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Stable cosmological solutions and superpotential method in two-field models

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The stability of isotropic cosmological solutions for two-field models in the Bianchi I metrics is considered. We have proved that the sufficient conditions for Lyapunov stability in the Friedmann–Robertson–Walker metric are sufficient for the stability under anisotropic perturbations in the Bianchi I metric as well. The standard way to construct cosmological models with exact solutions in the Friedmann–Robertson–Walker metric is the superpotential method. We have used the superpotential method to construct stable kink-type solutions and obtained conditions on superpotential, which are sufficient conditions for the Lyapunov stability. We analyze the stability of isotropic kink-type solutions for quintom models related to the string field theory.

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1. Introduction

Contemporary observations [1] give strong support that the uniformly distributed cosmic fluid with negative pressure, the so-called dark energy, currently dominates in the Universe. The dark energy state parameter[1] $w_{\rm DE} = -1 \pm 0.2$. Strong restrictions on the anisotropy were found using observations [2, 3], and it was also shown that the Universe is spatially flat at large scales. The recent analysis of the observation data [4, 5] (see also reviews [6, 7]) indicates that the varying in time $w_{\rm DE}$ gives a better fit than $w_{\rm DE} \equiv -1$, corresponding to the cosmological constant.

The standard way to obtain an evolving state parameter is to include scalar fields into a cosmological model. Two-field models with the crossing of the cosmological constant barrier $w_{\rm DE} = -1$ are known as quintom models and include one phantom scalar field and one ordinary scalar field. Quintom models are being actively studied at present time [6, 7, 8, 9, 10, 11, 12].

The cosmological models with the crossing of the cosmological constant barrier violate the null energy condition (NEC). The NEC violating models can admit classically stable solutions in the Friedmann–Robertson–Walker (FRW) cosmology. In particular, there are classically stable solutions for self-interacting ghost models with minimal coupling to gravity. Moreover, there exists an attractor behavior in a class of the phantom cosmological models [13, 10]. The standard way to analyse the stability of quintom models [8, 10, 11] (see also [7]) includes the change of variables. In the case of exponential potentials this change is useful [8], because it transforms some depending on time solutions into fixed points of new system. For an arbitrary potential it is possible to get the stability conditions, obtained in [10], without any change of variables [14]. We show that the obtained conditions are sufficient for stability not only in the FRW metric, but also in the Bianchi I metric. For one-field models in the Bianchi I metric, the sufficient conditions for stability of isotropic solutions, which tend to fixed points, have been obtained in [15].

In [9, 16] the superpotential method has been used to construct quintom models with exact solutions. In this paper we express the stability conditions in terms of the superpotential and use this method for construction of two-field models with stable exact solutions. We also check the stability of solutions, obtained in the string field theory (SFT) inspired quintom models [9, 16].

2. The Bianchi I cosmological model with scalar and phantom scalar fields

We consider a cosmological model with two scalar fields ϕ_1 and ϕ_2 described by the action

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}-\left(\frac{C_1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{C_2}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi-V(\phi,\xi)\right)\right), \qquad (2.1)$$

where the potential $V(\phi,\xi)$ is a twice continuously differentiable function, which can include the cosmological constant Λ, G_N is the Newtonian gravitational constant, ϕ and ξ are either scalar or phantom scalar fields in dependence on signs of constants C_1 and C_2. The metric tensor $g_{\mu\nu} = \mathrm{diag}(-1, a_1^2(t), a_2^2(t), a_3^2(t))$ in the Bianchi I metric [17]. It is convenient to express the functions

[1]To specify different components of the cosmic fluid one typically uses a phenomenological relation $p = w\rho$ between the pressure (Lagrangian density) p and the energy density ρ corresponding to each component of the fluid. The function w is called the state parameter.

