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Shock wave in the Friedmann-Robertson-Walker space-time

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According to 't Hooft shock waves in the Minkowski space-time, as well as in (A)dS can be used to describe ultra relativistic particles collisions. In this talk the generalization of this construction for the ultrarelativistic particles in the Friedman-Robertson-Walker space-time is presented.

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1. Introduction

The shock gravitation waves can be obtained by Lorentz transformations in the ultrarelativistic limit of the stationary solutions of classical gravitation theory [1]. At 1987 G. 't Hooft [2] proposed to use the Aichelburg-Sexl shock wave [1] as ultrarelativistic particle moving in Minkowski space-time. The problem of shock waves construction in different metrics is related with consideration of ultrarelativistic particles collision models in these metrics. Shock waves in (anti) de Sitter ((A)dS) space-time have been constructed in [3], [4] and used to describe ultra relativistic particles collisions [5], [6] in these backgrounds. Subtleties of obtaining shock waves in dS space are discussed in [7].

The goal of the present paper is to get shock waves in the Friedmann-Robertson-Walker (FRW) metric

$$ds_0^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{1.1}$$

Our starting point is the McVittie metric [8]

$$ds^2 = -\frac{(1-\mu)^2}{(1+\mu)^2}dt^2 + a^2(t)\,(1+\mu)^4\,(dx^2+dy^2+dz^2), \quad \text{where} \tag{1.2}$$

$$\mu = \frac{m}{2a(t)r}, \quad r = \sqrt{x^2+y^2+z^2}, \quad m \text{ is a parameter} \tag{1.3}$$

by choosing $a(t) = e^{Ht}$ we can get the Schwarzschild-de Sitter metric.

For McVittie metric (1.2) with $a(t) = t^n$ the Hubble parameter $H \equiv \dfrac{\dot a}{a} = \dfrac{n}{t}$ [9]. The Ricci scalar $R = 12H^2 + 6\dfrac{1+\mu}{1-\mu}\dot H$. If $\mu = 1$, then we have singularity, which can be interpreted such as cosmological Big Bang singularity. Note, that if $t \to \infty$, the metric (1.2) reduces to the Schwarzschild black hole of mass m [10].

In the present paper we construct the shock wave in FRW space-time with $a(t) = t^n$. In Section 2 we consider relations between coordinates of five-dimensional (5D) Minkowski space-time on non-stationary hyperboloid and FRW space-time. In Section 3 we apply the small mass approximation to the McVittie metric with $a(t) = t^n$. In Section 4 we make boost of the McVittie metric in the small mass approximation. The boosted metric has a divergence and in Section 5 we consider this divergence in the generalized function setting. The shock wave metric is presented in Section 6. In Section 7 we shortly summarize our results.

2. Coordinates relations

Let us remind, that in the five-dimensional (5D) Minkowski space-time

$$dS_{5M}^2 = -dZ_0^2 + dZ_1^2 + dZ_2^2 + dZ_3^2 + dZ_4^2, \tag{2.1}$$

the stationary hyperboloid

$$-Z_0^2 + Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 = A^2 = const \tag{2.2}$$

is 4D de Sitter space-time.

This relation can be generalized for FRW case. The explicit coordinate transformations, which show the equivalence between a four-dimensional spatially flat cosmology and an appropriate sub-manifold in the flat 5D Minkowski space-time, have been presented in [11].

In the present Section we consider the analog to the de Sitter metric (2.1) with non-stationary hyperboloid condition

$$-Z_0^2+Z_1^2+Z_2^2+Z_3^2+Z_4^2=b^2(t) \tag{2.3}$$

and FRW metric (1.1) with

$$a(t)=t^n. \tag{2.4}$$

We suppose that coordinates related by the expressions:

$$\begin{aligned} Z_0 &= \frac{1}{2}a(t)-\frac{1}{2}\frac{b^2(t)}{a(t)}+\frac{1}{2}a(t)(x^2+y^2+z^2), \\ Z_4 &= \frac{1}{2}a(t)+\frac{1}{2}\frac{b^2(t)}{a(t)}-\frac{1}{2}a(t)(x^2+y^2+z^2), \\ Z_1 &= a(t)x, \quad Z_2=a(t)y, \quad Z_3=a(t)z. \end{aligned} \tag{2.5}$$

