

Neutral mesons' mixings and rare decays in the framework of MSSM

Mikhail Dubinin

DTHEP, Skobeltsyn Institute of Nuclear Physics, MSU, Moscow

E-mail: dubinin@theory.sinp.msu.ru

Alexey Sukachev*

DEPNI, Skobeltsyn Institute of Nuclear Physics, MSU, Moscow

E-mail: salex-82@yandex.ru

Extensions of the standard model (*SM* hereinafter) is a natural way to resolve some issues, which cannot be fully dealt with in the theoretical framework of the conventional high energy physics. In these proceedings the minimal supersymmetric standard model (*MSSM*) is considered through its applications to mixing processes and rare decays in neutral mesons' systems with a special emphasis put on the former.

Selected results of symbolical calculations and their numerical simulations for a *CP* violating (non-direct) parameter ε and a mass splitting value Δm_{LS} in K^0 -, $B_{d,s}^0$ - and D^0 -meson systems, due to charged Higgs and chargino exchanges, are presented. The aforementioned results are shown to constrain MSSM scenario-defining parameters (m_{H^\pm} and $\text{tg}\beta$) helping to exclude certain regions at the MSSM plane of parameters (namely for low values of $\text{tg}\beta < 4$ and for high values of $\text{tg}\beta > 35$ with light charged scalars $m_{H^\pm} < 150\text{GeV}$). Prospects of future calculations for rare decays are mentioned as well.

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*Speaker.

1. Introduction

The standard model, which is verified perfectly by a great variety of precision experimental data, is probably the finest creation of a human's mind, at least in high energy physics. However, as in the case of every profound physical theory, SM has its own constraints and limitations as well as some problems it struggles to resolve and some subtle effects which are yet to be understood fully (if ever) in terms of the conventional approach. Those problems include (but aren't limited to): the Higgs sector, where the fundamental scalar boson is yet to be discovered experimentally; an exact number of fermion generations; an excess of arbitrary parameters of the model under consideration; a mass hierarchy problem; possible extra dimensions etc.

Aforementioned problems is a reason, why various SM extensions are presently being thoroughly reviewed and investigated. MSSM [1] is a natural way to understand some of these problems while its predictions at a considerable span of defining parameters don't contradict to any experimental data. There are also such physical systems and such phenomena, which can hardly be described by a common version of the MSSM and thus require some much needed constraints over MSSM parameter space.

One bunch of such systems — neutral K^0 -, $B_{d,s}^0$ - and D^0 -mesons — is being considered in this work. Two well-known physical quantities: a mass splitting Δm_{LS}^i and a measure of the (indirect) CP violation ε — were carefully re-evaluated in the framework of a specific MSSM scenario with an emphasis on charged Higgs contributions. The purpose of the work was to get constraints on the MSSM parameter plane ($\tan\beta$, m_{H^\pm}) by comparing theoretical results with experimental values of evaluated physical quantities.

A scene of the work is set in this short **Introduction**. The Model itself is briefly described in the **second paragraph**. The **third paragraph** contains the most important results of symbolical calculations as well as a tiny mention of various QCD corrections. Numerical results are presented (in pictorial form) in the **fourth part** of the article. Prospects and conclusions of the work are being summed up in the **fifth paragraph**.

2. The Model

1. The MSSM of the basic type has been used thus providing us with two scalar doublets (we shall refer to this fact as “THDM”). The Yukawa sector, featured in the model under consideration, is of the second type [2]:

$$-L_Y^H = g_{ij}^{u1} \bar{Q}'_{iL} \tilde{\Phi}_1 u'_{jR} + g_{ij}^{d2} \bar{Q}'_{iL} \Phi_2 d'_{jR} + \text{lept. sec.} + \text{h.c.}, \quad (2.1)$$

which means Φ_1 scalar field generates upper quark masses while scalar field Φ_2 does the same for down-type quarks.

