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The evolution of phase states of Universe to the present inert phase

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We assume that current state of the Universe is described by the Inert Doublet Model, containing two scalar doublets, one of which is responsible for EWSB and masses of particles and the second one having no couplings to fermions and being responsible for dark matter. We consider possible evolutions of the Universe to this state during cooling down of the Universe after inflation. We found that in the past Universe could pass through phase states having no DM candidate. In the evolution via such states in addition to a possible EWSB phase transition (2-nd order) the Universe sustained one 1-st order phase transition or two phase transitions of the 2-nd order.

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1. Introduction

One of the widely discussed models for Dark Matter (DM) particles is the Inert Doublet Model (IDM) [1]. The model contains "standard" scalar (Higgs) doublet ϕ_S, responsible for electroweak symmetry breaking and masses of fermions and gauge bosons as in the Standard Model (SM), and scalar doublet, ϕ_D, which doesn't receive vacuum expectation value (v.e.v.) and doesn't couple to fermions. *(Our notations are similar to those in the 2HDM with $\phi_1 \to \phi_S$, $\phi_2 \to \phi_D$.) More complete version of this report is given in [2].* Here four degrees of freedom of the Higgs doublet ϕ_S are as in the SM: three Goldstone modes become longitudinal components of the EW gauge bosons and one mode becomes the Higgs boson h_S. All the components of the scalar doublet ϕ_D are realized as massive scalar D-particles: two charged $D^\pm$ and two neutral ones D_H and D_A. By construction, they possess a conserved multiplicative quantum number and therefore the lightest particle among them can be considered as a candidate for DM particle. Assuming that DM particles are neutral, we have

$$M_{D^\pm}, M_{D_A} \geq M_{D_H} \ \text{ or } \ M_{D^\pm}, M_{D_H} \geq M_{D_A}\,. \tag{1.1}$$

Possible masses of D-particles are constrained by the accelerator and astrophysical data (see e.g. [3]).

Assuming that the current state of the Universe is described by IDM, we discuss possible variants of the history of the phase states of Universe during its cooling down after inflation. In some respects, this analysis can be considered as particular case of analysis [4], [5]. We use below some results and notations from [4]-[6].

2. The Lagrangian

The electroweak symmetry breaking via the Higgs mechanism is described by the Lagrangian

$$\mathscr{L} = \mathscr{L}^{SM}_{gf} + T - V + \mathscr{L}_Y(\psi_f, \phi_S)\,. \tag{2.1}$$

Here, $\mathscr{L}^{SM}_{gf}$ describes the $SU(2)\times U(1)$ Standard Model interaction of gauge bosons and fermions, which is independent on the realization of the Higgs mechanism, T is the standard kinetic term for two scalar doublets ϕ_S and ϕ_D and the potential V with these two scalars. The $\mathscr{L}_Y$ describes the Yukawa interaction of fermions ψ_f with only one scalar doublet ϕ_S in the same form as in the SM.

Potential. The potential must be Z_2 symmetric in order to describe IDM. Without loss of generality it can be written in the form with all real parameters[1]

$$\begin{gathered} V = -\frac{1}{2}\left[m_{11}^2(\phi_S^\dagger\phi_S) + m_{22}^2(\phi_D^\dagger\phi_D)\right] + \frac{\lambda_1}{2}(\phi_S^\dagger\phi_S)^2 + \frac{\lambda_2}{2}(\phi_D^\dagger\phi_D)^2 + \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D) + \\ + \lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S) + \frac{\lambda_5}{2}\left[(\phi_S^\dagger\phi_D)^2 + (\phi_D^\dagger\phi_S)^2\right], \qquad \lambda_5 < 0\,. \end{gathered} \tag{2.2}$$

The IDM is realized in some regions of parameters of this potential. To study thermal evolution, we will consider also other possible vacuum states of such potential, at another values of parameters.

[1] In the general Z_2 symmetric potential the last term has a form $\left[\tilde\lambda_5(\phi_S^\dagger\phi_D)^2 + \tilde\lambda_5^*(\phi_D^\dagger\phi_S)^2\right]$. The physical content of theory cannot be changed by the global phase rotation $\phi_a \to \phi_a e^{i\alpha_a}$ $(a = S, D)$. Starting with an arbitrary complex $\tilde\lambda_5 = |\tilde\lambda_5| e^{i\rho}$ we select $\alpha_S - \alpha_D = \rho/2 + \pi/2$, to get (2.2) with negative $\lambda_5 = -|\tilde\lambda_5|$.