Substituting (2.5) to (2.1) we get the following condition to $a(t)$ and $b(t)$:

$$-\left(\frac{da(t)}{dt}\frac{b(t)}{a(t)}\right)^2+2\frac{da(t)}{dt}\frac{db(t)}{dt}\frac{b(t)}{a(t)}+1=0. \tag{2.6}$$

For considering case (2.4), from (2.6) we obtain $b^2(t)$:

$$b^2(t)=\frac{t^2}{n(n-2)}. \tag{2.7}$$

3. McVittie metric in the small mass approximation

Let us consider the small mass approximation of (1.2) with (2.4). Let m is a small parameter. To zero and first orders in m^2 the coefficients of (1.2) are as follows:

$$\frac{(1-\mu)^2}{(1+\mu)^2}\approx 1-4\mu, \quad (1+\mu)^4\approx 1+4\mu. \tag{3.1}$$

Thus we can consider

$$ds^2=-(1-4\mu)dt^2+a^2(t)(1+4\mu)(dx^2+dy^2+dz^2) \quad \text{or} \tag{3.2}$$

$$ds^2=ds_0^2+4\mu(ds_0^2+2dt^2). \tag{3.3}$$

Such as $da(t)=nt^{n-1}dt$ and (2.7), dt^2 can be represented by the form:

$$dt^2 = \frac{(da(t))^2}{n^2(n(n-2)b^2(t))^{n-1}}. \tag{3.4}$$

According to (2.5) we have the following expressions for dt^2 and μ:

$$dt^2 = \frac{(d(Z_0+Z_4))^2}{n^2(n(n-2)(-Z_0^2+Z_i^2+Z_4^2))^{n-1}}, \quad \mu = \frac{m}{2\sqrt{Z_i^2}}, \quad i=\overline{1,3}. \tag{3.5}$$

Thus, the metric (3.3) can be considered in the plane coordinates (2.5):

$$ds^2 = dS_{5M}^2 + \frac{2m}{\sqrt{Z_i^2}}\left(dS_{5M}^2 + \frac{2d(Z_0+Z_4)^2}{n^2(n(n-2)(-Z_0^2+Z_i^2+Z_4^2))^{n-1}}\right) \tag{3.6}$$

4. Boost

Consider the Lorentz transform in 5D Minkowski space-time:

$$Z_0 = \gamma(\tilde{Z}_0 + v\tilde{Z}_1), \quad Z_1 = \gamma(\tilde{Z}_1 + v\tilde{Z}_0), \quad \gamma = \frac{1}{\sqrt{1-v^2}}. \tag{4.1}$$

The hyperboloid condition (2.3) is invariant under the Lorentz transformations

$$-Z_0^2 + Z_i^2 + Z_4^2 = -\tilde{Z}_0^2 + \tilde{Z}_i^2 + \tilde{Z}_4^2 \quad \text{and} \quad b^2(t) = \tilde{b}^2(\tilde{t}). \tag{4.2}$$

Let us apply the Lorentz transform (4.1) to metric (3.6):

$$ds_\gamma^2 = d\tilde{S}_{5M}^2 + \frac{2\tilde{m}}{\gamma\sqrt{\gamma^2(v\tilde{Z}_0+\tilde{Z}_1)^2+\tilde{Z}_2^2+\tilde{Z}_3^2}}\left(dS_{5M}^2 + 2\frac{d(\gamma(\tilde{Z}_0+v\tilde{Z}_1)+\tilde{Z}_4)^2}{n^2(n(n-2))(-\tilde{Z}_0^2+\tilde{Z}_i^2+\tilde{Z}_4^2)^{n-1}}\right), \tag{4.3}$$

where $d\tilde{S}_{5M} = -d\tilde{Z}_0^2 + d\tilde{Z}_i^2 + d\tilde{Z}_4^2$, $\tilde{m} = m\gamma$.
The ultrarelativistic limit $\gamma \to \infty$, $v \to 1$ to (4.3) is