2. As mentioned above, the Higgs sector contains two scalar doublets [3]. The MSSM effective potential of the most common type [4, 5] was used at m_t energy scale:

$$U(\Phi_1, \Phi_2) = -\mu_1^2 (\Phi_1^\dagger \Phi_1) - \mu_2^2 (\Phi_2^\dagger \Phi_2) - \mu_{12}^2 (\Phi_1^\dagger \Phi_2) - \mu_{12}^{*2} (\Phi_2^\dagger \Phi_1) + \quad (2.2)$$

$$+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) +$$

$$\begin{aligned}
& + \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \frac{\lambda_5^*}{2} (\Phi_2^\dagger \Phi_1) (\Phi_2^\dagger \Phi_1) + \\
& + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_6^* (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \lambda_7^* (\Phi_2^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1).
\end{aligned}$$

Parameters μ_{12} , λ_5 , λ_6 and λ_7 are importantly complex after the SUSY breaking. They are all equal to zero above the SUSY breaking scale, which we set at $M_{SUSY} = 500 \text{ GeV}$, but obtain imaginary contributions from radiative corrections due to “triangle” and “box” one-loop diagrams with an interaction between Higgs bosons and squarks of the third generation.

3. After the diagonalization we obtain a mass spectrum of scalar fields in the model [5, 6]. By using the THDM we shall have three neutral and two charged scalar fields in addition to three goldstone particles to be eaten by W^\pm - and Z^0 -bosons. In case of an explicit CP violation with a pre-set relationship between parameters, governing radiative corrections ($\mu = 2 A_{t,b} = 4 M_{SUSY}$ — corresponds to a so-called ‘ CPX ’-scenario [7]), neutral scalar modes mix into three non- CP eigenstates h_1, h_2, h_3 , so that a VEV’s ratio $\text{tg}\beta$ and a mass of the charged Higgs become MSSM defining parameters at the phenomenological level. Meanwhile, the latter quantity proves to be different from MSSM models with the CP preservation [6]:

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v^2}{2} (\text{Re}\Delta\lambda_5 - \Delta\lambda_4), \quad (2.3)$$

where m_A is the mass of pseudoscalar in CP -preservation limit (radiative corrections $\Delta\lambda_{4,5}$ can be seen in [6] as well), thus allowing lighter scalar fields in a wider $\text{tg}\beta$ range than is usually expected.

3. Symbolical Calculations

We shall mainly concentrate on a mass splitting value for K^0 - and $B_{d,s}^0$ -meson systems. The common formula can be presented the following way:

$$\Delta m_{LS} = \Delta m_{LS}^{LD} + \Delta m_{LS}^{SD} = \Delta m_{LS}^{LD} + B \cdot \Delta m_{LS}^{SD'}, \quad (3.1)$$

where the first summand corresponds to non-perturbative long-distance contributions, while the second one is related to short-distance contributions from various SM, charged Higgs and chargino “box” diagrams (see fig.1), including a set (η_i) of perturbative QCD corrections from hard gluon exchanges. B denotes non-perturbative QCD corrections via different intermediate hadron states at low energies.

3.1 Standard model

In a framework of the SM, a (“short-distance”) mass splitting in K -mesons is defined by the following well-known formula (by virtue of the GIM-mechanism — fig.1-1):

$$\begin{aligned}
\Delta m_{LS}^{SD-WW} &= \frac{G_F^2 f_K^2 m_K B_K}{6\pi^2} \text{Re} \left[(V_{cd}^*)^2 V_{cs}^2 m_c^2 \eta_1 I(\xi_1) + \right. \\
& \left. + (V_{td}^*)^2 V_{ts}^2 m_t^2 \eta_2 I(\xi_2) + 2 V_{td}^* V_{cd}^* V_{ts} V_{cs} \eta_3 m_c m_t I(\xi_2, \xi_3) \right], \quad (3.2)
\end{aligned}$$

where $I(\xi_i, \xi_j)$ and $I(\xi_i)$ is a couple of well-known Inami-Lim-Vysotski (ILV) functions [8, 9] and $\xi_1 = (\frac{m_c}{m_W})^2$, $\xi_2 = (\frac{m_t}{m_W})^2$, $\xi_3 = (\frac{m_t}{m_c})^2$.