To make some equations shorter, we use the abbreviations:

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \frac{\lambda_{345}}{\sqrt{\lambda_1 \lambda_2}}. \tag{2.3}$$

Discrete symmetries. This potential (2.2) is invariant under two discrete symmetry transformations of a Z_2 type, called respectively *S-transformation* and *D-transformation* (here SM denote the SM fermions and gauge bosons):

$$S: \ \phi_S \xrightarrow{S} -\phi_S, \quad \phi_D \xrightarrow{S} \phi_D, \quad SM \xrightarrow{S} SM, \tag{2.4}$$
$$D: \ \phi_S \xrightarrow{D} \phi_S, \quad \phi_D \xrightarrow{D} -\phi_D, \quad SM \xrightarrow{D} SM. \tag{2.5}$$

For the EW symmetric phase (with $\langle\phi_S\rangle = \langle\phi_D\rangle = 0$ this invariance results in the *D-parity* and *S-parity* conservation in the processes involving only scalars and gauge bosons. The Yukawa term violates S-symmetry, while it respects D-symmetry in any order of perturbation theory.

Positivity constraints. To have a stable vacuum, the potential must be positive at large quasi–classical values of fields $|\phi_i|$ (*positivity constraints*), for an arbitrary direction in the (ϕ_S, ϕ_D) plane. These conditions limit possible values of λ_i (see e.g. [7]). In terms of parameters (2.3) positivity constraints which are needed in our analysis, can be written as

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad R+1 > 0. \tag{2.6}$$

3. Thermal evolution

Potential. Since the Hubble constant is small, we assume a statistical equilibrium at every temperature T. In this approximation, at the finite temperature, the ground state of system is given by a minimum of the Gibbs potential

$$V_G = Tr\left(Ve^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right). \tag{3.1}$$

In the first nontrivial approximation and high enough temperature the obtained Gibbs potential has the same form as the basic potential V(2.2), i. e. as the potential at zero temperature. The coefficients $\lambda's$ of the quartic terms in the potential V_G and V coincide, while the mass terms vary with temperature T, as follows

$$\begin{gathered} m_{11}^2(T) = m_{11}^2 - c_1 T^2, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2, \\ c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32} + \frac{g_t^2 + g_b^2}{8}, \quad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32}. \end{gathered} \tag{3.2}$$

Here g and g' are the EW gauge couplings, $g_t \approx 1$ and $g_b \approx 0.03$ are values of the SM Yukawa couplings for t and b quarks, respectively.

Generally each of coefficients c_1 and c_2 can be either positive or negative. However, in virtue of positivity conditions (2.6) their sum is positive (even neglecting positive contributions from gauge bosons W/Z and fermions),

$$c_2 + c_1 > 0. \tag{3.3}$$

.

We will show later on that for a realization of the present inert vacuum with neutral dark matter particle one needs $\lambda_4+\lambda_5<0$ (5.4). Therefore, at $R>0$ we have $\lambda_3>0$. Taking into account that $\lambda_5<0$ (2.2), we obtain that $c_1>0$, $c_2>0$. At $R<0$ there are no constraints on signs of $c_{1,2}$:

$$\begin{array}{ll} R>0: & c_1>0, \quad c_2>0; \\ R<0: & \text{arbitrary signs of } c_{1,2}. \end{array} \tag{3.4}$$

Yukawa interaction. The form of Yukawa interaction and values of Yukawa couplings don't vary during thermal evolution.

4. Extrema of the potential

Following [4] we first consider extrema of the potential (2.2) at arbitrary values of parameters. The extrema conditions:

$$\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle}=0\,,\ \partial V/\partial\phi_i^\dagger\Big|_{\phi_i=\langle\phi_i\rangle}=0 \qquad (i=S,D) \tag{4.1}$$

define the extremum values $\langle\phi_S\rangle$ and $\langle\phi_D\rangle$ of the fields ϕ_S and ϕ_D, respectively. The extremum with the lowest energy (*the global minimum of the potential*) realizes *the vacuum state* of the system. Other extrema are saddle points, maxima or local minima of the potential.