$a_i(t)$ in terms of new functions a and β_i (we use notations from [18]), which are subject to the constraint $\beta_1+\beta_2+\beta_3=0$. One has the following relations

$$a_i(t)=a(t)e^{\beta_i(t)}, \qquad \text{hence,} \qquad a(t)=(a_1(t)a_2(t)a_3(t))^{1/3}. \tag{2.2}$$

Following [18], we introduce the shear variable $\sigma^2\equiv\dot\beta_1^2+\dot\beta_2^2+\dot\beta_3^2$ and get the Einstein equations:

$$\ddot\beta_i=-3H\dot\beta_i, \qquad \frac{d}{dt}\left(\sigma^2\right)=-6H\sigma^2, \tag{2.3}$$

$$3H^2-\frac{1}{2}\sigma^2=8\pi G_N\rho, \qquad 2\dot H+3H^2+\frac{1}{2}\sigma^2=-8\pi G_N p, \tag{2.4}$$

where $H\equiv\dot a/a$, a dot denotes a time derivative and

$$\rho=\frac{C_1}{2}\dot\phi^2+\frac{C_2}{2}\dot\xi^2+V(\phi,\xi), \qquad p=\frac{C_1}{2}\dot\phi^2+\frac{C_2}{2}\dot\xi^2-V(\phi,\xi). \tag{2.5}$$

We also obtain from action (2.1) the following equations:

$$\dot\phi=\psi, \quad \dot\psi=-3H\psi-\frac{1}{C_1}\frac{\partial V}{\partial\phi}, \qquad \dot\xi=\zeta, \quad \dot\zeta=-3H\zeta-\frac{1}{C_2}\frac{\partial V}{\partial\xi}. \tag{2.6}$$

Summing equations (2.4) we obtain

$$\dot H=-3H^2+8\pi G_N V(\phi,\xi). \tag{2.7}$$

3. Sufficient conditions for Lyapunov stability of fixed point

Let us assume that the fields ϕ and ξ tend to finite limits at $t\to+\infty$. System (2.6)-(2.7) has a fixed point $y_f=(H_f,\phi_f,\psi_f,\xi_f,\zeta_f)$ if and only if $\psi_f=0$, $\zeta_f=0$,

$$V'_\phi\equiv\frac{\partial V}{\partial\phi}(\phi_f,\xi_f)=0, \quad V'_\xi\equiv\frac{\partial V}{\partial\xi}(\phi_f,\xi_f)=0, \quad H_f^2=\frac{8\pi G_N}{3}V(\phi_f,\xi_f). \tag{3.1}$$

The stability of a kink or lump solution means the stability of the fixed point that the solution tends to. To analyse the stability of y_f we study the stability of this fixed point for the corresponding linearized system of equations and use the Lyapunov theorem [19]. In the neighborhood of y_f

$$H(t) = H_f+\varepsilon h_1(t)+\mathscr{O}(\varepsilon^2), \tag{3.2}$$

$$\phi(t) = \phi_f+\varepsilon\phi_1(t)+\mathscr{O}(\varepsilon^2), \quad \psi(t)=\varepsilon\psi_1(t)+\mathscr{O}(\varepsilon^2), \tag{3.3}$$

$$\xi(t) = \xi_f+\varepsilon\xi_1(t)+\mathscr{O}(\varepsilon^2), \quad \zeta(t)=\varepsilon\zeta_1(t)+\mathscr{O}(\varepsilon^2), \tag{3.4}$$

where ε is a small parameter. To first order in ε we obtain the following system of equations

$$\dot h_1(t) = -6H_f h_1(t), \tag{3.5}$$

$$\dot\phi_1(t) = \psi_1(t), \quad \dot\psi_1(t)=-3H_f\psi_1(t)-\frac{1}{C_1}\left(V''_{\phi\phi}\phi_1(t)+V''_{\phi\xi}\xi_1(t)\right), \tag{3.6}$$

$$\dot\xi_1(t) = \zeta_1(t), \quad \dot\zeta_1(t)=-3H_f\zeta_1(t)-\frac{1}{C_2}\left(V''_{\xi\phi}\phi_1(t)+V''_{\xi\xi}\xi_1(t)\right), \tag{3.7}$$

