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# Simple and robust method for measuring of properties of Dark Matter particles at ILC for various models of DM

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In a number of models Dark Matter (DM) consists of particles similar to those in SM. I suggest simple method for the search of Dark Matter particles and some related particles which allows to measure reliably their masses and spins in a wide class of such models for Dark Matter.

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## 1. Introduction

There are a number of models in which Dark Matter (DM) consists of particles similar to those in SM with new discrete quantum number, which I denote here as D-parity. For known particles $D = 1$, for DM particles (DMP) $D = -1$, and D-parity conservation ensures stability of the lightest particle with $D = -1$. We consider such models for DM, in which there is more than one D-odd particle, in particular, neutral DMP D and its more heavy lightest charged counterpart $D^{\pm}$ and all these D-particles have identical spin $s_D$ (1/2 or 0).

• The well known examples of such models for DM are given by MSSM and NMSSM. Here D-parity is another name for R-parity, D is neutralino and $D^{\pm}$ is chargino, here $s_D = 1/2$ [1].

• The second example is given by inert doublet model (IDM) [2]. In notations [3], this model contains one standard Higgs doublet $\phi_S$, responsible for electroweak symmetry breaking and generation of fermions and gauge bosons masses as in the Standard Model (SM), and another scalar doublet $\phi_D$, which doesn't receive vacuum expectation value and doesn't couple to fermions. Four degrees of freedom of the Higgs doublet $\phi_S$ are as in the SM: three Goldstone modes and one mode which becomes the Higgs boson $h$. All the components of the second scalar doublet $\phi_D$ are realized as massive scalar $D$-particles: two charged $D^{\pm}$ and two neutral ones $D$ and $D^A$. By construction, they possess a conserved multiplicative quantum number – D-parity, the lightest particle among them is considered as a candidate for DM particle, here $s_D = 0$.

Limitations for masses of DM-particles from cosmology and collider physics are discussed in many papers (see e.g. [1], [4]). These limitations allow existence of discussed particles with masses lower than electron beam energy of ILC/ CLIC. We don't discuss here the case of mass difference $M_{D^{\pm}} - M_D$ or $M_{D^{\pm}} - M_{D^A}$ lower than about 10 GeV.

In the considered models the interaction of $D$-particles with SM particles appears only from the covariant derivative in the kinetic term of the Lagrangian, that are gauge interactions

$$D^+D^-\gamma, \quad D^+D^-Z, \quad D^+DW^-, \quad D^+D^AW^-, \quad D^ADZ \tag{1.1}$$

with standard electroweak gauge couplings $g$ and $g'$.

We consider production of such $D$-particles at ILC with electron beam energy $E_e \equiv \sqrt{s}/2$, provided that the family of $D$-particles with smaller mass than $E_e$ is confined to $D$, $D^{\pm}$ and no more than one additional neutral $D^A$. All processes below we treat as basic reactions

$$(a)\quad e^+e^- \to D^+D^-\,, \qquad (b)\quad e^+e^- \to DD^A \tag{1.2}$$

with subsequent decay of $D^{\pm}$ or $D^A$.

The problems which should be solved in these experiments are the following.
(1) To observe unambiguously the process with the production of $D$-particles.
(2) To evaluate the mass of DMP $M_D$ and masses of other $D$-particles.
(3) To evaluate the spin of $D$-particles $s_D$.
(4) To obtain some additional information about interactions of $D$-particles.

In all presented cross sections we neglect quantity $1/4 - \sin^2\theta_W$, describing $\gamma - Z$ interference.



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## 2. Two types of $D$-particles, $D$ and $D^{\pm}$.

In this section we consider the case when the set of $D$-particles in the energy range of ILC is confined to $D^{\pm}$ and $D$ with only decay channel $D^{\pm} \to DW^{\pm}$ (with either on shell or off shell [1] $W$'s).

