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Black holes in N>4 gravity

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We consider geodesic equations for a black hole solution in the Randall-Sundrum II scenario presented in [1]. This solution is a generalization of the Schwarzschild one and has the mathematical form of the Reissner-Nördstrom solution, but with an additional "tidal charge" instead of the electric charge. We examine the behavior of geodesic parameters and show that the solution does not contradict the observational data and does not predict any fundamentally new effects. A more serious restriction on the "tidal charge" value can be extracted from the circular orbit equation.

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1. Introduction

Impossibility of direct quantization of the general relativity, like it is done in the electrodynamics, leads to serious problems in attempts to build an universal theory of all physical interactions. On one hand we have general relativity, that fully describes all modern observational data and gives basis for understanding of the Universe at large scales. On the other hand we have quantum mechanics, that describes microscopical scales. Unfortunately, these two theories can not be combined with each other. Any attempt to directly quantize gravity leads to an infinite amount of counterterms during renormalization because of the nonflat background presence. This fact does not create any problems for experimental physics: gravitational interaction becomes significant only for energies near 10^9 GeV, this means that the role of the gravitational interaction in modern experimental physics vanishes. However, it is important to consider gravity to understand and describe such phenomena as Big Bang, Early Universe, last stages of Hawking evaporation [2, 3, 4]. To describe these phenomena, a generalized theory of gravity is required, for example, the string theory in its low energy limit. When we turn to four-dimensional spacetime, we arrive at the so-called "braneworld model", which main idea is that our world is four-dimensional slice of some higher dimensional spacetime — bulk. Matter and fields are localized on a brane and only gravity can reach extra dimensions. Such models are remarkable with the fact that they can solve the hierarchy problem: why electroweak scale is so much different from the great unification one. In this case, four-dimensional Planck scale ceases to be fundamental and becomes effective [5]. A truly fundamental characteristic is the multidimensional Planck scale, that is related to four-dimensional one via extra dimensional properties. In braneworld models fundamental Planck scale can lower from 10^{19}GeV to energies of 1TeV, making search for experimental manifestation of extra-dimension not hopeless.

2. Black hole solution

New types of black hole solutions, which arise in braneworld models, have new unusual properties due to presence of extra dimensions. One of the first localized on the brane black hole solutions in Randall-Sundrum scenario was obtained in [1]. Authors [1] considered 5-dimensional field equations:

$$\widetilde{G}_{AB} = \tilde{\kappa}^2 \left(-\widetilde{\Lambda}\tilde{g}_{AB} + \delta(\chi)\left(-\lambda g_{AB} + T_{AB}\right) \right), \tag{2.1}$$

where tidels and large latins denote 5-dimensional quantities, small greek letters are used for 4-dimensional quantities.$\widetilde{G}_{AB}$ — Einstein tensor, $\tilde{\kappa}^2 = 8\pi/\widetilde{M}^3_{Pl}$, M_{Pl} — Planck mass, brane tension — λ, $\widetilde{\Lambda}$ cosmological constant, $g_{AB} = \tilde{g}_{AB} - n_A n_B$ — induced metric on the brane (i.e. metric, obtained via projection of 5-dimensional metrik onto the brane), n_A — spacelike unit normal to the brane. T_{AB}- energy-momentum tensor on the brane. Corresponding field equations on the brane include terms, carrying bulk effects onto the brane:

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \tilde{\kappa}^4 S_{\mu\nu} - \mathscr{E}_{\mu\nu}, \tag{2.2}$$

where $\kappa^2 = 8\pi/M_{Pl}^2$, $S_{\mu\nu}$ - squared energy-momentum, $\mathscr{E}_{\mu\nu}$ - 5-dimensional Weyl tensor projection. The vacuum solution is under consideration, which means after several transformation equations reduce to:

$$R_{\mu\nu} = -\mathscr{E}_{\mu\nu}, \qquad R_\mu{}^\mu = 0 = \mathscr{E}_\mu{}^\mu, \qquad \nabla^\mu \mathscr{E}_{\mu\nu} = 0.$$