where $V''_{\phi\phi}\equiv\frac{\partial^2V}{\partial\phi^2}(\phi_f,\xi_f)$, $V''_{\xi\xi}\equiv\frac{\partial^2V}{\partial\xi^2}(\phi_f,\xi_f)$, $V''_{\phi\xi}\equiv\frac{\partial^2V}{\partial\phi\partial\xi}(\phi_f,\xi_f)$. Analyzing solutions of (3.5)–(3.7) we get the sufficient stability conditions of the fixed point [14]:

$$H_f>0, \qquad \frac{V''_{\xi\xi}}{C_2}+\frac{V''_{\phi\phi}}{C_1}>0, \qquad \frac{V''_{\xi\xi}V''_{\phi\phi}}{C_1C_2}>\frac{{V''_{\phi\xi}}^2}{C_1C_2}. \tag{3.8}$$

4. Construction of stable solutions via the superpotential method

Let us consider the superpotential method for two-field models [20, 9, 16] in the FRW metric. We assume that the Hubble parameter $H(t)$ is a function (superpotential) of $\phi(t)$ and $\xi(t)$: $H(t) = W(\phi(t), \xi(t))$, and that functions $\phi(t)$ and $\xi(t)$ are solutions of the following system of two ordinary differential equations $\dot{\phi} = F(\phi, \xi)$, $\dot{\xi} = G(\phi, \xi)$, where $F(\phi, \xi)$ and $G(\phi, \xi)$ are such continuously differentiable function that $\frac{\partial F}{\partial \xi} = \frac{C_2}{C_1}\frac{\partial G}{\partial \phi}$. It is easy to check that equations (2.6)–(2.7) are solved provided the following relations are satisfied:

$$\frac{\partial W}{\partial \phi} = -4\pi G_N C_1 \dot{\phi}, \qquad \frac{\partial W}{\partial \xi} = -4\pi G_N C_2 \dot{\xi}, \tag{4.1}$$

$$V = \frac{3}{8\pi G_N} W^2 - \frac{1}{32\pi^2 G_N^2}\left(\frac{1}{C_1}\left(\frac{\partial W}{\partial \phi}\right)^2 + \frac{1}{C_2}\left(\frac{\partial W}{\partial \xi}\right)^2\right). \tag{4.2}$$

The goal of this subsection is to obtain conditions on the superpotential W, which are equivalent to conditions (3.8) for the corresponding potential V. At the fixed point $y_f = (H_f, \phi_f, \psi_f, 0, 0)$

$$W_f \equiv W(\phi_f, \xi_f) = H_f, \qquad W'_\phi \equiv \frac{\partial W}{\partial \phi}(\phi_f, \xi_f) = 0, \qquad W'_\xi \equiv \frac{\partial W}{\partial \xi}(\phi_f, \xi_f) = 0. \tag{4.3}$$

It is easy to see that from (4.3) it follows that $V'_\phi = 0$, $V'_\xi = 0$. The condition $H_f > 0$ is equivalent to $W_f > 0$. If $W''_{\phi\xi} = 0$, then $V''_{\phi\xi} = 0$ and conditions (3.8) are

$$\left(12\pi G_N C_1 W_f - W''_{\phi\phi}\right) W''_{\phi\phi} > 0, \qquad \left(12\pi G_N C_2 W_f - W''_{\xi\xi}\right) W''_{\xi\xi} > 0. \tag{4.4}$$

In the general case we have the sufficient stability conditions in the following form

$$12 C_1 C_2 \pi G_N (C_2 W''_{\phi\phi} + C_1 W''_{\xi\xi}) W_f > C_2^2 \left(W''_{\phi\phi})^2 + 2C_1 C_2 (W''_{\phi\xi}\right)^2 + C_1^2 (W''_{\xi\xi})^2. \tag{4.5}$$

$$\begin{aligned}&\left(144 C_1 C_2 \pi^2 G_N^2 W_f^2 - 12 C_1 C_2 \pi G_N (C_2 W''_{\phi\phi} + C_1 W''_{\xi\xi}) W_f + W''_{\xi\xi} W''_{\phi\phi} - (W''_{\phi\xi})^2\right) \times \\ &\times \left(W''_{\xi\xi} W''_{\phi\phi} - (W''_{\phi\xi})^2\right) > 0.\end{aligned} \tag{4.6}$$