### 2.1 Production, decay, signature

We suggest to use reaction (1.2a) with decay $D^{\pm} \to dW^{\pm}$:

$$e^+e^- \to D^+D^- \to DDW^+W^- \,. \tag{2.1}$$

In the lab system (coincident with c.m.s. for $e^+e^-$) energies, $\gamma$-factors and velocities of $D^{\pm}$ are

$$E_{\pm} = E_e, \;\; \gamma_{\pm} = E_e/M_{D^{\pm}}, \;\; \beta_{\pm} = \sqrt{1-M_{D^{\pm}}^2/E_e^2}. \tag{2.2}$$

The cross section of this process reads

$$\sigma(e^+e^- \to D^+D^-) = \begin{cases} \dfrac{2\pi\alpha^2}{3s}\beta_{\pm}(3-\beta_{\pm}^2)\left(1+\dfrac{R_Z^{(1/2)}s^2}{(s-M_Z^2)^2}\right) & at \;\; s_D = \dfrac{1}{2}; \quad (f) \\ \dfrac{\pi\alpha^2}{3s}\beta_{\pm}^3\left(1+\dfrac{R_Z^{(0)}s^2}{(s-M_Z^2)^2}\right) & at \;\; s_D = 0; \quad (s) \end{cases} \tag{2.3}$$
$$R_Z^{(1/2)} = \frac{1}{16\sin^4(2\theta_W)}, \qquad R_Z^{(0)} = \frac{\cot^2(2\theta_W)}{4\sin^2(2\theta_W)}.$$

(In different models for DMP the relative value of $Z$ contributions $R_Z^{(s)}$ can differ from these values by a simple numerical factor). These cross sections are of the same order as $\sigma(e^+e^- \to \mu\mu)$ (which is given by (2.3f) at $\beta = 1$). For ILC these cross sections are huge.

**If $M_{D^{\pm}} - M_D > M_W$**, the produced $D^{\pm}$ decays to $DW^{\pm}$ with on-mass shell $W$ only. We suggest to observe the following final state systems:

• Two dijets from $q\bar{q}$ decay of $W^+$ and $W^-$, with effective mass $M_W$. For this channel the cross section is $\left[0.676^2 \approx 0.45\right] \cdot \sigma(e^+e^- \to D^+D^-)$.

• One dijet from $q\bar{q}$ decay of $W^+$ or $W^-$ plus $\mu$ or $e$ from $\mu\nu$ or $e\nu$ or $\tau\nu \to \mu\nu\nu\nu$ or $\tau\nu \to e\nu\nu\nu$ decay of $W^-$ or $W^+$. For this channel the cross section is $[2\cdot 0.676 \cdot 2 \cdot (1+0.17)0.108 \approx 0.33]\cdot$ $\sigma(e^+e^- \to D^+D^-)$ (here 0.17 is a fraction of $\mu$ or $e$ from the decay of $\tau$).

*Typical event will have the large missing transverse energy $E_{\perp}$ carried out by neutral and stable D-particles.* The background is given by SM processes with the same kinematics and with large missed transverse energy $E_{\perp}$, carried off by neutrino(s). The value of corresponding cross section is at least by one electroweak coupling squared $g^2/4\pi$ or $g'^2/4\pi$ less than $\sigma(e^+e^- \to \mu\mu)$ with $g^2/4\pi \sim g'^2/4\pi \sim \alpha$. Therefore, cross sections for such background processes are by about two orders less than the cross section of the process under discussion. The same estimate is valid for all reactions considered below.

[1]The interactions (1.1) form complete set of non-diagonal interactions of $D$-particles. Therefore, the probability of discussed decay is independent on what is observed in the final state $W$ or $W^*$, where $W^*$ is of shell state of $W$, i. e. $q\bar{q}$ jets or $\ell\nu$ system with quantum numbers of $W$ and lower effective mass $M^*$. In the lists of reactions below we don't distinguish $W$ and $W^*$, $Z$ and $Z^*$.



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**If $M_{D^\pm}-M_D<M_W$**, the only mode for charged $D^\pm$ decays is $DW^{*+}$. The structure of the final state and the value of cross sections are the same as in the previous case with only difference – dijet effective mass $M^*$ is now not peaked around resonance value $M_W$ but it is distributed in some interval below $M_{D^\pm}-M_D$. The form of this distribution depends on the spin of $D$-particles $s_D$.