$$ds_\gamma^2 \,|_{\upsilon\to 1} \to d\tilde{S}_{5M}^2 + \frac{4\tilde{m}\gamma}{\sqrt{\gamma^2(\tilde{Z}_0+\tilde{Z}_1)^2+\tilde{Z}_2^2+\tilde{Z}_3^2}}\left(\frac{d(\tilde{Z}_0+\tilde{Z}_1)^2}{n^2(n(n-2)(-\tilde{Z}_0^2+\tilde{Z}_i^2+\tilde{Z}_4^2))^{n-1}}\right). \tag{4.4}$$

In the next section we consider the following limit

$$\lim_{\gamma\to\infty} \frac{\tilde{m}\gamma d(\tilde{Z}_0+\tilde{Z}_1)^2}{\sqrt{\gamma^2(\tilde{Z}_0+\tilde{Z}_1)^2+\tilde{Z}_2^2+\tilde{Z}_3^2}} \tag{4.5}$$

in the generalized function meaning.

5. Regularization

For consideration of the limit $\lim\limits_{\gamma\to\infty} \int\limits_{-\infty}^{\infty} \frac{\gamma}{\sqrt{\gamma^2 U^2 + X^2}} f(U)dU$, we can use the following trick:

$$\int\limits_{-\infty}^{\infty} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}} = \int\limits_{-\infty}^{-1} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}} + \int\limits_{1}^{\infty} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}} + \int\limits_{-1}^{1} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}},$$

$$\lim_{\gamma\to\infty}\int\limits_{-\infty}^{\infty} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}} \to \int\limits_{-\infty}^{-1} \frac{f(U)dU}{|U|} + \int\limits_{1}^{\infty} \frac{f(U)dU}{|U|} + \lim_{\gamma\to\infty}\int\limits_{-1}^{1} \frac{\gamma f(U)dU}{\sqrt{\gamma^2U^2+X^2}}$$

Examine the last integral. It is evidently that

$$\int_{-1}^{1} \frac{\gamma f(U)dU}{\sqrt{\gamma^2 U^2+X^2}} = \int_{-1}^{1} \frac{\gamma(f(U)-f(0))dU}{\sqrt{\gamma^2 U^2+X^2}} + \int_{-1}^{1} \frac{\gamma f(0)dU}{\sqrt{\gamma^2 U^2+X^2}}. \tag{5.1}$$

We can just write

$$\lim_{\gamma\to\infty} \int_{-1}^{1} \frac{\gamma}{\sqrt{\gamma^2 U^2+X^2}}[f(U)-f(0)]\,dU \to \int_{-1}^{1} \frac{f(U)-f(0)}{|U|}dU. \tag{5.2}$$

Direct calculation gives that

$$\int_{-1}^{1} \frac{\gamma f(0)dU}{\sqrt{\gamma^2 U^2+X^2}} = \ln\frac{\gamma^2+\gamma\sqrt{\gamma^2+X^2}}{-\gamma^2+\gamma\sqrt{\gamma^2+X^2}} f(0). \tag{5.3}$$

Using the Taylor series for large γ in logarithm argument

$$\ln\frac{\gamma^2+\gamma\sqrt{\gamma^2+X^2}}{-\gamma^2+\gamma\sqrt{\gamma^2+X^2}} \to \ln\frac{4\gamma^2}{X^2}, \tag{5.4}$$

we can write

$$\lim_{\gamma\to\infty}\left[\int_{-\infty}^{\infty} \frac{\gamma}{\sqrt{\gamma^2 U^2+X^2}} f(U)dU - f(0)\ln\gamma^2\right] = -f(0)\ln\frac{X^2}{4} + \int_{-\infty}^{\infty}\left(\frac{1}{|U|}\right)_{reg} f(U)\,dU + \mathscr{O}(\frac{1}{\gamma^2})$$

where

$$\int_{-\infty}^{\infty}\left(\frac{1}{|U|}\right)_{reg} f(U)\,dU \equiv \int_{-1}^{1}\frac{f(U)-f(0)}{|U|}\,dU + \int_{-\infty}^{-1}\frac{1}{|U|}f(U)dU + \int_{1}^{\infty}\frac{1}{|U|}f(U)dU \tag{5.5}$$

or

$$\lim_{\gamma\to\infty}\left[\frac{\gamma}{\sqrt{\gamma^2 U^2+X^2}} - \delta(U)\ln\gamma^2\right] = -\delta(U)\ln\frac{X^2}{4} + \left(\frac{1}{|U|}\right)_{reg} \tag{5.6}$$