5. String field theory inspired cosmological models

An interest in cosmological models coming from open string field theories [21] is caused by a possibility to get solutions rolling from a perturbative vacuum to the true one. The dark energy model [21] assumes that our Universe is a slowly decaying D3-brane and its dynamics is described by the open string tachyon mode. For the open fermionic NSR string with the GSO$(-)$ sector in a reasonable approximation, one gets the Mexican hat potential for the tachyon field [22]. Rolling of the tachyon from the unstable perturbative extremum towards this minimum describes, according to the Sen conjecture [22], the transition of an unstable D-brane to a true vacuum. In fact one gets a nonlocal potential with a string scale as a parameter of nonlocality. After a suitable field redefinition the potential becomes local, meanwhile, the kinetic term becomes non-local. This nonstandard kinetic term has a so-called phantomlike behavior and can be approximated by a phantom kinetic term. It has been found that the open string tachyon behavior is effectively modelled by a scalar field with a negative kinetic term [23]. The back reaction of this brane is incorporated in the dynamics of the closed string tachyon. The scalar field ξ comes from the closed string sector and its effective local description is given by an ordinary kinetic term.

In the papers [9, 16] quintom models ($C_1 = -C_2 < 0$) with effective the sixth degree polynomial potentials $V(\phi,\xi)$ have been considered:

$$V(\phi,\xi) = \sum_{k=0}^{6}\sum_{j=0}^{6-k} c_{kj}\phi^k\xi^j, \qquad V(\phi,\xi) = V(-\phi,-\xi). \tag{5.1}$$

From the SFT we can also assume asymptotic conditions for solutions. We assume that the phantom field $\phi(t)$ smoothly rolls from the unstable perturbative vacuum ($\phi = 0$) to a nonperturbative one, for example $\phi = A$, and stops there. The field $\xi(t)$ corresponds to close string and is expected to go asymptotically to zero in the infinite future. In other words we plan to analyse the stability of solutions, which tends to a fixed point with $\phi_f = A$ and $\xi_f = 0$.

To construct the sixth degree even polynomial potential $V(\phi,\xi)$ we can choose $W(\phi,\xi)$ as an arbitrary third degree odd polynomial:

$$W_3(\phi,\xi) = 4\pi G_N\left(a_{1,0}\phi + a_{3,0}\phi^3 + a_{0,1}\xi + a_{0,3}\xi^3 + a_{2,1}\phi^2\xi + a_{1,2}\phi\xi^2\right), \tag{5.2}$$

where $a_{i,j}$ are constants. Using asymptotic conditions: $\phi(+\infty) = A$, $\xi(+\infty) = 0$, $\dot\phi(+\infty) = \dot\xi(+\infty) = 0$ we obtain $a_{1,0} = -3a_{3,0}A^2$, $a_{0,1} = -a_{2,1}A^2$. So, we get the following system of equations:

$$\dot\phi = \frac{1}{C_2}\left(3a_{3,0}(\phi^2 - A^2) + 2a_{2,1}\phi\xi + a_{1,2}\xi^2\right), \quad \dot\xi = -\frac{1}{C_2}\left(a_{2,1}(\phi^2 - A^2) + 3a_{0,3}\xi^2 + 2a_{1,2}\phi\xi\right). \tag{5.3}$$