• The signature for the process in both cases is:

| | |
|---|---|
| **Two dijets one dijet plus *e* or $\mu$** with large missing energy and large a-collinearity + *nothing*, with cross section $\sim\sigma(e^+e^-\to\mu^+\mu^-)$. Typically these dijets (or dijet and lepton) move in the opposite hemispheres. | (2.4) |

## 2.2 Parameters of $D^\pm$ and $D$

We denote

$$\Delta(s;s_1,s_2)=\sqrt{s^2+s_1^2+s_2^2-2ss_1-2ss_2-2s_1s_2}. \tag{2.5}$$

*The case $M_{D^\pm}>M_{D^0}+M_W$*. In the rest frame of $D^\pm$ we deal with 2-particle decay $D^\pm\to DW^\pm$. In this frame the energy and momentum of $W^\pm$, observed as $q\bar{q}$ dijet with effective mass $M_W$, are

$$E_W^r=\frac{M_{D^\pm}^2+M_W^2-M_D^2}{2M_{D^\pm}},\qquad p^r=\frac{\Delta(M_{D^\pm}^2,M_W^2,M_D^2)}{2M_{D^\pm}}. \tag{2.6}$$

Denoting the $W$ escape angle in $D^+$ rest frame relative to the direction of $D^+$ motion in the lab system by $\theta$ and $c=\cos\theta$, we have energy of $W^+$ in the lab system

$$E_W^L=\gamma_\pm(E_W^r+c\beta_\pm p^r).$$

Therefore, energies of dijets from W's are distributed within the interval

$$E_{(-)}=\gamma_\pm(E_W^r-\beta_\pm p^r)\ \leqslant E_W^L\leqslant\ E_{(+)}=\gamma_\pm(E_W^r+\beta_\pm p^r). \tag{2.7}$$

**Masses**. *The end point values $E_{(\pm)}$ give two equations for evaluation of masses $D^\pm$ and $D^0$.* In particular, $E_{(-)}E_{(+)}=\gamma_\pm^2M_W^2+(p^r)^2$ and $E_{(-)}+E_{(+)}=2\gamma_\pm E_W^r$. Therefore at large enough electron energy (at $\gamma\gg1$), $M_{D^\pm}^2\approx E_e^2M_W^2/[E_{(-)}E_{(+)}]$, $M_D^2=M_W^2+\dfrac{E_e-E_{(-)}-E_{(+)}}{E_e}M_{D^\pm}^2$. At finite $\gamma$ the exact equations are more complex.

The accuracy of this procedure is determined by the accuracy of measurement of dijet energy together with its effective mass and by a width of $D^\pm$ (if the latter is large). In particular, at $s_D=0$ the decay $D^\pm\to DW^\pm$ width is

$$\Gamma=\frac{\alpha}{2\sin^2\theta_W}\cdot\frac{(p^r)^3}{M_W^2}. \tag{2.8}$$

The $\Gamma/M_{D^\pm}$ ratio is below 0.1 at $M_{D^\pm}\le500$ GeV.

The distribution of these dijets in energy is uniform, $dN(E)\propto dE$ since there is no correlation between escape angle of $W$ in the rest frame of $D^\pm$ and production angle of $D^\pm$. When the width of $D^\pm$ is not small, this distribution become non-uniform near the end points. The measuring of fine structure of this distribution near the end point will give, at least roughly, the total $D^\pm$ width.

*If $M_{D^\pm}>M_{D^0}+M_W$*, the only decay channel is $D^+\to DW^{*+}$. All discussed above results are valid *for each separate value of dijet effective mass $M^*$*, with evident change in all equations



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$M_W \to M^*$. The energy and $M^*$ distributions for each pair of dijets are independent from each other. The masses of $M_{D^\pm}$ and $M_D$ are evaluated in this case for each measured value of $M^*$ even with the best accuracy than in the previous case since in this case the proper width of $D^\pm$ is low enough.

**Spin**. After evaluation of $M_{D^\pm}$, the cross section of $e^+e^- \to D^+D^-$ process is calculated precisely for each $s_D$. The cross section for $s_D = 0$ (2.3s) is more than four times less than that for $s_D = 1/2$ (2.3f). This big difference allows to make a definite conclusion about spin of $D$-particles.

**Other properties**. At $s_D = 1/2$ the spins of $D^+$ and $D^-$ are correlated with longitudinal polarization of colliding electrons. In each event $e^+e^- \to D^+D^- \to DDjj\ell + \nu$'s we know the sign of dijet charge $W = q\bar{q}$ . It allows to study the charge and polarization asymmetries for accessing of more detail properties of $D$-particles (e.g. ratio of $D^+D^-\gamma$ to $D^+D^-Z$ couplings).

