PROCEEDINGS OF SCIENCE

Parity violation in QCD motivated models at extreme conditions

Vladimir Andrianov*
St.Petersburg State University
E-mail: v.andriano@rambler.ru

Alexander A. Andrianov[a,b] **and Domenec Espriu**[b]
[a] *St.Petersburg State University, Russia*
[b] *Departament d'ECM and ICCUB-Institut de Ciencies del Cosmos, University of Barcelona, Spain*
E-mail: andrianov@icc.ub.edu, espriu@ecm.ub.edu

The possibility of parity breaking in QCD motivated hadron models at finite baryon density and temperature is investigated. QCD is approximated by a generalized hadron σ model with two multiplets of scalars and pseudoscalars fields. The mechanism of parity breaking is based on interplay between condensates of lightest and heavier fields. We show that in nuclear matter of a few normal densities and at moderate temperatures, parity breaking may be the rule rather than an exception and its occurrence is well compatible with the existence of a stable bound state of normal nuclear matter.

The XIXth International Workshop on High Energy Physics and Quantum Field Theory, QFTHEP2010
September 08-15, 2010
Golitsyno, Moscow, Russia

*Speaker.

1. Introduction

The possibility of parity (P)- breaking via pseudoscalar condensation for sufficiently large values of temperature and/or chemical potential has been attracting much interest during last decade to search it both in dense nuclear matter (in neutron/quark stars and heavy ion collisions at intermediate energies) and in strongly interacting quark-gluon matter ("quark-gluon plasma" in heavy ion collisions at very high energies). We note that at finite baryon density pion condensation was conjectured by A. Migdal in [1] long ago . Recently signatures of local $(C)P$-parity violation have been analyzed in hot nuclear matter [2]. Finally P violation might conceivably accompany the transitions to open color phases[3] , but they are beyond the scope of our talk .

Parity breaking in QCD would lead to rather remarkable experimental signals [4] such as the same in-medium resonance being able to decay into even and odd number of pions, the presence of additional Goldstone bosons, changes in the nuclear equation of state, isospin breaking effects in the pion decay constant and substantial modification of the weak decay constant $F_{\pi'}$ for massless charged pions, giving an enhancement of electroweak decays. A most striking effect of parity breaking could be the abnormal enhancement of dilepton production due to triggering of photon and vector meson decays (see the talk by A.Andrianov and [5]).

In our talk we examine the interesting possibility of spontaneous parity breaking using effective Lagrangian techniques in the range of nuclear densities where the hadron phase persists and quark percolation does not occur yet. Our effective Lagrangian is a realization of the generalized linear σ model, including the two lowest lying resonances in each channel and this is the minimal model where possibility parity breaking can be realized. We show that the parity breaking phase persists in some finite domain in the $\mu - T$ plane.

Previously several approaches have been used to study QCD in extreme conditions: from meson-nucleon [1, 6] or quark-meson [8] Lagrangians to models of Nambu-Jona-Lasinio type [9, 10]. However, for different reasons, the above mentioned hadronic models lack some essential ingredient, namely, an additional degree of freedom responsible for a P-breaking effect.

The range of intermediate nuclear densities (from 3 to 10 times the usual nuclear density) where we expect parity breaking to occur is of high interest as it may be reached both in heavy-ion collisions [11] and compact stars [8].

2. A generalized sigma model for QCD

The conventional linear σ-model[13] contains a multiplet of the lightest isoscalar σ and isotriplet pseudoscalar π^a fields. Spontaneous chiral symmetry breaking emerges due to a non-zero value for $\langle\sigma\rangle \sim \langle\bar{q}q\rangle/F_\pi^2$. In order to relate this model to QCD one has to choose a real condensate for the scalar density, with its sign opposite to current quark masses. Adding a chemical potential does not rotate the condensate in chiral space and does not trigger P breaking.

Thus too simple phenomenological models retaining only the lightest degrees of freedom are not capable to explore all the different phases that the presence of manifest $(C)P$ violation due to the non-zero chemical potential opens.

The minimal generalization of σ model for QCD to explore the possibility of spontaneous parity breaking (SPB) contains two multiplets of scalar/pseudoscalar fields $H_j = \tilde{\sigma}_j\mathbf{I} + i\hat{\pi}_j, \quad j =$

$1,2$, with $\hat{\pi}_j \equiv \tilde{\pi}_j^a \tau^a$ where τ^a are Pauli matrices. We require an exact $SU(2)_L \times SU(2)_R$ symmetry in the chiral limit. These two chiral multiplets represent the two lowest-lying radial states. Thus the effective potential of this generalized σ model,

$$\begin{aligned} V_{\rm eff} = \frac{1}{2}\text{local} \Bigg\{ &- \sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \\ &+ \frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \\ &+ \frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \Bigg\} + \mathscr{O}(\frac{|H|^6}{\Lambda^2}) \end{aligned} \tag{2.1}$$

contains 9 leading vertexes of dimension ≤ 4 with real constants [4]. The higher-order terms are suppressed by inverse powers of the chiral symmetry breaking (CSB) scale $\Lambda \simeq 1.2$ GeV.