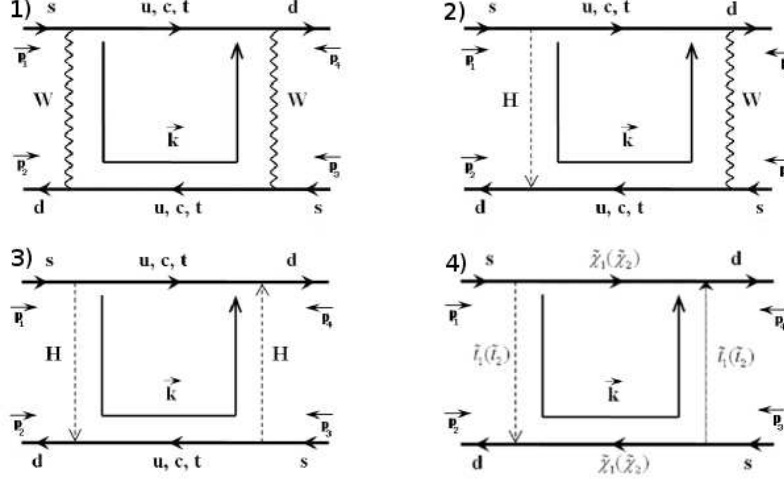


Figure 1: $K^0-\bar{K}^0$ vacuum transitions in the model under consideration. 1). WW -box of the SM; 2).-3). MSSM contributions due to charged Higgs exchanges; 4). MSSM contributions due to chargino ($\tilde{\chi}_{1,2}$) and 3rd-generation squark $\tilde{t}_{1,2}$ exchanges.

3.2 Charged Higgs exchanges

By using the defined Model (sec. 2) we obtain additional contributions to a mass splitting value from diagrams with charged Higgs (H^\pm — fig.1-2 and fig.1-3) and charginos ($\chi_{1,2}$). The corresponding results (see [10] for detailed calculations) are presented below ('t-Hooft-Feynman gauge has been used resulting in contributions from non-physical scalars G):

$$\Delta m_{LS}^{SD-HH} = \frac{C_H^2 f_K^2 m_K B_K}{384\pi^2 m_W^4} \cdot \left[\frac{\text{tg}^4 \beta m_s^2 m_d^2}{4 \cdot m_H^2} D_1(J_{11}^{HH}, J_{12}^{HH}) - \frac{m_s m_d}{2} \cdot D_2(J_{21,22}^{HH}) + \frac{m_H^2}{4 \cdot \text{tg}^4 \beta} \cdot D_3(J_{31}^{HH}, J_{32}^{HH}) + \frac{5}{8} \cdot \frac{B_K^S}{B_K} \cdot m_s^2 \cdot D_4(J_{41}^{HH}, J_{42}^{HH}) \right], \quad (3.3)$$

$$\Delta m_{LS}^{SD-HW} = \frac{G_F C_H f_K^2 m_K B_K m_H^2}{24\pi^2 \cdot m_W^3} \left[\frac{m_W}{2 \cdot \text{tg}^2 \beta} E_1(J_{1j}^{HW}) - \frac{\text{tg}^2 \beta m_s m_d}{m_W} E_2(J_{2j}^{HW}) \right], \quad (3.4)$$

$$\Delta m_{LS}^{SD-HG} = \frac{G_F C_H f_K^2 m_K B_K m_H^2}{96\pi^2 m_W^8} \left[m_d^2 m_s^2 \text{tg}^2 \beta \cdot F_1(J_{11}^{HG}, J_{12}^{HG}) + m_s m_d m_W^2 \cdot F_2(J_{21}^{HG}, J_{22}^{HG}) + \frac{m_W^4}{2 \cdot \text{tg}^2 \beta} \cdot F_3(J_{31}^{HG}, J_{32}^{HG}) - \frac{5}{4} \cdot \frac{B_K^S}{B_K} \cdot m_W^2 \cdot \left(m_s^2 + m_d^2 - \frac{m_s m_d}{\text{tg}^2 \beta} - m_s m_d \text{tg}^2 \beta \right) \cdot F_4(J_{41}^{HG}, J_{42}^{HG}) \right], \quad (3.5)$$