For each electroweak symmetry violating extremum with $\langle\phi_S\rangle\neq 0$, one can choose the z axis in the weak isospin space so that $\langle\phi_S\rangle\sim\begin{pmatrix}0\\ v_S\end{pmatrix}$, with real, nonnegative v_S (choosing a "neutral direction" in the weak isospin space). Therefore, the most general solution of (4.1) can be written as

$$\langle\phi_S\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_S\end{pmatrix}, \quad \langle\phi_D\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}u\\ v_D\end{pmatrix}. \tag{4.2}$$

Neutral extrema. The solutions of (4.1) with $u=0$ are called neutral extrema, as they respect $U(1)$ symmetry of electromagnetism. For these extrema the conditions (4.1) can be written as a system of two degenerate cubic equations with four solutions:

$$v_S(-m_{11}^2+\lambda_1 v_S^2+\lambda_{345}v_D^2)=0\,, \qquad v_D(-m_{22}^2+\lambda_2 v_D^2+\lambda_{345}v_S^2)=0\,, \quad v_S^2,\, v_D^2\geqslant 0\,. \tag{4.3}$$

This system has four solutions (here $\mathscr{E}_a$ are extrema energies):

$$\boldsymbol{EWs}: \ EW symmetric \ \ v_D=0, \quad v_S=0, \qquad \mathscr{E}_{EWs}=0; \tag{4.4}$$

$$\boldsymbol{I_1}: \quad inert \quad v_D=0,\ v^2\equiv v_S^2=\frac{m_{11}^2}{\lambda_1}, \quad \mathscr{E}_{I_1}=-\frac{m_{11}^4}{8\lambda_1}; \tag{4.5}$$

$$\boldsymbol{I_2}: \quad inert-like \quad v_S=0,\ v^2\equiv v_D^2=\frac{m_{22}^2}{\lambda_2}, \quad \mathscr{E}_{I_2}=-\frac{m_{22}^4}{8\lambda_2}; \tag{4.6}$$

$$\boldsymbol{M}: \quad mixed \quad \begin{cases} v_S^2=\dfrac{m_{11}^2\lambda_2-\lambda_{345}m_{22}^2}{\lambda_1\lambda_2-\lambda_{345}^2}, \quad v_D^2=\dfrac{m_{22}^2\lambda_1-\lambda_{345}m_{11}^2}{\lambda_1\lambda_2-\lambda_{345}^2}, \\ \mathscr{E}_M=\dfrac{-m_{11}^4\lambda_2+2\lambda_{345}m_{11}^2m_{22}^2-m_{22}^4\lambda_1}{8(\lambda_1\lambda_2-\lambda_{345}^2)}. \end{cases} \tag{4.7}$$

If one of these equations gives $v_S^2 < 0$ or $v_D^2 < 0$, the corresponding extremum is absent.

The energy differences between $I_{1,2}$ and M extrema are

$$\mathscr{E}_{I_1} - \mathscr{E}_M = \frac{\left(m_{11}^2\lambda_{345} - m_{22}^2\lambda_1\right)^2}{8\lambda_1^2\lambda_2(1-R^2)}; \qquad \mathscr{E}_{I_2} - \mathscr{E}_M = \frac{\left(m_{22}^2\lambda_{345} - m_{11}^2\lambda_2\right)^2}{8\lambda_1\lambda_2^2(1-R^2)}. \tag{4.8}$$

Charge breaking extremum. For $u \neq 0$ the extremum violates not only EW symmetry but also the $U(1)$ electromagnetic symmetry, leading to the electric charge non-conservation. This extremum can realize vacuum state only if $\lambda_4 + \lambda_5 > 0$ [9, 4]. We will see later on that at this condition the DM particle can not be neutral, that contradicts (1.1).

5. Vacuum states

Below we describe briefly the properties of neutral extrema, listed above, provided in each case that it realizes a true vacuum.

5.1 Electroweak symmetric vacuum *EWs*, $\langle\phi_S\rangle = \langle\phi_D\rangle = 0$

The electroweak symmetric extremum exists for all values of parameters of the potential (2.2). This extremum is a minimum, realizing vacuum state, at

$$m_{11}^2 < 0, \qquad m_{22}^2 < 0. \tag{5.1}$$

In this case, gauge bosons and fermions are massless, while scalar doublets ϕ_S and ϕ_D have masses equal to $|m_{11}|/\sqrt{2}$ and $|m_{22}|/\sqrt{2}$, respectively.

5.2 Inert vacuum I_1, $\langle\phi_D\rangle = 0$

In the case when I_1 extremum realizes vacuum, the Inert Doublet Model describes reality. The standard field decomposition near I_1 extremum has a form

$$\phi_S = \begin{pmatrix} G^+ \\ \dfrac{v + h_S + iG}{\sqrt{2}} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} D^+ \\ \dfrac{D_H + iD_A}{\sqrt{2}} \end{pmatrix}, \tag{5.2}$$

where $G^\pm$ and G are Goldstone modes, while h_S and $D = D_H, D_A, D^\pm$ are scalar particles. Here the Higgs particle h_S interacts with the fermions and gauge bosons just as the Higgs boson in the SM. D-particles don't interact with fermions. Neither there are interactions of D-particles with gauge bosons V of the type $D_iV_1V_2$.