At the fixed point $\phi_f = A$, $\xi_f = 0$ we have $W_f = -8\pi G_N a_{3,0}A^3$. So, the conditions (3.8) are as follows:

$$a_{3,0}A < 0, \tag{5.4}$$

$$2a_{2,1}^2 - a_{1,2}^2 - 12\pi G_N C_2 A^2 a_{3,0}a_{1,2} + 9(4\pi G_N C_2 A^2 - 1)a_{3,0}^2 > 0, \tag{5.5}$$

$$(3a_{3,0}a_{1,2} - a_{2,1}^2)\left(3(4\pi G_N A^2 C_2 - 1)a_{3,0}a_{1,2} - 36\pi G_N A^2 C_2(4\pi G_N A^2 C_2 - 1)a_{3,0}^2 + a_{2,1}^2\right) < 0. \tag{5.6}$$

If $a_{2,1} = 0$, then $V''_{\phi\xi} = 0$ and the sufficient conditions for the stability have the following form

$$a_{3,0}A < 0, \qquad 4\pi G_N C_2 A^2 > 1, \qquad a_{1,2}(a_{1,2} + 12\pi G_N C_2 a_{3,0}A^2) < 0. \tag{5.7}$$

The case of superpotential $W_3(\phi,\xi)$ with $a_{2,1} = 0$ and $a_{0,3} = 0$ has been considered in [9, 16]. In this case the system (5.3) has the following form

$$\phi = -\frac{C_2}{2a_{1,2}}\frac{\dot\xi}{\xi}, \qquad \ddot\xi = \frac{(2a_{1,2} - 3a_{3,0})}{2a_{1,2}\xi}\dot\xi^2 + \frac{2a_{1,2}}{C_2^2}\xi\left(3a_{3,0}A^2 - a_{1,2}\xi^2\right). \tag{5.8}$$

For some values of parameters the general solution of (5.8) can be presented in the explicit form [16]. Indeed, at $a_{3,0}/a_{1,2} = -1/3$ system (5.8) has the following general solution:

$$\phi_s(t) = \frac{A\left(C_2^2 e^{4a_{1,2}At/C_2} - 64a_{1,2}^4 C_2^2 A^2 D_1^2 - 4a_{1,2}^2 A^2 D_2^2\right)}{\left(C_2 e^{2a_{1,2}At/C_2} - 2D_2 a_{1,2}A\right)^2 + 64D_1^2 a_{1,2}^4 C_2^2 A^2}, \tag{5.9}$$

$$\xi_s(t) = \frac{16 D_1 C_2^2 a_{1,2}^2 A^2 e^{2a_{1,2}At/C_2}}{\left(C_2 e^{2a_{1,2}At/C_2} - 2D_2 a_{1,2} A\right)^2 + 64 D_1^2 a_{1,2}^4 C_2^2 A^2}. \tag{5.10}$$

Let us analyse the stability of the exact solution. One can see that $\phi_s(t)$ and $\xi_s(t)$ are continuous functions, which tend to a fixed point at $t \to \infty$. Therefore, the obtained exact solution is attractive if and only if the fixed point is asymptotically stable. At $a_{3,0} = -a_{1,2}/3$ we obtain that three stability conditions (5.7) transform into two independent conditions $a_{1,2}A > 0$ and $4\pi G_N A^2 C_2 > 1$.

6. Conclusion

We have analysed the stability of isotropic solutions for two-field models. Using the Lyapunov theorem we have found sufficient conditions of stability of kink-type and lump-type isotropic solutions for two-field models in the Bianchi I metric. The obtained results allow us to prove that the exact solutions, found in string inspired phantom models [9, 16], are stable.

We have presented the algorithm for construction of kink-type and lump-type isotropic exact stable solutions via the superpotential method. In particular we have formulated the stability conditions in terms of superpotential.

Our study of the stability of isotropic solutions for quintom models in the Bianchi I metric shows that the NEC is not a necessary condition for classical stability of isotropic solutions. In this paper we have shown that the models [9, 16] have stable isotropic solutions and that large anisotropy does not appear in these models. It means that considered models are acceptable, because they do not violate limits on anisotropic models, obtained from the observations [2, 3].

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