal{O}_2$.

Practically no one argues with the above formulation. However, the situation changes radically when we try to supplement the above requirement with the following one. There exists a certain physical reality, which determines the results of a local measurement.

Many people object to such an extension of the causality requirement. The arguments on this matter began a long time ago. One can recall the famous debates between Einstein [3] and Bohr [4]. Einstein was in favor of the above extension, while Bohr was against it.

Later on, the majority's opinion within the physics scientific community leaned towards the Bohr side. The results of many modern experiments related to this problem are currently considered as proof that the physical reality mentioned above does not exist.

However, if we abandon the extension formulated above, we almost completely lose the physical foundation behind the locality axiom accepted in the quantum field theory. This rejection would force us to assume that neither local fields, nor their combinations describe a local reality (because it does not exist). Then, it is not clear why these combinations must commute in space-like separated domains.

Thus we have a deadlock situation. The assumption of the existence of a local physical reality contradicts the mathematical formalism of the quantum theory. At the same time, the rejection of this assumption denies the physical foundation one of the main axioms in the mathematical formalism of the quantum field theory.

However, the mathematical formalism of the quantum theory can be compatible with the assumption of the existence of physical reality determining the results of local measurements [5, 6]. The often-produced incompatibility proofs have the following two flaws. First, these proofs often point out toward a contradiction between the experimental data and certain mathematical assumptions, which are used in the construction of mathematical formalism. The questions of physical validity of these assumptions and their necessity are usually not discussed. Second, the interpretation given to the obtained experimental data is far from being always adequate.

The so-called de Broglie waves can be considered as one of the most striking examples of inadequate interpretation. In the beginning of practically any textbook on quantum mechanics it is

said that a de Broglie wave with the wavelength

$$\lambda = \frac{2\pi\hbar}{k}. \tag{1}$$

is associated with any quantum particle having the momentum k. The results of electron interference are mentioned as examples supporting the above statement. In agreement with (1) a clear interference pattern was observed in the experiment.

Equation (1) became the basis of subsequent assertions, that the distinctive feature of quantum particles is the presence of both corpuscular and wave properties.

These assertions seem to be quite well supported experimentally. Nevertheless, we would like to examine if this is indeed the case.

Let us turn to the results of more recent experiments performed by Tonomura [7]. These experiments investigated electron beam scattering by a biprism, which by its physical properties is analogous to a double-slit screen. The beam intensity was so low, that on average there was less than one electron in the experimental apparatus at any single moment. This allowed one to neglect the influence of electron interaction on the results of the experiment. Moreover, it was possible to register the results of passage of a small number of electrons in this experiment.

The experimental results are shown in Fig. 1. The individual photographs correspond to different exposure times. The photograph (1) registered traces of 10 electrons, (2) — 200, (3) — 6000, (4) — 40000, (5) — 140000.

When only a small number of electrons are registered (the photographs (1) and (2)) the interference is not showing through. A pattern appears only after a very large number of electrons were registered (the photographs (4) and (5)). If we try to determine the electron wavelength with a help of the photographs (1) and (2), we do not obtain anything similar to de Broglie Eq. (1).

These results speak in favor of the fact that wave properties are not revealed by a single electron. They become apparent only in a specially prepared ensemble of electrons. In the considered case, all electrons had approximately the same momentum.

Just as interference pattern, quantum state is not the characteristic of an individual physical object. It describes ensemble of such objects. Therefore, the commonly used in textbooks formulation of the mathematical formalism of the quantum theory, with wave functions or state vectors as the basic elements, is not ideal for discussions of the locality problem, because these objects themselves are obviously nonlocal.

The so-called algebraic approach [8, 9] is much better suited for these purposes. Unlike the traditional approach, the Hilbert's space of state vectors is no longer a primary object of the theory within the algebraic approach, and observables are no longer defined as operators in the Hilbert space.

Observables, more specifically, local observables are considered as the primary elements of the theory. Heuristically, an observable is defined as such an attribute of the investigated physical system for which one can obtain some numerical value with the help of a certain measuring procedure. Accordingly, for local observables one can obtain numerical values with the help of local measurements.