Here we have:

$$D_i(J_{ij}^{HH}) = \text{Re}[(V_{cd}^* V_{cs})^2 m_c^4 \eta_7 J_{i1}^{HH}(\xi_4) + (V_{td}^* V_{ts})^2 m_t^4 \eta_8 J_{i1}^{HH}(\xi_5) +$$

$$\begin{aligned}
& + 2 V_{td}^* V_{cd}^* V_{ts} V_{cs} m_c^2 m_t^2 \eta_9 J_{i2}^{HH}(\xi_3, \xi_4, \xi_5)] \quad (i = 1, 2, 3, 4; j = 1, 2), \\
E_i(J_{ij}^{HW}) & = \text{Re}[(V_{cd}^* V_{cs})^2 m_c^2 \eta_4 J_{i1}^{HW}(\xi_1, \xi_4, \xi_6) + (V_{td}^* V_{ts})^2 m_t^2 \eta_5 J_{i1}^{HW}(\xi_2, \xi_5, \xi_6) + \\
& + 2 V_{td}^* V_{cd}^* V_{ts} V_{cs} m_c m_t \eta_6 J_{i2}^{HW}(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)] \quad (i = 1, 2; j = 1, 2), \\
F_i(J_{ij}^{HG}) & = \text{Re}[(V_{cd}^* V_{cs})^2 m_c^4 \eta_4 J_{i1}^{HW}(\xi_1, \xi_4, \xi_6) + (V_{td}^* V_{ts})^2 m_t^4 \eta_5 J_{i1}^{HW}(\xi_2, \xi_5, \xi_6) + \\
& + 2 V_{td}^* V_{cd}^* V_{ts} V_{cs} m_c^2 m_t^2 \eta_6 J_{i2}^{HW}(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)] \quad (i = 1, 2, 3, 4; j = 1, 2).
\end{aligned}$$

A set of $J_{ij}^{Hk}(\xi_l)$ functions is fully similar to ILV functions of the standard model (as is the case of ξ_i variables). Their exact expressions are rather complicated; they are listed in [10]. $C_H = \frac{4\sqrt{2}m_W^2}{v^2 m_{H^\pm}^2}$ is an analogue of Fermi constant for Yukawa interaction.

3.3 Chargino — squark exchanges

Contributions from the SM superpartner exchanges, represented in fig.1-1, can be evaluated in a similar way. Retaining only the leading order impacts (details can be found in [10]) we obtain:

$$\Delta m_{LS}^{SD-\tilde{\chi}i} = \frac{m_t^4 f_K^2 m_K B_K \cdot |Z_{p22}|^4}{48\pi^2 v^4 \cdot m_{\tilde{\chi}_1}^2 \cdot \sin^4 \beta} \cdot \text{Re}[(V_{td}^* V_{ts})^2 \cdot Q], \quad (3.6)$$

where

$$Q = (Z_{u63}^* Z_{u33})^2 \cdot J_{\tilde{t}_1, \tilde{t}_1}^{11} + (Z_{u66}^* Z_{u36})^2 \cdot J_{\tilde{t}_2, \tilde{t}_2}^{11} + 2 \cdot Z_{u63}^* Z_{u33} Z_{u66}^* Z_{u36} \cdot J_{\tilde{t}_1, \tilde{t}_2}^{12}.$$

Here $Z_{p22} = 0.98$ is a matrix element of chargino mass matrix and $Z_{u_{ij}}$ are matrix elements of squark mixing matrix. J_{kl}^{11} and J_{kl}^{12} are identical to corresponding $J_{ij}^{Hk}(\xi_l)$ functions, mentioned above.

3.4 QCD corrections and a measure of CP Violation

The results, listed above, can be easily adjusted to $B_{d,s}^0$ -meson systems by replacing corresponding indices and CKM-matrix elements in aforementioned expressions with their B -analogues. Expressions for D^0 mixings are harder to esteem but a four-fermion approximation works well in this case helping us to simplify the formulae. See [10] for details.

Perturbative QCD corrections η_i due to hard gluon exchanges were previously evaluated in [11, 12], while non-perturbative corrections were re-evaluated in a manner of [13, 14] with a use of finite energy sum rules formalism thus giving us values for B^{K, B_s, B_d} and B_K . Finally, long-distance contributions Δm^{LD} can be extracted from experimental data with a use of normalization procedure.