The solution on the brane has no any additional assumptions about 5-dimensional metric structure, the only important point is that it satisfies (2.2). Metric has the form:

$$ds^2 = \Delta(r)dt^2 - \frac{dr^2}{\Delta(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2.3}$$

where

$$\Delta(r) = 1 + \frac{\alpha}{r} + \frac{\beta}{r^2}, \tag{2.4}$$

α and β are constants. This metric has mathematical form of Reissner-N$\ddot{o}$rdstrom type, where $\alpha = -2M/M_{Pl}^2$, M — black hole mass, and $\beta = q/\widetilde{M}_{Pl}^2$, where q is "tidal charge", arising from bulk Weyl tensor, which projection on a brane is formally identified with energy-momentum tensor. So, the tidal charge is an "imprint" of the bulk free gravitational field. This metric gives rise to two types of black hole solutions. One is the classical Reissner-N$\ddot{o}$rdstrom solution with two horizons. This solution corresponds to $\beta < 0$ (this case is absent in general relativity). The solution with $\beta < 0$ has only one horizon, lower temperature and greater entropy, comparing to it's Schwarzschild counterpart:

$$r_h = \frac{M}{M_P l^2}\left[1 + \sqrt{1 - q\frac{M_P l^4}{M^2 \widetilde{M}_P l^2}}\right]. \tag{2.5}$$

Gravitational potential in the Schwarzschild metric $\Phi = \frac{M}{M_{Pl}^2 r}$ changes to:

$$\Phi = -\frac{M}{M_{Pl}^2 r} + \frac{Q}{2r^2}. \tag{2.6}$$

Authors of [1] have shown that the case $q < 0$ is physically more natural than $q > 0$ one. For $q < 0$ the effective energy density on the brane is negative, just like energy density of isolated massive source gravitational field in the Newton theory. Negative q provides spacelike singularity (like in Schwarzschild case), while positive value of q makes the singularity timelike, leading to qualitative change of the Schwarzshild solution nature. An estimate on q can be made, if we require the correction term in the modified potential to be much less than the Schwarzschild term:

$$|q| \ll 2\left(\frac{\widetilde{M}_P l}{M_P l}\right)^2 M_\odot R_\odot, \tag{2.7}$$

The authors argue, that this restriction anywhere allows q to be large enough to affect the spacetime geometry in strong-gravity regime.

In this work we are looking for geodesic equations in metric (2.3) and compare them with Swarzschild geodesic equation. Reissner-N$\ddot{o}$rdstrom solution is unstable, because macroscopic electrical charge is neutralized due to the surrounding plasma. Tidal charge arises in metric from geometrical considerations, hence, is not constrained by the above argument. Reissner-N*ö*rdstrom metric is well studied for positive values of charge parameter. Cases with negative values of charge parameter were not studied, due to absence of this opportunity in classical general relativity. This may give rise to new obsevrable effects.

3. Bound orbits

3.1 Timelike geodesic

We now turn to the orbits, which have an upper bound on r. This is the case, when $E^2 < 1$. We rewrite geodesic equations as:

$$\left(\frac{du}{d\phi}\right)^2 = f(u). \tag{3.1}$$

Equation (3.1) shows, that for the Schwarzshild case $\beta = 0$ $f(u)$ is third-order function on u, which means, it has in general three roots , for $\beta \neq 0$ $f(u)$ is fourth-order function.

Different pairs of E^2 and L lead to five different cases. Chandrasekar [13] sets two types of orbits. Orbits of the first type oscillate between two values of r. Second type orbits start at some distance and terminate in singularity. The first orbit is an analogue of keplerian orbit, but there is no analogue for the second type in general relativity.