Initially the observables are not related to operators in a Hilbert space at all. The Hilbert space itself is constructed with the help of observables as some secondary object. After that a connection between the observables and the operators in this space is established.

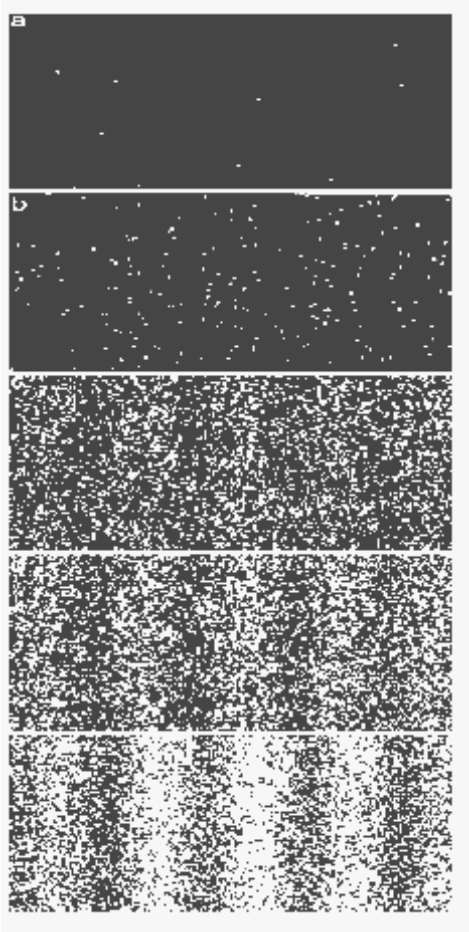

Figure 1: Interference pattern in electron scattering: 10; 200; 6000; 40000; 140000 events

We will conduct the subsequent examination in the framework of a special version of the algebraic approach.

We begin from stating the basic properties of observables. The main property is the following one. The observables can be multiplied by real numbers, added to each other, and multiplied by one another. This property is formulated as the following postulate.

POSTULATE 1. The observables $\hat{A}$ of a physical system are Hermitian elements of some C^*-algebra [10].

Postulate 1 (and all the subsequent ones) is valid for classical systems as well. The set of observables will be denoted $\mathfrak{A}_+$ ($\mathfrak{A}_+ \subset \mathfrak{A}$). In classical systems all observables are compatible with each other (can be measured simultaneously). In a quantum system they can be either compatible or incompatible.

POSTULATE 2. The set of compatible with each other observables is a maximal real associative commutative subalgebra $\mathfrak{Q}_\xi$ of the algebra $\mathfrak{A}$ ($\mathfrak{Q}_\xi \subset \mathfrak{A}_+$).

The index ξ, which runs through the set Ξ, distinguishes one such subalgebra from another. For a classical system the set Ξ contains just a single element, for a quantum system Ξ contains infinitely many elements.

The set of observables $\mathfrak{A}_+$ can be considered as a mathematical model of a quantum system. Accordingly, the subset $\mathfrak{Q}_\xi$ can be considered as observables of some classical subsystem. This subsystem is open, because the quantum-system's degrees of freedom corresponding to observables from different subsets $\mathfrak{Q}_\xi$ can interact with each other.

Moreover, these classical subsystems may not have their own dynamics, because the generalized coordinates and momenta corresponding to the same degree of freedom, may belong to different subsets of $\mathfrak{Q}_\xi$. Therefore, the traditional approach for defining the state as a point of a phase space is not suitable for such subsystems. But, specifying a point in the phase space is equivalent to setting initial conditions for the equations of motion. This allows one to fix the values of all observables of the considered system.

However, one can avoid using equations of motion and the initial condition, and fix the values of all observables directly. Such an approach is suitable for open systems as well.

Measuring the sum of observables in any concrete classical system yields the sum of the values of the individual observables, and measuring the product of observables yields the product of their individual values. In other words, specifying the values of all observables is equivalent to specifying some homomorphic map of the algebra of observables into the set of real numbers. For commutative associative algebra, such a map is called a character. Therefore we accept the following postulate.

POSTULATE 3. The state of a classical subsystem, whose observables are elements of a subalgebra $\mathfrak{Q}_\xi$, is described by a character of this subalgebra.