Symmetry properties. This vacuum is invariant under the D-transformation (2.5). Therefore the D-parity is conserved. Hence, the lightest D-particle is stable, being a good DM candidate. (In this state the S-symmetry (2.4) is broken.)

Allowed region of parameters. The inert *extremum* exists if only $m_{11}^2 > 0$ (4.5). In accordance with (4.5) and (4.6), the extremum I_1 can be a *vacuum* only if $m_{11}^2/\sqrt{\lambda_1} > m_{22}^2/\sqrt{\lambda_2}$. Additional condition arises from a comparison of I_1 and M extrema. In virtue of (4.8) at $R^2 > 1$ the extremum M can exist but its energy is larger than energy of I_1 extremum – so that the extremum I_1 realizes vacuum. At $R^2 < 1$ the inert extremum still can be a vacuum, in the case when the mixed extremum does not exist, i. e. if at least one of quantities v_S^2, v_D^2 defined by eq. (4.7) is negative. Note, that due to the positivity constraint $1+R>0$ (2.6) in the case when $R^2>1$ we have $R>1$. For the opposite case, with $R^2<1$, the quantity R can be either positive or negative.

Particle properties. The quadratic part of the potential written in terms of physical fields h_S, D_H, D_A and $D^\pm$ (5.2) gives the masses of scalars:

$$\begin{aligned} M_{h_S}^2 = \lambda_1 v^2 = m_{11}^2\,, \qquad M_{D^\pm}^2 = \frac{\lambda_3 v^2 - m_{22}^2}{2}\,,\\ M_{D_A}^2 = M_{D^\pm}^2 + \frac{\lambda_4-\lambda_5}{2}v^2\,, \quad M_{D_H}^2 = M_{D^\pm}^2 + \frac{\lambda_4+\lambda_5}{2}v^2\,. \end{aligned} \tag{5.3}$$

The requirement that lightest D-particle is a neutral one (1.1) results in the condition

$$\lambda_4+\lambda_5<0. \tag{5.4}$$

As in the standard 2HDM, scalars D_H and D_A have opposite P-parities but since they don't couple to fermions, there is no way to assign to them a definite value of P-parity. However, their relative parity does matter and for example, vertex ZD_HD_A is allowed while vertices ZD_HD_H and ZD_AD_A are forbidden. Since $\lambda_5<0$ (2.2) the "scalar" D_H is lighter than "pseudoscalar".

5.3 Inert-like vacuum I_2, $\langle\phi_S\rangle = 0$

The inert-like vacuum I_2 looks as "mirror-symmetric" to the inert vacuum I_1. The interactions among scalars and between scalars and gauge bosons in both cases are identical in form with the change $\phi_S \leftrightarrow \phi_D$. The only difference between I_2 and I_1 is given by the Yukawa interaction.

Main formulae for this state are similar to those for the vacuum I_1 with obvious replacements. The corresponding field decomposition is given by

$$\phi_S = \begin{pmatrix} S^+ \\ \dfrac{S_H+iS_A}{\sqrt{2}} \end{pmatrix}, \qquad \phi_D = \begin{pmatrix} G^+ \\ \dfrac{v+h_D+iG}{\sqrt{2}} \end{pmatrix}, \tag{5.5}$$

with one Higgs particle h_D and four S-particles: $S_H, S_A, S^\pm$.

Symmetry properties. The inert-like vacuum I_2 violates D-symmetry (2.5). Moreover, here S-parity is also violated by the Yukawa interaction (in contrast to the D-parity in the inert vacuum).

Allowed regions of parameters. The *inert-like extremum* exists only for $m_{22}^2>0$. It can be vacuum if $m_{11}^2/\sqrt{\lambda_1} < m_{22}^2/\sqrt{\lambda_2}$. For $R^2>1$ there are no additional demands. If $R^2<1$ inert-like extremum can be a vacuum only if at least one of quantities v_S^2, v_D^2, defined by eq. (4.7), appears to be negative. Both these conditions are similar to those for the inert vacuum I_1.