The principal difference in the case of fourth-order equation is that $f(u)$ can have in general one more root and changes it's behavior when $r \to -\infty$. This can give rise to new types of orbits connected with u_4. Such case is only possible when all roots of $f(u) = 0$ are positive. We expand $f(u)$ like: $f(u) = (u-u_1)(u-u_2)(u-u_3)(u-u_4)$ and suggest all roots to be positive. After opening the brackets and comparing with (3.1), one obtains:

$$-u_1 - u_2 - u_3 - u_4 = \frac{\alpha}{\beta}, \tag{3.2}$$

hence, there is a contradiction, bacause $\alpha/\beta > 0$. Therefore at least one root should be negative, so the presence of β *does not cause an appearance of new type of orbits*. We would like to point out that this result does not depend on L and E, which means it also remains correct for unbound orbits.

3.2 Radial geodesics

Radial geodesics corresponding to $L = 0$ are:

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 - \Delta. \tag{3.3}$$

We consider particles starting at some initial distance with zeroth motion and falling into singularity. Initial conditions are: $r - r_i$ when $\dot{r} = 0$.

$$r_i^{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta(1-E^2)}}{2(1-E^2)}, \tag{3.4}$$

$$\frac{dt}{d\tau} = \frac{r^2 E}{r^2 + \alpha r + \beta}. \tag{3.5}$$

for bound orbits there is only one positive value of r_i, corresponding to positive sign (discriminant in (3.4) for $E^2 < 1$ and $\beta < 0$ becomes larger than α^2 in absolute value). This value exceeds the corresponding one of the Schwarzshild case:

$$r_i^{Sch} = -\frac{\alpha}{1-E^2}. \tag{3.6}$$

Coordinate time of reaching singularity is infinite, like in the Schwarzschild case. For the proper time one obtains a complicated expression, which does not allow to estimate it's value or compare it to the Schwarzshield one, without substituting parameter values.

So, one can arrive to a conclusion, that the presence of tidal charge leads only to quantitative changes in radial geodesics: increasement of initial distance r_i and change of the equation for proper time.

3.3 Circular orbits

Values of energy E and momentum L for circular orbits can be found from [13]:

$$f'(u) = -\frac{\alpha+2\beta}{L^2} - 2u - 3\alpha u^2 - 4\beta u^3 = 0, \tag{3.7}$$

$$f(u) = \frac{E^2-1}{L^2} - \frac{\alpha u+\beta u^2}{L^2} - u^2 - \alpha u^3 - \beta u^4 = 0. \tag{3.8}$$

On a corcular orbit of radius $r_c = \frac{1}{u_c}$ one has:

$$E^2 = \frac{2(1+\alpha u_c+\beta u_c^2)^2}{2+3\alpha u_c+\beta u_c^2}, \qquad L^2 = \frac{-\alpha-2\beta u_c}{u_c(2+3\alpha u_c+4\beta u_c^2)}. \tag{3.9}$$

Values of E^2 and L^2 in the limit $\beta \to 0$ correspond to Schwarzschild geometry. For $\beta \neq 0$ value of L^2 are larger. For negative β numerator in L^2 increases, denumerator decreases. Based on (3.9), one can obtain the following inequality (with $E^2 > 0, L^2 > 0$):

$$2+3\alpha u_c+4\beta u_c^2 > 0. \tag{3.10}$$

When $\beta < 0$ equation (3.10) reads:

$$0 < u_c < \frac{-3\alpha+\sqrt{9\alpha^2-32\beta}}{8\beta},$$

$$\frac{-3\alpha+\sqrt{9\alpha^2-32\beta}}{4} < r_c < \infty.$$

When $\beta = 0$ inequality (3.10) gives rise to a condition for the Schwarzschild circular orbits

$$r_c > -\frac{3}{2}\alpha.$$

If E and L take values of (3.9) correspondingly, equation (3.1) reads:

$$\left(\frac{du}{d\phi}\right)^2 = -(u-u_c)^2\left[\beta u^2+(\alpha+2u_c\beta)u+\left(u_c\beta+\frac{\alpha}{2}\left(1-\frac{1}{L^2u_c^2}\right)\right)u_c\right].$$