Using the global invariance of the model we parameterize,

$$H_1(x) = \sigma_1(x)\xi^2(x) = \sigma_1(x) \exp\left(i \frac{\pi_1^a \tau_a}{F_0} \right); \quad H_2(x) = \xi(x)\Big(\sigma_2(x) + i\hat{\pi}_2(x)\Big)\xi(x). \tag{2.2}$$

The parities of $\sigma_2(x)$ and $\hat{\pi}_2$ are even and odd, respectively (in the absence of SPB). In these variables the corresponding gap equations are,

$$\begin{aligned} 2\Delta\sigma_1 &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 + \rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\ 2\Delta\sigma_2 &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 + \rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \end{aligned} \tag{2.3}$$

$$0 = \rho\Big(-\Delta + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2 \Big), \tag{2.4}$$

where the following notation has been introduced in anticipation of neutral pseudoscalar condensate: $\langle \pi_1^a \rangle = \langle \pi^0 \rangle \delta^{0a}, \langle \pi_2^a \rangle = \rho\,\delta^{0a}$.

The above effective potential must exhibit the usual chiral symmetry breaking pattern at $\mu = T = 0$. For this to happen $\langle \sigma_1 \rangle$ must acquire a real and positive v.e.v. .

The set of gap equations may have several solutions for σ_1 and σ_2, but since we know that in normal conditions QCD does not break parity, ρ must vanish. For the potential to be well defined one takes $\lambda_2 > 0$ and a *sufficient* (and necessary [4]) condition for the absence of SPB is,

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta. \tag{2.5}$$

3. Inclusion of chemical potential and temperature into the model

The baryon chemical potential μ is transmitted to the meson sector (in the leading order of chiral expansion) via a local quark-meson coupling. In the large N_c limit one can neglect the temperature dependence due to meson collisions and assume that the temperature T is induced with the help of the imaginary time Matsubara formalism for Green functions - Matsubara frequencies for quarks $\omega_n = (2n+1)\pi/\beta$ with $\beta = 1/kT$.

We take the chiral multiplet to have local couplings with the quark fields as being H_1. Thus μ and T are transmitted to the boson sector by the term,

$$\Delta\mathscr{L} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R) \longrightarrow -\bar{q}\sigma_1 q, \tag{3.1}$$

PoS(QFTHEP2010)054

where $q_{L,R}$ are constituent quarks.

After averaging over the constituent quarks, to the leading orders in chiral expansion, the first gap Eq. (2.3) is modified in the hot and dense matter to,

$$2\Delta\sigma_1 = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 + \rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathscr{N}\sigma_1\mathscr{A}(\sigma_1,\mu,\beta), \qquad \mathscr{N} \equiv \frac{N_c N_f}{4\pi^2}, \tag{3.2}$$

$$\mathscr{A}(\sigma_1,\mu,\beta) = 2\int_{\sigma_1}^{\infty} dE\,\sqrt{E^2-\sigma_1^2}\,\frac{\cosh(\beta\mu)+\exp(-\beta E)}{\cosh(\beta\mu)+\cosh(\beta E)}\,, \tag{3.3}$$

where the Fermi distribution has been introduced. All the dependence on the environment is in the function $\mathscr{A}$ which originates from the one-loop contribution to $V_{\rm eff}$.

At $T=0$, the value of the effective potential at its minima is given by the compact expression,

$$V_{\rm eff}(\mu) = -\frac{1}{2}\Delta\sum_{j=1}^{2}\Big(\sigma_j(\mu)\Big)^2 - \frac{1}{2}\Delta\rho^2(\mu) - \frac{\mathscr{N}}{3}\mu\Big(\mu^2-\sigma_1(\mu)^2\Big)^{3/2}\theta\Big(\mu-\sigma_1(\mu)\Big). \tag{3.4}$$

Thermodynamically the system is described by the pressure p and the energy density, ε. The pressure is determined by the potential density difference with and without the presence of chemical potential, $dp = -dV$,

$$p(\sigma_j(\mu),\mu) \equiv V_{\rm eff}\Big(\sigma_j^0\Big) - V_{\rm eff}\Big(\sigma_j(\mu),\rho(\mu),\mu\Big), \tag{3.5}$$

where the dependence of σ_j and $\rho(\mu)$ on μ has been shown explicitly and $\sigma_j^0 \equiv \sigma_j(0)$. The energy density is related to the pressure by $\varepsilon = -p + N_c\mu\rho_B$. The chemical potential is defined as,$\partial_{\rho_B}\varepsilon = N_c\mu$, with the entropy and volume held fixed. Therefore $\partial_\mu p = N_c\rho_B$. The factor N_c is introduced to relate the quark and baryon chemical potentials. Thus the relation between baryon density, Fermi momenta and the chemical potential is for quark matter,

$$\rho_B = -\frac{1}{N_c}\partial_\mu V_{\rm eff} = \frac{N_f}{3\pi^2}p_F^3 = \frac{N_f}{3\pi^2}(\mu^2-\sigma_1(\mu)^2)^{3/2}. \tag{3.6}$$

This set of identities provides a functional relation between ρ_B and μ.