Particle properties. The masses of the Higgs boson h_D and S-scalars are given by (cf. (5.3))

$$\begin{aligned} M_{h_D}^2 = \lambda_2 v^2 = m_{22}^2\,, \qquad M_{S^\pm}^2 = \frac{\lambda_3 v^2 - m_{11}^2}{2}\,,\\ M_{S_A}^2 = M_{S^\pm}^2 + \frac{\lambda_4-\lambda_5}{2}v^2\,, \quad M_{S_H}^2 = M_{S^\pm}^2 + \frac{\lambda_4+\lambda_5}{2}v^2\,. \end{aligned} \tag{5.6}$$

The Higgs boson h_D couples to gauge bosons just as the Higgs boson of the SM, however it does not couple to fermions at the tree level. The S-scalars do interact with fermions. All fermions, by construction interacting only with ϕ_S with vanishing v.e.v. $\langle\phi_S\rangle = 0$, are massless. (Small mass can appear only as a loop effect.) *Here there are no candidates for dark matter particles.*

5.4 Mixed vacuum M, $\langle\phi_S\rangle$, $\langle\phi_D\rangle \neq 0$

The mixed extremum M violates both D- and S-symmetries. In this vacuum we have massive fermions and no candidates for DM particle, like in the SM. The decomposition around the mixed vacuum looks as follows:

$$\phi_S = \begin{pmatrix} \rho_S^+ \\ \dfrac{v_S+\rho_S+i\chi_S}{\sqrt{2}} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} \rho_D^+ \\ \dfrac{v_D+\rho_D+i\chi_D}{\sqrt{2}} \end{pmatrix}, \tag{5.7}$$

where the ρ_S^+ and ρ_D^+ lead to two orthogonal combinations G^+ and H^+, while ρ_S and ρ_D (χ_S and χ_D) – to two orthogonal combinations h and H (G and A), respectively. There are here five Higgs bosons - two charged $H^\pm$ and three neutral ones: the CP-even h and H and CP-odd A.

Allowed regions of parameters. In accordance with (4.7) and (4.8) the mixed extremum is global minimum of potential, i. e. *vacuum*, if and only if $v_S^2>0$, $v_D^2>0$ and $R^2<1$. For v.e.v.'s squared given by eqs. (4.7) the latter conditions can be transformed to the relations between mass parameters m_{11}^2 and m_{22}^2:

$$\begin{aligned} &\text{at}\quad 1>R>0: \quad 0<R\frac{m_{11}^2}{\sqrt{\lambda_1}}<\frac{m_{22}^2}{\sqrt{\lambda_2}}<\frac{m_{11}^2}{R\sqrt{\lambda_1}};\\ &\text{at}\ \ 0>R>-1: \ \frac{m_{22}^2}{\sqrt{\lambda_2}}>R\frac{m_{11}^2}{\sqrt{\lambda_1}},\ \frac{m_{22}^2}{\sqrt{\lambda_2}}>\frac{m_{11}^2}{R\sqrt{\lambda_1}}. \end{aligned} \tag{5.8}$$

Particle properties. Masses of charged and axial scalars together with mass matrix for CP-even scalars $\mathscr{M}$ are (see, e.g. [6, 4])

$$M_{H^\pm}^2 = -\frac{\lambda_4+\lambda_5}{2}v^2, \quad M_A^2=-v^2\lambda_5 \quad \left(v^2=v_S^2+v_D^2\right), \quad \mathscr{M}=\begin{pmatrix} \lambda_1 v_S^2 & \lambda_{345}v_Sv_D \\ \lambda_{345}v_Sv_D & \lambda_2 v_D^2 \end{pmatrix}. \tag{5.9}$$

The masses of the neutral CP-even Higgs bosons are given by eigenvalues of this mass matrix.

The extremum can be minimum only if both diagonal elements of mass matrix and its determinant are positive, i. e. $\lambda_1\lambda_2 v_S^2 v_D^2(1-R^2)>0$, in agreement with the above mentioned conditions. It means also that in the case if mixed extremum is minimum, it is global minimum – vacuum.

Couplings of the physical Higgs bosons to fermions and gauge bosons have standard forms as for the 2HDM, with the Model I Yukawa interaction.

6. Evolution of phase states of the Universe

In this section we consider possible phase history of the Universe, leading to the inert vacuum I_1 today, based on the thermal evolution described in sec. 3. To summarize properties of different vacua and to classify all possible ways of evolution of the Universe we will use phase diagrams in the $(\mu_1(T),\mu_2(T))$ plane, where

$$\mu_1(T)=m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T)=m_{22}^2(T)/\sqrt{\lambda_2}\,. \tag{6.1}$$

Let us remind (sect. 3) that in our approximation during cooling down of Universe parameters λ_i are fixed, while mass parameters m_{ii}^2 vary. These variations result in modification of vacuum state and a possible change of its nature. Possible types of evolution depend on value of parameter R (2.3) and are depicted in the figures 1, 2 and 3. The possible current states of Universe are represented in these figures by small black dots $P = (\mu_1, \mu_2)$. Since currently we are in the inert phase, we have[2] $\mu_1 > 0$ for each point P (see sect. 5.2). The parameter μ_2 can be both positive (points $P1$ and $P3$) and negative (points $P2$, $P4$ and $P5$).