This definition of the state of a classical subsystem has an important advantage, that it can be generalized to the quantum case. Each quantum observable belonging to $\mathfrak{A}_+$, simultaneously belongs to some subalgebra $\mathfrak{Q}_\xi$. This allows one to consider a quantum system as a family of classical subsystems. If we knew the states of all these subsystems, we could have predicted the result of measuring any observable of the quantum system. This gives us grounds for accepting the following postulate.

POSTULATE 4. The result of measuring any observable of a physical system is determined by its elementary state φ.

Here, φ is a family $\varphi = [\varphi_\xi]$ of characters φ_ξ of all subalgebras $\mathfrak{Q}_\xi$. Each subalgebra $\mathfrak{Q}_\xi$ in the family is represented by a single character.

At first it may seem that the last postulate contradicts the fact that one cannot predict the measurement results for all observables of a quantum system. However, there is no contradiction here. The point is that we can measure simultaneously (that is in a compatible way) only compatible observables. These observables belong to a certain subalgebra $\mathfrak{Q}_\xi$. Lets say for instance they belong to the subalgebra with the index $\xi = \eta$. Then, from the complete set $[\varphi_\xi]$ we can specify only one character φ_η.

Endowed with such information we can predict only the measurement results for observables belonging to $\mathfrak{Q}_\eta$. We will not be able to say anything certain about the values of other observables. Additional measurements, if they are not compatible with the previous ones, will not improve the situation. They will produce new information about the quantum system; however, simultaneously the additional measurements will disturb the state of our system and will make the information obtained earlier worthless.

Figuratively speaking, an elementary state is a holographic image of the system under investigation. Using classical measuring devices we can look at it from one side only, and, hence, obtain a two-dimensional image. Moreover, the measurement will disturb the system and will change its original holographic image.

Therefore, if later we will look at the system from another side, we will see a two-dimensional projection of the new holographic image. Thus, we will never be able to see the entire holographic image.

In connection with the above it is useful to introduce the notion of φ_η-equivalence. Two elementary states $\varphi = [\varphi_\xi]$ è $\varphi' = [\varphi'_\xi]$ will be called φ_η-equivalent, if $\varphi_\eta = \varphi'_\eta$. The relations between the remaining characters φ_ξ and φ'_ξ can be arbitrary. The class of φ_η-equivalent elementary states will be denoted $\{\varphi\}_{\varphi_\eta}$. The most that one can possibly learn about an elementary state φ is that it belongs to some equivalence class $\varphi \in \{\varphi\}_{\varphi_\eta}$.

There is one more obstacle preventing unambiguous predictions of measurement results.

One and the same observable $\hat{A}$ may belong simultaneously to several subalgebras $\mathfrak{Q}_\xi$: $\hat{A} \in \mathfrak{Q}_\xi \cap \mathfrak{Q}_{\xi'}$ $(\xi \neq \xi')$.

Therefore, it is not clear which of the functionals (characters) φ_ξ or $\varphi_{\xi'}$ will describe the results of a particular measurement.

At first it may seem that this additional ambiguity can be easily eliminated with the help of the additional condition

$$\varphi_\xi(\hat{A}) = \varphi_{\xi'}(\hat{A}), \text{ åñëè } \hat{A} \in \mathfrak{Q}_\xi \cap \mathfrak{Q}_{\xi'}. \tag{2}$$

However, this condition leads to numerous contradictions. On the other hand, one can show that the condition (2) is not a necessary one. Indeed, the measurement result may depend not only on the system under investigation, but on the characteristics of the measuring device as well.

From the observer's point of view such dependence is extremely objectionable, and experimentalists try to minimize it as much as possible.

We have come to think that measurement results are virtually independent of the characteristics of "good" measuring devices. However, for this to be true all the devices used for measuring the observable of interest must at least be calibrated in a consistent way. One can show that the existence of incompatible measurements in the quantum case makes such calibration far from being always possible. In particular, if we assign a certain type of measuring devices (ξ-type) to every subalgebra $\mathfrak{Q}_\xi$, then, as it turns out, the devices of different types cannot be calibrated consistently. Therefore, one cannot get rid of a possible dependence of the measurement results on the device type (or, on the index ξ).