## 3. Three types of $D$-particles, $D$, $D^\pm$ and $D^A$

In the IDM together with $D$ and $D^\pm$ the one more neutral scalar particle $D^A$ exists (with mass $M_{D^A} > M_D$) [3]. Here CP parities of $D$ or $D^A$ cannot be defined separately since they do not interact with fermions, but their relative parity is fixed, they have opposite CP-parities. Complete set of interactions in this case is given in (1.1). Similar particle can also exist in some models with $s_D = 1/2$. So, we discuss now the case when additional neutral $D$-particle is $D^A$.

• **If $M_{D^A} < M_{D^\pm}$.**

1. The lowest energy threshold for $D$-particle production has the process (1.2b) with decay $D^A \to DZ$

$$e^+e^- \to DD^A \to DDZ. \tag{3.1}$$

Instead of (2.2), $D^A$ energy, $\gamma$-factor and velocity are

$$E_A = \frac{4E_e^2 + M_{D^A}^2 - M_D^2}{4E_e}, \quad \gamma_A = \frac{E_A}{M_{D^A}}, \quad \beta_A = \frac{\Delta(4E_e^2, M_{D^A}^2, M_D^2)}{4E_e E_A}. \tag{3.2}$$

In the IDM (at $s_D = 0$) the cross section is of the same order of value as $\sigma(e^+e^- \to \mu^+\mu^-)$:

$$\sigma(e^+e^- \to DD^A) = \frac{\pi\alpha^2 s R_Z^{(0)}}{3(s - M_Z^2)^2}\beta_A^3 \frac{E_A}{2E_e - E_A}. \tag{3.3}$$

The signature of this process is similar to that given by eq. (2.4):

| |
|---|
| One $q\bar{q}$ dijet or $e^+e^-$ or $\mu^+\mu^-$ pair with identical effective mass and energy distributions + *nothing* and with large missing $E_\perp$. |

(3.4)

If $M_{D^A} - M_D > M_Z$, the observable final state is $Z$, which is seen as hadronic dijet or $e^+e^-$ or $\mu^+\mu^-$ with effective mass equal to $M_Z$. Energy distribution of this $Z$ is given by equations similar to (2.6),(2.7). End points of this distribution allow to evaluate masses $M_D$ and $M_{D^A}$.

If $M_{D^A} - M_D < M_Z$, the observable final state is $Z^*$, which is seen as hadronic dijet or $e^+e^-$ or $\mu^+\mu^-$ with identical spectra of effective mass. For each value of this effective mass $M^*$, the energy distribution of this $Z^*$ is given by equations similar to (2.6),(2.7). End points of this distribution allow to evaluate masses $M_D$ and $M_{D^A}$.



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2. After the study of process (3.1) one must to study process (1.2a) and cascade reactions

$$e^+e^- \to D^+D^- \to \begin{cases} DDW^+W^-, & (a)\\ D^AW^\pm DW^\mp \to DDW^+W^-Z, & (b)\\ D^AW^+D^AW^- \to DDW^+W^-ZZ. & (c) \end{cases} \tag{3.5}$$

The decay $D^\pm \to D^AW^\pm$ is described by the same equation as the decay $D^\pm \to DW^\pm$. Its probability is lower than that for decay $D^\pm \to DW^\pm$ due to smaller final phase space volume. Therefore, $\sigma(a) > \sigma(b) > \sigma(c)$.

The signature of the process (3.5(a)), just as the process (3.5(b), (c)) for invisible decays of $Z$, is given by eq. (2.4). Each decay $D^\pm \to DW^\pm$ and $D^\pm \to D^AW^\pm$ is described by identical equations, the only new point is that the end points $E_{(-)}$ and $E_{(+)}$ for the energy distribution of $W$'s from decay $D^\pm \to DW^\pm$ are given by equations (2.7) while the end points $E^A_{(-)}$ and $E^A_{(+)}$ for energy distribution of $W$'s from decay $D^\pm \to D^AW^\pm$ are given by the same equations but with the change $M_D$ to $M_{D^A}$. It is evident that $E_{(-)} < E^A_{(-)} < E^A_{(+)} < E_{(+)}$. Therefore in this case the same procedure as in sect. 2 allows to obtain masses $M_{D^\pm}$ and $M_D$ (cross check for measuring of $M_D$).