In accordance with (3.2) a particular evolution leading to a given physical vacuum state P is represented by a ray, that ends at a point P. Arrows on these rays are directed towards a growth of time (decreasing of temperature). The direction of the ray is determined by parameters (cf. (3.2))

$$\tilde{c}_1 = c_1/\sqrt{\lambda_1}, \qquad \tilde{c}_2 = c_2/\sqrt{\lambda_2}, \qquad \tilde{c} = \tilde{c}_2/\tilde{c}_1. \tag{6.2}$$

R	$\tilde{c}$	*initial state of Universe*
>0	>0	EWs
<0	>0	EWs
	<0	EWv

According to (3.2), for $\tilde{c} > 0$ in the initial state of Universe ($T \to \infty$) $m_{11}^2 < 0$ and $m_{22}^2 < 0$, i. e. the initial phase state of the Universe is electroweak symmetric. If $\tilde{c} < 0$ in the initial state of Universe either m_{11}^2 or m_{22}^2 is positive, i. e. the initial state of Universe is electroweak-symmetry violating [3] – EWv. Taking into account (3.4) the list of opportunities is presented in the Table here.

For different possible positions of today's point P we consider typical evolutions for different possible values of parameter $\tilde{c}$. In figures below all representative rays are shown; they are labeled by two numbers, with the first one corresponding to the label of the final point P.

6.1 The case $R > 1$

Phase diagram for this case is presented in Fig. 1. It contains one quadrant with EWs phase and two sectors, describing the I_1 and I_2 phases. These sectors are separated by *the phase transition line* $\mu_1 = \mu_2$ (thick black line). Two typical positions of today's state are represented by points $P1$ ($\mu_2 > 0$) and $P2$ ($\mu_2 < 0$). Since (according to (3.4)), both $\tilde{c}_1, \tilde{c}_2 > 0$ ($\tilde{c} > 0$), all possible phase evolutions are represented by rays 11 and 12 for the today's point $P1$ and by a ray 21 which leads to the today's point $P2$.

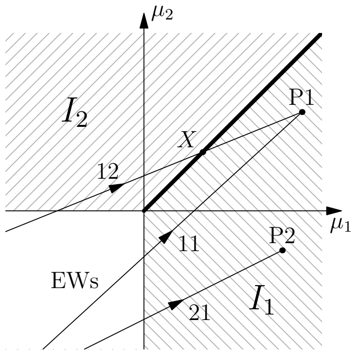

Figure 1: $R > 1$ case.

Ray 11: $\tilde{c} > \mu_2/\mu_1 > 0$. The Universe started from the EWs state and after the second-order EWSB transition at $m_{11}^2(T) = 0$, i. e. it has entered to the present inert phase I_1 at the temperature

$$T_{EWs,1} = \sqrt{m_{11}^2/c_1} = \sqrt{\mu_1/\tilde{c}_1}\,. \tag{6.3}$$

Ray 12: $0 < \tilde{c} < \mu_2/\mu_1$. The Universe started from the EWs state. Then it went through the EWSB second-order phase transition into the inert-like phase I_2 at $m_{22}^2(T) = 0$, i. e. at the temperature

$$T_{EWs,2} = \sqrt{m_{22}^2/c_2} = \sqrt{\mu_2/\tilde{c}_2}. \tag{6.4}$$

[2] We distinguish present day values of parameters μ_i and their values $\mu_i(T)$ at some temperature T.

[3] Such opportunity is not ruled out [10], but it contradicts a key idea of modern approach – the initial state of Universe has high symmetry which is broken at cooling down. In this sense such opportunity is unnatural.

The next transition is the phase transition from the inert-like phase I_2 into the today's inert phase I_1 at the point R, where $\mu_2(T) = \mu_1(T)$, i. e. at the temperature

$$T_{2,1} = \sqrt{(\mu_1 - \mu_2)/(c_1 - \tilde{c}_2)}\,. \tag{6.5}$$

That is the first-order phase transition with the latent heat given by

$$Q_{I_2 \to I_1} = T_{2,1}\,(\partial \mathcal{E}_{I_2}/\partial T - \partial \mathcal{E}_{I_1}/\partial T)_{T=T_{2,1}} = (\mu_2 \tilde{c}_1 - \mu_1 \tilde{c}_2) T_{2,1}^2/4\,. \tag{6.6}$$

Ray 21: $\boldsymbol{\mu_2 < 0}$. The Universe started from EWs state and after the second-order EWSB transition at the temperature (6.3) has entered the today's phase.