Thus, value of an observable is not attribute of physical system. Such attribute (local physical reality) is the elementary state.

The above assertion does not exclude that for some elementary states φ Eq. (2) will be valid for all $\mathfrak{Q}_\xi$, $\mathfrak{Q}_{\xi'}$, containing the observable $\hat{A}$. In this case we shall say that the elementary state φ is stable with respect to the observable $\hat{A}$.

Measurements allow one to establish that the elementary state φ of the system under investigation belongs to some equivalence class $\varphi \in \{\varphi\}_{\varphi_\eta}$. Thereafter, we can make the following predictions. Measuring devices of the η-type will yield the value $A = \varphi_\eta(\hat{A})$ for the observable $\hat{A} \in \mathfrak{Q}_\eta$. From now on the measurement result is denoted by the same symbol as the observable itself, but without the "hat."

If the elementary state φ is stable with respect to the observables $\hat{A} \in \mathfrak{Q}_\eta$, then the same result will be obtained by using measuring devices of any type ξ. One cannot say anything definite about measurement results for observables $\hat{A} \notin \mathfrak{Q}_\eta$, because we will obtain different values for different elementary states $\varphi \in \{\varphi\}_{\varphi_\eta}$.

Within the standard mathematical formalism of quantum mechanics all the physical properties mentioned above are exhibited by quantum states specified by particular values of a complete set of commuting observables. This allows one to state the following definition of a quantum state within the proposed approach.

DEFINITION. A quantum state Ψ_{φ_η} is the class $\{\varphi\}_{\varphi_\eta}$ φ_η- equivalent elementary states, which are stable with respect to the observables $\hat{A} \in \mathfrak{Q}_\eta$.

It is usually assumed that a quantum state Ψ_{φ_η} appears as a result of measuring the observables $\hat{A} \in \mathfrak{Q}_\eta$, where a specified value is registered for each of the observables $\hat{A}$ Of course, this is not always true, at least, because some particles of the investigated system can be absorbed by the device in the measuring process. In order for a measurement to be simultaneously a preparation of a quantum state, it must be reproducible. If repeated measurements of an observable $\hat{A}$ give identical results, we shall mean the measurements reproducible. Note that the repeated measurements are not necessarily performed by measuring devices of the same type.

Within the standard mathematical formalism of quantum mechanics pure states are defined as vectors $|\Phi\rangle$ of some Hilbert state $\mathfrak{H}$.

These vectors are used for calculating the average values of observables in the corresponding quantum states. This definition works very well for applied purposes; however, it does not have an intuitively clear physical interpretation. Within the approach proposed in the present work the average value of an observable is connected in a natural way with the probability distribution of the elementary states φ within the equivalence class $\varphi \in \{\varphi\}_\eta$.

One has to bear in mind that the elementary states satisfy the standard properties of elementary events from the classical Kolmogorov probability theory [11]. Namely, each random experiment results in one and only one elementary event. Different elementary events are mutually exclusive.

Note that the standard approach to quantum mechanics does not have such an ingredient. This became an insurmountable obstacle for application of the classical probability theory to quantum mechanics. Such an obstacle is absent within the approach used here. Therefore, there is no need for creating some artificial quantum probability theory. Instead one can use the well-developed formalism of the classical probability theory. Therefore, the following postulate appears to be fairly natural.

POSTULATE 5. The equivalence class $\{\varphi\}_{\varphi_\eta}$ corresponding to the quantum state Ψ_{φ_η} can be equipped with the structure of a probability space.

Then, the mean value of the observable $\hat{A}$ in the quantum state Ψ_{φ_η} is given by the formula

$$\Psi_{\varphi_\eta}(\hat{A}) = \int_{\varphi\in\Psi_{\varphi_\eta}} P_{\hat{A}}(d\varphi)\,\varphi_\xi(\hat{A}), \tag{3}$$

where

$$P_{\hat{A}}(d\varphi) = P(\varphi : \varphi_\xi(\hat{A}) \le A + dA) - P(\varphi : \varphi_\xi(\hat{A}) \le A),$$

and $P(\varphi : \varphi_\xi(\hat{A}) \le A)$ is probability measure corresponding to the event $\varphi : \varphi_\xi(\hat{A}) \le A$.