A following expression was also applied for evaluation of a non-direct CP violation value ϵ via amplitudes of various SM and MSSM contributions:

$$|\epsilon^{tot}| = \frac{1}{2\sqrt{2}} \frac{M_{LS}^{WW} + \sum_{i=1}^{i=2} M_{LS}^{HWi} + \sum_{j=1}^{j=7} M_{LS}^{HGj} + \sum_{k=1}^{k=4} M_{LS}^{HHk}}{N_{LS}^{WW} + \sum_{i=1}^{i=2} N_{LS}^{HWi} + \sum_{j=1}^{j=7} N_{LS}^{HGj} + \sum_{k=1}^{k=4} N_{LS}^{HHk}}. \quad (3.7)$$

It's to be noted, that the amplitudes of box diagrams, listed above, are also the amplitudes for different rare meson decays induced partially by the same type of Feynman graphs.

4. Numerical results

Various tables and graphs are thoroughly presented in [10] and in other articles of the same authors, which are listed in [10] bibliography. As a sample of obtained constraints we can address fig.2, which selects certain regions of $(\text{tg}\beta, m_{H^\pm})$ MSSM plane, where deviations from the experimental value of $|\varepsilon_K|$ exceed $1, 3, 5\sigma$ respectively, thus excluding (at different CL) low values of $\text{tg}\beta$ for almost all possible m_{H^\pm} , higher levels of $\text{tg}\beta$ for light charged scalars, and a certain region of big $\text{tg}\beta$ (from B_s data) also for the lightest charged Higgs bosons.

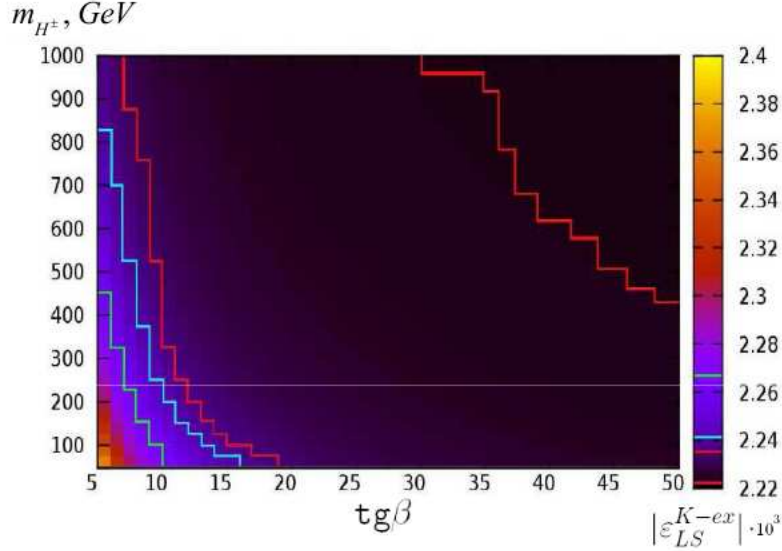


Figure 2: Non-direct CP violation parameter $|\varepsilon_K|$, defined for neutral K^0 -mesons, as a function of the charged Higgs mass m_{H^\pm} and $\text{tg}\beta$. Coloured lines correspond to a deviation of $|\varepsilon_K^{\text{theory}}|$ from $|\varepsilon_K^{\text{exp}}|$: red — 1σ , blue — 3σ , and green — 5σ — if precision of measurements is raised by an order of magnitude.

A specific statistical method [15], developed by S. Bitukov and N. Krasnikov, to assess a distinguishability between the new (MSSM) physics and the SM was tested in this work. Two statistical estimators have been implemented — the distinguishability κ and the confidence level ζ (not directly related to the CL). The following constraints on the $(\text{tg}\beta, m_{H^\pm})$ MSSM plane were obtained:

1. “Mild exclusion” ($\zeta > 1.05\sigma$ and $\kappa > 86\%$). The following regions are excluded with these values of estimators: 1). $\text{tg}\beta < 5$ (for all m_{H^\pm} at considered range); 2). $5 < \text{tg}\beta < 10$ for $m_{H^\pm} < 325$ GeV; 3). $\text{tg}\beta > 30$ for $m_{H^\pm} < 175$ GeV.
2. “Normal exclusion” ($\zeta > 2\sigma$ and $\kappa > 97.5\%$). The following regions are excluded with these values of estimators: 1). $\text{tg}\beta < 2$ (for all m_{H^\pm}); 2). $\text{tg}\beta > 42$ for $m_{H^\pm} < 125$ GeV.