6.2 The case $1 > R > 0$

The phase diagram for this case – Fig. 2 is obtained from that for previous case (Fig. 1) by adding in the upper right quadrant the new sector – the mixed phase M, described in accordance with (5.8) by equation

$$0 < R\mu_1 < \mu_2 < \mu_1/R. \tag{6.7}$$

As before, since $R > 0$ we have $\tilde{c} > 0$.

Since currently we are in the inert vacuum, the possible today's states are of type of points $P3$ and $P4$, for which

$$\mu_2 < R\mu_1. \tag{6.8}$$

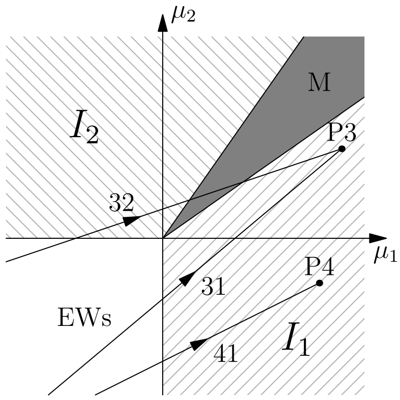

Figure 2: $1 > R > 0$ case.

All possible phase evolutions are represented by three rays in Fig. 2, with rays 31 and 32 having the today's endpoint $P3$ while the ray 41 is pointing $P4$.

For the rays 31 and 41, phase evolutions are as for the rays 11 and 21, respectively. New situation appears for the ray 32.

Ray 32: $\boldsymbol{0 < \tilde{c} < \mu_2/\mu_1}$. The Universe started from the EWs state. Then at the temperature given by (6.4) it went through the EWSB second-order phase transition into the inert-like phase I_2. At the subsequent cooling down the Universe goes through the mixed phase M into the present inert phase I_1. The second-order phase transitions $I_2 \to M$ and $M \to I_1$ happened at the temperatures

$$T_{phtr}: \quad \begin{aligned} T_{2,M} &= \sqrt{(\mu_1 - R\mu_2)/(\tilde{c}_1 - R\tilde{c}_2)}, \\ T_{M,1} &= \sqrt{(R\mu_1 - \mu_2)/(R\tilde{c}_1 - \tilde{c}_2)}\,. \end{aligned} \tag{6.9}$$

In accordance with equations in sect. 5, at the transition point $I_2 \to M$ masses of S_H and h vanish, while at the transition point $M \to I_1$ masses of h and D_H become 0. At small distance from the transition point with temperature T_{phtr} these masses grow as a function of the temperature T as $M_a^2 = A_a|T^2 - T_{phtr}^2|$, with different coefficients A_a.

6.3 The case $0 > R > -1$

The phase diagram is presented in Fig. 3. In this case, as follows from (5.8), the mixed phase M region is realized in a wider region than in Fig. 2, even beyond an upper right quadrant of this plane, namely:

$$\mu_2 > \mu_1/R, \quad \mu_2 > \mu_1 R. \tag{6.10}$$

Since currently we are in the inert vacuum ($\mu_1 > 0$), for the today's point $P5$ we have $\mu_2 < R\mu_1$ ($\mu_2 < 0$). New opportunities appear due to larger freedom for temperature coefficients c_i, as in accordance with (3.4) in this case $\tilde{c}$ can be negative.

All possible phase evolutions leading to the point $P5$ are represented in Fig. 3 by four rays 51, 52, 53 and 54.

The ray 51 with $\tilde{c} > 0$ describes similar evolution as rays 21 and 41. New are rays 52, 53 and 54 (at $\tilde{c} < 0$) with common feature, which is a lack of electroweak symmetry in very early stages of the Universe.

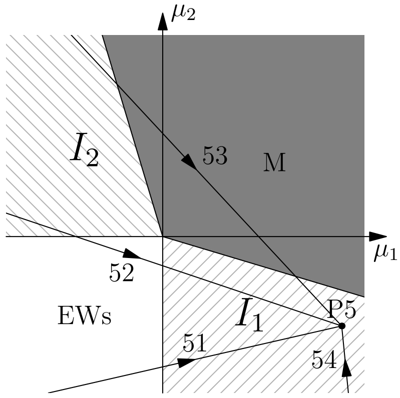

Figure 3: $1 > R > 0$ case.