The pressure is an increasing function of the density and vanishes at zero density. In turn, infinite nuclear matter is stable implying zero pressure too. Therefore the phase diagram in the p,ρ_B plane must necessarily exhibit a discontinuity (first order transition at some critical value μ^*).

The stabilization of nuclear matter requires not only attractive scalar forces but also repulsive ones (vector-mediated) [14]. Conventionally, the latter ones are associated to the interactions mediated by the iso-singlet vector ω meson. Let us supplement our action with,

$$\Delta\mathscr{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_{\omega\bar{q}q}\bar{q}\gamma_\mu\omega^\mu q, \tag{3.7}$$

with a coupling constant $g_{\omega\bar{q}q} \sim \mathscr{O}(1/\sqrt{N_c})$. In bosonization of QCD, in the quark sector the time component ω_0 interplays with the chemical potential. Let us assign a constant v.e.v. for this component $g_{\omega\bar{q}q}\langle\omega_0\rangle \equiv \bar{\omega}$, $\mu \to \mu + \bar{\omega} \equiv \bar{\mu}$. Then $\bar{\mu}$ can be determined via the variation of the extended $V_{\rm eff}$,

$$\frac{\bar{\mu}-\mu}{G_\omega} = -N_c\rho_B(\mu) = -\frac{N_cN_f}{3\pi^2}(\bar{\mu}^2-\sigma_1(\bar{\mu})^2)^{3/2}. \tag{3.8}$$

A viable model of dense baryon matter must describe the phase transition to a stable bound state at the usual density of infinite nuclear matter, to the so called "saturation point".

The saturation point where nuclear matter forms is characterized by vanishing pressure. The energy crossing condition $p = 0$ at $\bar{\mu}^* < \sigma_1^0, \sigma_j^* \equiv \sigma_j(\bar{\mu}^*)$ is given by,

$$\sum_{j,k=1}^{2} \left(\sigma_j^0 \Delta_{jk} \sigma_k^0 - \sigma_j^* \Delta_{jk} \sigma_k^*\right) = \frac{N_c}{2} \bar{\mu}^* \rho_B(\mu^*) + G_\omega N_c^2 \rho_B^2(\mu^*), \tag{3.9}$$

where $\bar{\mu}^*$ is related to the physical value of μ^* by Eq.(3.8). This relation represents the condition for the existence of symmetric nuclear matter.

Taking this at face value we can derive a relation between $\mu^* \simeq 300$ MeV and $\sigma_1^* \simeq 170$ MeV: $p_F = \sqrt{(\bar{\mu}^*)^2 - (\sigma_1^*)^2} = 1.3\ \mathrm{fm}^{-1} = 250$ MeV that corresponds to $\rho_0 = 0.13\ \mathrm{fm}^{-3}$.

4. The SPB phase transition

We shall consider from now on the solution corresponding to the most stable minima for $\mu > \mu^*$ (the formation of nuclear matter at some critical value μ^*, details see in [12]).

The possibility of local SPB is controlled by the inequality (2.5). In order to approach a SPB phase transition when the chemical potential is increasing we need to diminish it [4]. Let us examine the possible existence of a region of μ where $\rho \neq 0$. Then,

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \left(\sigma_2^2 + \rho^2\right) = \Delta, \tag{4.1}$$

and,

$$\lambda_5 \sigma_1^2 + 4\lambda_4 \sigma_1 \sigma_2 + \lambda_6 \left(\sigma_2^2 + \rho^2\right) = 0, \tag{4.2}$$

where we have taken into account that $\sigma_1 \neq 0$. Together with (4.1) this completely fixes the relation between the two v.e.v.'s of the scalar fields $\sigma_{1,2}$ throughout the SPB phase independently of μ and ρ.