To evaluate $M_{D^A}$ from this reaction let us remind that for each type of decay the energy distribution of $W$ in the lab system is uniform. The energy distribution of $W$ in the lab system is the sum of two uniform distributions with the described above end points. The end points $E^A_{(-)}$, $E^A_{(+)}$ are marked by the steps in the density of event energy distribution. They can be used for new evaluation of $M_{D^A}$.

The distribution of dijets in the effective mass can be different. If all masses are peaked around $M_W$, it means that $M_{D^\pm} - M_D > M_{D^\pm} - M_{D^A} > M_W$. If there are dijets with effective mass $M_W$ and those with lower effective mass, the former appear from $D^\pm \to DW^\pm$ decay and the latter – from $D^\pm \to D^AW^\pm$ decay. If the effective masses of all dijets are below $M_W$, we have $M_W > M_{D^\pm} - M_D > M_{D^\pm} - M_{D^A}$. In this case the above mentioned steps in the dijet energy distribution at each $M^*$ will be added to steps in the distribution in $M^*$.

The signature of processes (3.5(b)) and (3.5(c)) for visible decays of $Z$ is similar to (2.4) with adding of dijet or $\ell^+\ell^-$ pairs which represent $Z$ or $Z^*$. If one can distinguish dijets from $Z$ and those from $W$, the energy distribution of $W$ in these processes can be used to enhance data massive for evaluation of masses, discussed above.

• **If $M_{D^A} > M_{D^\pm}$, $M_{D^A} + M_D < 2E_e$**, the analysis of sect. 2 is valid completely for the final states with signature (2.4) – reaction $e^+e^- \to D^+D^-$.

The second series of processes is (1.2b) with two channels of $D^A$ decay and different signatures

$$e^+e^- \to DD^A \to \begin{cases} DDZ, & (a)\\ DD^\pm W^\mp \to DDW^+W^-. & (b) \end{cases} \tag{3.6}$$

The process (3.6(a)) is the process (3.1). It can be analyzed just as it was discussed earlier. The process (3.6(b)) is a cascade process. It can be eliminated from mass analysis of process (2.1) using the fact that in difference with the process (2.1) the observable decay products of this process move typically in one hemisphere.

Note that in the case when $M_{D^A} + M_D < 2M_{D^\pm}$ the process (3.6) has lower energy threshold than (1.2a). The operations in suitable energy interval allows to find masses $M_D$ and $M_{D^A}$ but meet



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difficulties in evaluation of $M_{D^\pm}$. This problem can be solved by the increasing of beam energy for observation of process (1.2a).

## 4. Summary

We present the simple and robust method for discovery candidates for DM particles and evaluation of their masses and spins at ILC/CLIC. The same analysis can be applied to the case when the set of $D$-particles with $M < E_e$ contains one additional neutral particle $D_1$ with the same CP.

These particles will be discovered via observation of processes with signature (2.4), (3.4) and with cross section of the order of $\sigma(e^+e^- \to \mu^+\mu^-)$, which is huge for LC. The masses of these particles will be obtained via measuring the end points of the energy distribution of dijets (representing $W^\pm$ or $Z$). The cross section measurements of processes with signature (2.4), (3.4) and similar signature for the derivative processes with cascade decay allow to determine the spin $s_D$ of considered candidate for DM particle by comparison with simple SM calculation (the cross sections for $s_D = 1/2$ is approximately 4 times larger than that for $s_D = 0$).

The process (1.2a) was considered earlier in respect of discovery of neutralino as DMP, etc. (see e.g. [5]). However, I never saw such approach for simultaneous evaluation of masses and spins of $D$-particles irrespective to details of model.

The advantages of presented approach are following:

*1. The cross section of each suggested process is a substantial part of the total cross section of $e^+e^-$ annihilation at considered energy (typically up to tens percents).*

*2. The signature is clear, the background is very small (typically, $\sim 1$ % from the observable effect).*

*3. Simple kinematics allows to extract reliably quantities under interest from the data.*

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