Ray 52: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} > \mu_2/\mu_1$. Here a high-temperature state of the Universe is the inert-like vacuum I_2. With cooling down the Universe goes through electroweak symmetric phase EWs into the present I_1 phase. The second-order phase transitions $I_2 \to EWs$ and $EWs \to I_1$ happened, respectively, at the temperatures

$$T_{2,EWs} = \sqrt{\mu_2/\tilde{c}_2}, \qquad T_{EWs,1} = \sqrt{\mu_1/\tilde{c}_1}. \tag{6.11}$$

Ray 53: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} < \mu_2/\mu_1$. Here a high-temperature state of the Universe is an inert-like vacuum I_2. With cooling down the Universe passes through the mixed phase M into the present I_1 phase. The phase transitions $I_2 \to M$ and $M \to I_1$ are of the second order; they happened at the temperatures given by eqs. (6.9).

Ray 54: $\tilde{c}_1 < 0$, $\tilde{c}_2 > 0$. For this ray the Universe stays in the inert vacuum I_1 during the whole evolution.

7. Results and discussion

Main results. The most important observation we made in this paper is as follows:
If current state of the Universe is described by IDM, then during the thermal evolution the Universe can pass through various intermediate phases, different from the inert one. These possible intermediate phases contain no dark matter, which appears only at the relatively late stage of cooling down of the Universe.

We find that in the considered approximation the thermal evolution of Universe can be studied effectively in the (μ_1, μ_2) plane, at fixed values of quartic parameters λ_i. Different types of such evolution represented as directed rays depend crucially on two parameters: R (2.3), describing the allowed for various vacua regions of the (μ_1, μ_2) plane, and $\tilde{c}$ (6.2), determining the direction of rays. The first one depends only on the ratios between coefficients of quartic part of potential λ_i, while the second depends both on mentioned parameters λ_i and on the known gauge and Yukawa couplings.

The starting point of evolution of Universe to the present day inert phase state can be either electroweak symmetric (EWs) state (if $\tilde{c} > 0$) or electroweak symmetry violating state (at $\tilde{c} < 0$). A complete set of possible ways of evolution of the Universe can be summarized as follows (symbol

I or II over arrows corresponds to the type of phase transition):

$$\boxed{EWs \xrightarrow{II} \begin{Bmatrix} I_1 & (rays\ 11, 21, 31, 41, 51) \\ I_2 \begin{cases} \xrightarrow{II} M & \xrightarrow{II} I_1 \quad (ray 32) \\ \xrightarrow{I} I_1 & (ray\ 12) \end{cases} \end{Bmatrix} : \tilde{c} > 0}$$
$$\boxed{EWv : \quad \left.\begin{matrix} I_2 \xrightarrow{II} \begin{Bmatrix} EWs \xrightarrow{II} I_1 \ (ray\ 52) \\ M \xrightarrow{II} I_1 \quad (ray\ 53) \end{Bmatrix}. \\ I_1 \to I_1 \qquad\qquad (ray\ 54) \end{matrix}\right\} : \tilde{c} < 0} \tag{7.1}$$

To find what scenario of evolution is realized in nature, one should measure all parameters of potential. The program how to measure these parameters at LHC and ILC is under preparation.

Outlook.

A. Extra phase transitions at lower temperature than EWSB temperature (and especially first order phase transition at the evolution like ray 12) can influence for baryogenesis even stronger than transformation of standard second order EWSB transition into the first order one due to term $\phi^3 T$ [11]. Moreover, in contrast to the standard picture, the considered scenarios allow for the phase transition to the current inert phase at relatively low temperature, giving new starting point for calculation of a today's abundance of the neutral DM components of the Universe and other phenomena.

B. In this paper we calculated thermal evolution of the Universe in the very high temperature approximation, i. e. for $T^2 \gg |m_{ii}^2|$. The most interesting effects are expected at lower temperatures, where more precise calculations are necessary. The simplest expected modifications of the presented description are:

1. Appearance of cubic terms like $\phi^3 T$ [11]. These terms are important near phase transition point, as they can transform some second-order phase transition into the first-order transition.
2. The parameters become depend on temperature in more complicated way than that given by (3.2). Therefore, the rays, depicted thermal evolutions in Figs. 1, 2 and 3, can become non-straight. The bending of these rays can be different in different points of our plots and at different λ_i. It can give possible spectrum of phase evolutions even reacher that discussed above.

However, we expect that the general picture will not change too much.

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