Radius of marginally stable orbit can be obtained from the requirement that $f(u)$ has an inflection point on a corresponding inverse radius [13]:

$$f''(u) = -\frac{2\beta}{L^2} - 2 - 6\alpha u - 12\beta u^2 = 0, \tag{3.11}$$

considering (3.9):

$$8\beta^2 u_c^3 + 9\alpha\beta u_c^2 + 3\alpha^2 u_c + \alpha = 0. \tag{3.12}$$

In the limit $\beta = 0$ one obtains Schwarzschild values $u_c = -1/3\alpha$ ore $r_c = -3\alpha$.

Finally, the presence of negative tidal charge leads for circular orbits, just like for radial geodesics, only to quantitative changes in the radii of marginally stable and unstable orbits.

4. Tidal charge contribution

The authors [1] assume, that the demand (2.7) allows for values of β to be large enough to affect space-time geometry in strong gravity regime. Switching to units of $M_\odot$ in alpha one obtains:

$$\alpha = -a\frac{M_\odot}{M_{pl}^2}, \tag{4.1}$$

Here a has the order of unity. Equating (2.4) to zero, one obtains tne equation for horizon radius, mass and "tidal charge" in the form:

$$r_h = \frac{-\alpha + \sqrt{\alpha^2 - 4\beta}}{2}. \tag{4.2}$$

It is convinient to redefine β similarly (4.1) as:

$$\beta = b\frac{M_\odot^2}{M_{pl}^4}. \tag{4.3}$$

Here we introduce the "normalized tidal charge" b, characterizing the value of β in solar mass scale.

Comparing the "normalized tidal charge"' (4.1) with the "tidal charge" found in (2.7) one can obtain a physical constraint on the "normalized tidal charge":

$$|b| \ll 2R_\odot \frac{M_{pl}^2}{M_\odot}. \tag{4.4}$$

Substituting solar mass and radius values ($R_\odot = 696000$ km, $M_\odot = 2\cdot 10^{30}$ kg, $M_{pl} = 10^{-8}$ kg) and converting this values to the Planckian system of units with $\hbar = c = 1$, we get:

$$|b| \ll 10^6.$$

One can see that the bound (2.7) is weaker than the one we can obtain from (3.12), because $a \sim 1$ corresponds to (4.1).

Indeed, the replacement $\tilde{u_c} = u_c M_\odot / M_{pl}^2$ in (3.12) gives rise to:

$$8b^2\tilde{u_c}^3 + 9ab\tilde{u_c}^2 + 3a^2\tilde{u_c} + a = 0. \tag{4.5}$$

Considering the demand the correction term containing b to be mutch smaller than the Schwarzschield one containing $a \sim 1$ one gets:

$$|b| \ll 1.$$

The solution appears to be self-consistent for these values of β and does not contradict to the general relativity and observation data [14].

Finally the "tidal charge" as five-dimensional theory contribution depends on the black hole mass. So one needs the exact solution to study the workability of the model under discussion. In this aproach a stronger bound on the tidal "charge" value can be found from circular orbits and the last stable circular orbit consideration.

5. Conclusions

Although the presence of the "tidal charge" β changes the geodesic equations, it should not affect the shape and possible types of geodesic for solar masses to avoid the contradiction withe the present observational data. The constraint to the "tidal charge" value β, introduced in [1] and in this paper makes all other possible effects, caused by β unobservable for solar mass and larger black holes. Probably, the "tidal charge" manifests itself in microphysics.

Finally the black hole solution [1] does not contradict to general relativity and observational data for choosen parameter values. A stronger bound on the tidal charge vcalue can be found considering circular orbits and the last stable circular orbit.

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