6. Shock wave

After the regularization we have the gravitational waves metric

$$ds_\gamma^2 = d\tilde{S}_{5M}^2 + F(\tilde{U})\delta(\tilde{U})d(\tilde{U})^2, \quad \bar{m} = \tilde{m}\ln\gamma^2, \quad \tilde{U} = \tilde{Z}_0 + \tilde{Z}_1, \tag{6.1}$$

where

$$F(\tilde{U}) = \frac{4\bar{m}}{n^2(n(n-2)\tilde{b}^2(\tilde{t}))^{n-1}}, \quad \tilde{b}^2(\tilde{t}) = -\tilde{U}\tilde{V} + \tilde{Z}_2^2 + \tilde{Z}_3^2 + \tilde{Z}_4^2, \quad \tilde{V} = \tilde{Z}_0 - \tilde{Z}_1. \tag{6.2}$$

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Conclusively, the shape of shock wave in 5D Minkowski space-time on non-stationary hyperboloid is

$$F(0)=\frac{4\bar{m}}{n^2(n(n-2)(\tilde{Z}_2^2+\tilde{Z}_3^2+\tilde{Z}_4^2))^{n-1}}. \tag{6.3}$$

Consider metric (6.1) in cosmological coordinates. Suppose that initially, before boost, we had some symmetry $Z_1=Z_2=Z_3$.
Applying the Lorenz transform (4.1) to Z_1, Z_2, Z_3 we obtain

$$\gamma(\tilde{Z}_1+v\tilde{Z}_0)=\tilde{Z}_2=\tilde{Z}_3. \tag{6.4}$$

From here we get

$$\tilde{b}^2(\tilde{t})=\tilde{a}^2(\tilde{t})\tilde{l} \quad \text{or} \quad \tilde{a}^2=\frac{\tilde{b}^2(\tilde{t})}{\tilde{l}}, \tag{6.5}$$

where $\tilde{b}^2(\tilde{t})=b^2(t)=\dfrac{t^2}{n(n-2)}$ and $\tilde{l}$ can has the following forms

$$\tilde{l}=\tilde{x}+\frac{v}{2}(1+\tilde{x}^2+\tilde{y}^2+\tilde{z}^2)-\frac{\tilde{y}+\tilde{z}}{2\gamma}, \tag{6.6}$$

$$\tilde{l}=\tilde{x}+\frac{v}{2}(1+\tilde{x}^2+2\tilde{y}^2)-\frac{\tilde{y}}{2\gamma}, \tag{6.7}$$

$$\tilde{l}=\tilde{x}+\frac{v}{2}(1+\tilde{x}^2+2\tilde{z}^2)-\frac{\tilde{z}}{\gamma}. \tag{6.8}$$

Accordingly, we obtain that $\tilde{U}=\dfrac{\pm t}{\sqrt{n(n-2)\tilde{l}}}\left(\tilde{x}+\dfrac{1+\tilde{r}^2-\tilde{l}}{2}\right)$, $\tilde{r}^2=\tilde{x}^2+\tilde{y}^2+\tilde{z}^2$.
Thus, the wave shape in cosmological coordinates has the form:

$$F(0)=\frac{4\bar{m}}{n^2t^{2(n-1)}}. \tag{6.9}$$

7. Summarizing

It is proposed to use the boosted McVittie metric such as model of ultrarelativistic particle in the Friedmann-Robertson-Walker space-time with $a(t)=t^n$. The shock wave corresponding ultrarelativistic particle in the Friedmann-Robertson-Walker space-time is constructed.

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