PROCEEDINGS OF SCIENCE

Equations of Markovian processes for compatible events and quantum physics.

A. A. Biryukov
Samara State University, Samara, Russia
E-mail: biryukov@ssu.samara.ru

The equations for a number of Markovian process models in combined event space are built. It is shown that equations of the pair - compatible events model are equivalent to to quantum mechanics equations.

The XIXth International Workshop on High Energy Physics and Quantum Field Theory, QFTHEP2010
September 08-15, 2010
in sense Golitsyno, Moscow, Russia

PoS(QFTHEP2010)085

1. The development of innovative technologies particularly in the questions of information processing and transmission stimulates further research of probabilistic model of physical systems and stochastic processes in them. The research of the mathematical structure of the quantum theory from the viewpoint of the probability theory provokes a big interest. A number of works can be offered where original results and surveys of this sphere are presented, for example [1] – [5].

At present, markovian processes are widely used for random process description [6] – [8]. Consider a certain physical system to construct the equations of Markovian processes, the state of which state at each moment of time t is characterized by a set N of random events, which can be realized with the certain probabilities. Let's designate these events at the moment $t=0$ as A_k, $k=1,\dots,N$ and at the moment $t>0$ as B_j, $j=1,2,\dots,N$.

The first rule of the Markovain process theory is the statement that the probability $P^t(B_j)$ of the event occurrence B_j at the moment t is defined by the realization of one of the events A_k at the previous moment $t=0$, that is

$$P^t(B_j) = P^t\big(B_j \cap \big(\bigcup_{k=1}^{N} A_k\big)\big). \tag{1}$$

The second rule is the condition, that the events A_k, $k=1,2,\dots,N$ are incompatible with each other, that is

$$P\big(\bigcup_{k=1}^{N} A_k\big) = \sum_{k=1}^{N} P(A_k). \tag{2}$$

Taking into account (2) we'll write down the equation (1) as

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j \cap A_k) \tag{3}$$

The formula (3) is the equation of Markovian process for incompatible state determined by events A_k, B_j. It can be conveniently presented as

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j|A_k)P^0(A_k). \tag{4}$$

Here $P^0(A_k)$ is the probability of event occurrence A_k at the moment $t=0$, where

$$P^t(B_j|A_k) = \frac{P^t(B_j \cap A_k)}{P^0(A_k)} \tag{5}$$

is the conditional probability of event realization B_j at the moment t under the condition of event realization A_k at the moment $t=0$ (the probability of transition from event A_k to event B_j for an interval of time t).

In this work the models of stochastic processes in space of random events are offered, in which the Markovian condition (1) is preserved and the condition (2) of events incompatibility with each other at each moment of time of system supervision is removed.

We'll show, that the equations for one of the presented models are compatible with the equations of the quantum mechanics. The construction of Markovian chains equations for compatible states was earlier considered in work [9].

2. Let's consider the evolving physical system, which is characterized by combined random events A_k, $k = 1,\ldots,N$. The probability of one of them at the time moment (specifically, at $t = 0$) is defined by (see e. g. [7],[8]):

$$P\Big(\bigcup_{k=1}^{N} A_k\Big) = \sum_{k=1}^{N} P(A_k) - \sum_{k<l}^{N} P\big(A_k \cap A_l\big) +$$

$$+ \sum_{k<m<l}^{N} P\big(A_k \cap A_l \cap A_m\big) + \ldots + (-1)^{N-1} P\Big(\bigcap_{k=1}^{N} A_k\Big). \tag{6}$$

Taking into account (6) the equation (1) gives the equation for Markovian processes with compatible states, which determine the possibility $P^t(B_j)$

$$P^t(B_j) = \sum_{k=1}^{N} P^t\big(B_j \cap A_k\big) - \sum_{k<l=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_j\big)\big) +$$

$$+ \sum_{k<l<m=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_l \cap A_m\big)\big) + \ldots + (-1)^{N-1} P^t\Big(B_j \cap \Big(\bigcap_{k=1}^{N} A_k\Big)\Big). \tag{7}$$

In accordance with definition of conditional probabilities it is convenient to write equation (7) in the form

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j|A_k) P^0(A_k) - \sum_{k<l=1}^{N} P^t(B_j|A_k A_l) P^0(A_k A_l) +$$

$$+ \sum_{k<l<m=1}^{N} P^t(B_j|A_k A_l A_m) P^0(A_k A_l A_m) -$$

$$- \ldots (-1)^{N-1} P^t(B_j|A_1 A_2 \ldots A_N) P^0(A_1 A_2 \ldots A_N), \tag{8}$$

where $P^0(A_k)$, $P^0(A_k A_l)$, $P^0(A_k A_l A_m \ldots)$, $P^0(A_1 \ldots A_N)$ are the probabilities of events A_k, $A_k A_l$, $A_k A_l A_m \ldots$, $A_1 \ldots A_N$ correspondingly at initial time moment $t = 0$;

$$\begin{aligned} P^t(B_j|A_k) &= \frac{P^t\big(B_j \cap A_k\big)}{P^0(A_k)} \\ P^t(B_j|A_k A_l) &= \frac{P^t\big(B_j \cap \big(A_k \cap A_l\big)\big)}{P^0\big(A_k \cap A_l\big)} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ P^t(B_j|A_1 \ldots A_N) &= \frac{P^t\big(B_j \cap \big(A_1 \cap \ldots \cap A_N\big)\big)}{P^0\big(A_1 \cap A_2 \cap \ldots \cap A_N\big)} \end{aligned} \tag{9}$$