5. Conclusions and prospects

5.1 Conclusions

1. It’s shown that HW -, HH - and HG - contributions are tiny in comparison with those of the SM in the largest region of the MSSM parameter plane as they also decrease with an increase of $\text{tg}\beta$ and m_{H^\pm} for K -mesons and B -mesons as well.

2. An entire set of Inami-Lim-Vysotski analogue functions is obtained. ILV analogues are shown to have common limits with the results of four-fermion low-energy approximation.
3. An estimation of possible constraints of the MSSM parameters space has been performed with the use of Bityukov-Krasnikov statistical approach. It's shown for the first time (based on B_s^0 -meson studies) that considerable deviations from the SM do exist in the region of large $\text{tg}\beta > 35$ and low values of the charged Higgs mass $m_{H^\pm} < 150$ GeV.

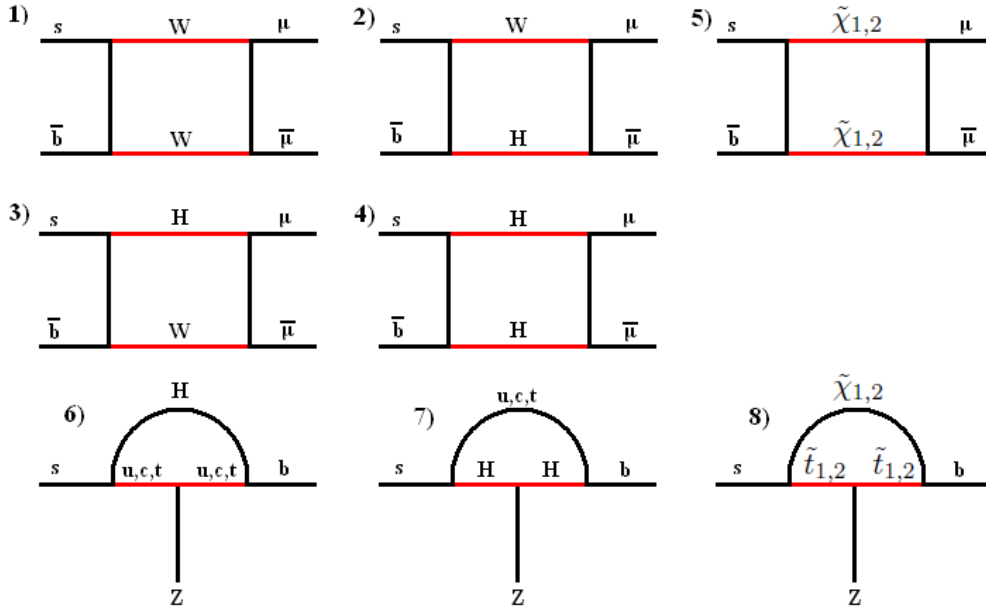


Figure 3: Various contributions of charged Higgs and chargino exchanges to the width of the $B_s^0 \rightarrow \mu\bar{\mu}$ rare decay via “box” (1–5) and “penguin” (6–8) diagrams in a framework of the MSSM. H denotes a charged Higgs, $\tilde{\chi}_{1,2}$ go for charginoes, while $\tilde{t}_{1,2}$ are squarks of the third generation.

5.2 Future prospects

The following questions are in progress or going to be addressed in subsequent works of these authors:

1. Mass splitting and non-direct CP -violation effects in neutral mesons due to chargino–stop exchanges can be large on the outskirts of the MSSM parameter plane.
2. Evaluation of penguin and box diagrams with charged Higgs and charginos for direct CP -violation quantities and asymmetries.
3. Box and penguin diagrams for rare decays in B -, K - and D -meson systems with scalar bosons and superpartners (look for some samples in fig.3).
4. Finite temperature effects and corresponding constraints for the MSSM parameter space.

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