Let us now determine the critical value of the chemical potential, namely the value μ_{crit} where $\rho(\mu_{crit}) = 0$,

$$\lambda_6 x^2 + 4\lambda_4 x + \lambda_5 = 0, \quad x = \frac{\sigma_2}{\sigma_1}. \tag{4.3}$$

Once we find x_{crit} we can immediately calculate $\sigma_{1,2}^{\pm}$,

Then using Eq. (3.2) one derives the boundary of the P-violation phase,

$$\mathcal{NA}(\sigma_1^{\pm}, \mu, \beta) = \Delta - 2\lambda_1 (\sigma_1^{\pm})^2 - \lambda_5 \sigma_1^{\pm} \sigma_2^{\pm} - (\lambda_3 - \lambda_4)(\sigma_2^{\pm})^2. \tag{4.4}$$

Thus for any nontrivial solution $\sigma_{1,2}^{\pm}$ the P-breaking phase boundary exists. If the phenomenon of P-violation is realized for zero temperature it will take place in a domain involving lower chemical potentials but higher temperatures.

Once a condensate for π_2^0 appears spontaneously the vector $SU(2)$ symmetry is broken to $U(1)$ and two charged excited π' mesons are expected to possess zero masses. Quantitatively the mass spectrum can be obtained only after kinetic terms are normalized.

5. Conclusions

Let us summarize our main results. Parity violation seems to be quite a realistic possibility in nuclear matter at moderate densities. We have arrived at this conclusion by using an effective Lagrangian for low-energy QCD that retains the two lowest lying states in the scalar and pseudoscalar sectors. Salient characteristics of this phase would be the spontaneous violation of isospin and the generation two additional massless charged pseudoscalar mesons. We have also examined departures from the chiral limit, i.e. allowing for non-zero quark masses. This leads to rather interesting results as in this case the usual pions are not exactly massless, but the new Goldstone bosons appearing at the transition point to the parity violating phase are.

We also find a strong mixing between scalar and pseudoscalar states that translate spontaneous parity violation into meson decays. The mass eigenstates will decay both in odd and even number of pions simultaneously. Isospin violation can also be visible in decay constants.

There is a possibility of the occurrence of local parity breaking in colliding nuclei due to generation of pseudoscalar, isosinglet or neutral isotriplet, background which is discussed in the talk of A.A.Andrianov.

We acknowledge the financial support from projects FPA2007-66665, 2009SGR502, CPAN (Consolider CSD2007-00042) and FLAVIANET. A.& V. Andrianov are supported also by Grants RFBR 09-02-00073-a and 10-02-00881-a and by SPbSU grant 11.0.64.2010.

References

[1] A.B. Migdal, *Zh. Eksp. Teor. Fiz.* **61** (1971) 2210 [*Sov. Phys. JETP* **36** (1973) 1052].

[2] D. Kharzeev, R.D. Pisarski and M.H.G. Tytgat, *Phys. Rev. Lett.* **81** (1998) 512; D. Kharzeev, *Phys. Lett.* **B 633** (2006) 260; D. Kharzeev and A. Zhitnitsky, *Nucl. Phys.* **A 797** (2007) 67; D.E. Kharzeev, L.D. McLerran and H.J. Warringa, *Nucl. Phys.* **A 803** (2008) 227 .

[3] M. Alford, K. Rajagopal and F. Wilczek, *Phys. Lett.* **B 422** (1998) 247. R. Rapp, T. Schaefer, E.V. Shuryak and M. Velkovsky, *Phys. Rev. Lett.* **81** (1998) 53.

[4] A. Andrianov and D. Espriu, *Phys. Lett.* **B 663** (2008) 450.

[5] A. Andrianov, V. Andrianov, D. Espriu and X. Plannels, `arXiv: 1010.4688 [hep-ph]`.

[6] G.E. Brown, M. Rho, *Phys. Rep.* **363** (2002) 85.

[7] O. Scavenius, Á. Mócsy, I.N. Mishustin and D.H. Rischke, *Phys. Rev.* **C 64**, 045202 (2001); S. Lawley, W. Bentz and A.W. Thomas, *J. Phys.* **G 32** (2006) 667 .

[8] S. Pal, M. Hanauske, I. Zakout, H. Stoecker and W. Greiner, *Phys. Rev.* **C 60**, 015802 (1999).

[9] V. Bernard, Ulf-G. Meissner and I. Zahed, *Phys. Rev* **D 36** (1987) 819;/ T. Hatsuda and T. Kunihiro, *Phys. Rep.* **247** (1994) 221.

[10] D. Ebert and K.G. Klimenko, *J. Phys.* **G 32** (2006) 599; *Eur. Phys. J.* **C 46** (2006) 771.

[11] P. Braun-Munzinger, K. Redlich and J. Stachel, `arXiv: nucl-th/0304013`.

[12] A. Andrianov, V. Andrianov and D. Espriu, *Phys. Lett.* **B 678** (2009) 416.

[13] M. Gell-Mann and M. Levy, *Nuovo Cim.* **16** (1960) 705.

[14] B.D. Serot and J.D. Walecka, *Int. J. Mod. Phys.* **E 16** (1997) 15.

PoS(QFTHEP2010)054