– conditional probability of event B_j at $t > 0$ when events A_k, $A_k \cap A_l$, $\ldots$ are correspondingly realized at $t = 0$ (probability of transition from events A_k, $A_k \cap A_l$, $\ldots$ to B_j during time interval t).

3. Let's consider physical system at any time moment, which similarly to the system from İ2, is characterized by compatible random events A_k, $k = 1,2,\ldots,N$. But there are conditions for events A_k^-, $k = 1,2,\ldots,N$ which are realized in the system. The events A_k^- are

$$A_k^- = A_k/\big(\bigcup_{l=1}^{N}(A_k \cap A_l)l \neq k\big), \quad k = 1,2,\ldots,N. \tag{10}$$

For the case of $N = 2$ we have

$$\begin{aligned} A_1^- &= A_1/\big(A_1 \cap A_2\big) = A_1/A_2 \\ A_2^- &= A_2/\big(A_2 \cap A_1\big) = A_2/A_1 \end{aligned} \tag{11}$$

The probability of events unification (11) is determined by equation [8]

$$P\big(A_1^- \cup A_2^-\big) = P(A_1) + P(A_2) - 2P\big(A_1 \cap A_2\big). \tag{12}$$

With mathematical induction method the probability of unification of N events can be proved to be presented like

$$\begin{aligned} P\big(\bigcup_{k=1}^{N} A_k^-\big) &= \sum_{k=1}^{N} P(A_k) - 2\sum_{k<l=1}^{N} P\big(A_k \cap A_l\big) + \\ &+ 4\sum_{k<l<m=1}^{N} P\big(A_k \cap A_l \cap A_m\big) - \ldots - (-2)^{N-1} P\big(\bigcap_{k=1}^{N} A_k\big) \end{aligned} \tag{13}$$

Consider Markovian stochastic process in this system, going under the next conditions. Random events B_j occur at $t > 0$, if one of the events A_k^- took place at $t = 0$. In accordance with the former condition the probability of B_j is defined as

$$P^t(B_j) = P^t\big(B_j \cap \big(\bigcup_{k=1}^{N} A_k^-\big)\big). \tag{14}$$

Using equation (13), the equation (14) can be written in the form

$$\begin{aligned} P^t(B_j) &= \sum_{k=1}^{N} P^t\big(B_j \cap A_k\big) - 2\sum_{k<l=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_l\big)\big) + \\ &+ 4\sum_{k<l<m=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_l \cap A_m\big)\big) + \ldots + \\ &+ (-2)^{N-1} P^t\big(B_j \cap \big(A_1 \cap \ldots \cap A_N\big)\big). \end{aligned} \tag{15}$$

Substituting transition probabilities (9) into equation (15) we derive

$$\begin{aligned} P^t(B_j) &= \sum_{k=1}^{N} P^t(B_j|A_k)P^0(A_k) - 2\sum_{k<l=1}^{N} P^t(B_j|A_kA_l)P^0(A_kA_l) + \\ &+ 4\sum_{k<l<m=1}^{N} P^t(B_j|A_kA_lA_m)P^0(A_kA_lA_m) + \ldots + \end{aligned}$$

$$+(-2)^{N-1}P^t(B_j|A_1\dots A_N)P^0(A_1\dots A_N), \tag{16}$$

where $P^0(A_k)$, $P^0(A_kA_l)$, $P^0(A_kA_lA_m)$, … are probabilities of events realization at initial time moment $t=0$.

Equations (15), (16) describe markovian processes in compatible events space of some physical system.

4. The model of the process, describing the system evolution, where both compatible events A_k, $k=1,2,\dots,N$ and events A_k^+ are realized, is an alternative to the model of Markovian process that is given in İ3.

Let us consider the system, where there are only two compatible events A_1 and A_2 and two events A_1^+, A_2^+ correspondingly, and specify the properties of introduced events. The probability of events unification $A_1^+\cup A_2^+$ is determined as

$$P\big(A_1^+\cup A_2^+\big)=P(A_1)+P(A_2)+2P\big(A_1\cap A_2\big). \tag{17}$$

On the basis of expression (17) formula for probability of unification of N events A_k^+ is found by the mathematical induction method

$$P\big(\bigcup_{i=1}^{N}A_i^+\big)=\sum_{k=1}^{N}P(A_k)+2\sum_{k<l=1}^{N}P\big(A_k\cap A_l\big)+$$

$$+4\sum_{k<l<m=1}^{N}P\big(A_k\cap A_l\cap A_m\big)+\dots+(2)^{N-1}P\big(\bigcap_{i=1}^{N}A_i\big). \tag{18}$$

The existence of the events A_k^+ and the expression (17) for the probability of their union cannot be explained in the framework of the Kolmogorov's axiomatics.This fact must be taken as a new statement.