In order for formula (3) to define the quantum average, the probabilistic measure must satisfy the following postulates.

POSTULATE 6. The functional Ψ_{φ_η} is linear over the algebra $\mathfrak{A}$

and

POSTULATE 7. The functional does not depend on the particular choice of ξ.

One can show [5] that such distribution actually exists.

With the C^*-algebra and a linear positive normalized functional $\Psi_{\varphi_\eta}(\cdot)$ defined over this algebra, we can construct a representation of the algebra $\mathfrak{A}$ by using the Gelfand-Naimark-Segal canonical construction [12]. In other words, we can construct Hilbert space $\mathfrak{H}$, in which there is an operator $\Pi(\hat{A})$ acting over a space $\mathfrak{H}$ that corresponds to each element $\hat{A}\in\mathfrak{A}$, while the expectation value $\langle\Phi|\Pi(\hat{A})|\Phi\rangle$, where $|\Phi\rangle\in\mathfrak{H}$ is the corresponding vector in Hilbert space — to the quantum average $\Psi_{\varphi_\eta}(\cdot)$. This is the way the standard mathematical apparatus of quantum mechanics is reproduced.

Thus, there are two paths leading to the same result. One can fix the algebra of observables, and build on it a set of elementary states corresponding to some quantum states. Then, one can endow this set by the structure of a probability space and, finally, calculate the probabilistic averages.

The alternative path is the following one. Fix a Hilbert space, define observables as linear operators in that space, while quantum states are either vectors of that space, or density matrices. The average values of observables are defined as the mathematical expectations of the corresponding operators with respect to either vectors of the Hilbert space, or density matrices.

Usually the second path turns out to be much more convenient from the pragmatic point of view. However, the first path has a better physical foundation. This allows one to create a more or less intuitively clear picture of the quantum world. In particular, our model allows one to present an intuitively appealing interpretation of quantum phenomena [6], whose traditional interpretation looks absolutely absurd from the classical physics point of view.

The list of such phenomena includes quantum particle scattering on a double-slit screen, the Einstein-Podolsky-Rosen paradox [13], the delayed choice experiment [14], and quantum teleportation [15].

References

[1] N.N. Bogolioubov and D.V. Shirkov, *Introduction to the Theory of Quantized Fields.* World Sci., Singapore, 1982.

[2] R. Streater and A.S. Wightman, *PCT, Spin and Statistics and All That*, Benjamin, New York, 1964.

[3] A. Einstein, *Journal of the Franklin Institute* **221** (313) 1936.

[4] N. Bohr, *Discussion with Einstein on Epistemological Problems in Atomic Physics*, in *Albert Einstein: Philosopher-Scientist*, Ed. by P.A.'Schlipp, La Salle, Open Court, (201) 1949.

[5] D.A. Slavnov, *Phys. Part. Nucl*, **38** (147) 2007.

[6] D.A. Slavnov, *Phys. Part. Nucl*, **41** (149) 2010.

[7] A. Tonomura, *PNAS* , **102** (14952) 2005.

[8] H. Araki, *Progr.Theor.Phys.*, **32** (844) 1964.

[9] R. Haag and D. Kastler, *J. Math. Phys.* **5** (848) 1964.

[10] J.S. Dixmier, C^**-Algebras*, Elsevier, North-Holland, Amsterdam, 1977.

[11] A.N. Kolmogorov, *Foundations of the Theory of Probability*, Chelsea, New York, 1956.

[12] M.A. Naimark, *Normed Algebras*, Wolters-Noordhoff, Groningen, 1972.

[13] A. Einstein, B. Podolsky, and N. Rozen, *Phys. Rev.* **47** (777) 1935.

[14] J.A. Wheeler, *Mathematical Foundation of Quantum Theory*, New York: Academic Press 1978.

[15] D. Bouwmeester, H. Weinfurter, and A. Zeilinger, *Protocol of Quantum Teleportation* in *Physics of Quantum Information, Ed. by D. Bouwmeester , A. Ekert, and A. Zeilinger,* Springer, Berlin, Heidelberg, 2001.