The equations (17), (18) may be derived taken into account the structure of events A_k^+ model without the limits of Kolmogorov's axiomatics. Let us define the events A_k^+

$$A_k^+=A_k\cup\big(\bigcup_{l=1}^{N}\big(A_k\cap A_l\big),l\neq k\big),\quad k=1,2,\dots,N, \tag{19}$$

under the conditions:

- The events A_k and $\big(\bigcup_{l=1}^{N}\big(A_k\cap A_l\big),l\neq k\big),\quad k=1,2,\dots,N$, are not compatible, i.e. the equations

$$A_k^+\neq A_k,\ (A_k\cap A_l)\neq 0,\ k\neq l=1,2,\dots,N \tag{20}$$

take place simultaneously;

- The events $(A_k\cap A_l),(A_l\cap A_k)$, under the condition $k\neq l$ are not compatible, i.e. the equations

$$(A_k\cap A_l)\cup(A_l\cap A_k)\neq A_k\cap A_l,\ (A_k\cap A_l)\cup(A_l\cap A_k)\neq(A_l\cap A_k),k\neq l=1,2,\dots,N \tag{21}$$

hold for their union;

- The probabilities of events $(A_k\cap A_l),(A_l\cap A_k)$, under the condition $k\neq l$ are invariably equal

$$P\big(A_k\cap A_l\big)=P\big(A_l\cap A_k\big),\ k,l=1,2,\dots,N. \tag{22}$$

One thing is clear, this definition of events A_k^+ should be considered as additional postulate in Kolmogorov axiomatics. Using the definition (19) and conditions (20) – (22) one can easily prove the formula (17) for $N=2$.

Let us consider in this model the Markovian process, where probability of event B_j at $t>0$ is realized if one or more events A_i^+, $i=1,2,\ldots,N$ took place at $t=0$, i.e.

$$P^t(B_j) = P^t\big(B_j \cap \big(\bigcup_{i=1}^{N} A_k^+\big)\big). \tag{23}$$

With using (18) the equation (23)can be written in the form

$$P^t(B_j) = \sum_{k=1}^{N} P^t\big(B_j \cap A_k\big) + 2\sum_{k<l=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_l\big)\big) +$$

$$+4\sum_{k<l<m=1}^{N} P^t\big(B_j \cap \big(A_k \cap A_l \cap A_m\big)\big) + \ldots + 2^{N-1} P^t\big(B_j \cap \big(A_1 \cap \ldots \cap A_N\big)\big). \tag{24}$$

Equation (24) (similarly to equation (15)) can be transformed by adding the event possibilities at the initial time and transition possibilities

$$P^t(B_j) = \sum_{i=1}^{N} P^t(B_j|A_i)P^0(A_i) + 2\sum_{i<j=1}^{N} P^t(B_j|(A_iA_j))P^0(A_iA_j) +$$

$$+4\sum_{i<j<k=1}^{N} P^t(B_j|(A_iA_jA_k))P^0(A_iA_jA_k) + \ldots +$$

$$+2^{N-1}P^t(B_j|(A_1\ldots A_N))P^0(A_1\ldots A_N). \tag{25}$$

Equations (24),(25) of Markovian processes for compatible events in this model of physical system are of the same structure as equations (15), (16) under the condition that all terms are positive.

5. Let us derive the equation of Markovian process in compatible event space for a physical system, associating the models mentioned before in İ 3 and İ 4.

Consider both compatible events A_k, $k=1,2,\ldots,N$ and events A_k^-, $k=1,2,\ldots,N_1$ and A_k^+, $k=1,2,\ldots,N_2$ $(N_1+N_2=N)$ in order to describe a physical system state at every time moment.

For the case of $N=2$ the possibilities of events unification $P(A_1^- \cup A_2^-)$, $P(A_1^+ \cup A_2^+)$ are defined by expressions (12), (17). It is provable that probability of events A_1^- and A_2^+ unification $(P(A_1^- \cup A_2^+))$ is given by the equation

$$P(A_1^- \cup A_2^+) = P(A_1^+ \cup A_2^+) = P(A_1) + P(A_2). \tag{26}$$

Denote $\tilde{A}_k$ the event, which can be either event A_k^- or event A_k^+. Using this denotation formulae (12), (17), (26) can be written as

$$P(\tilde{A}_1 \cup \tilde{A}_2) = P(A_1) + P(A_2) + 2g_{12}P(A_1 \cap A_2), \tag{27}$$

where $g_{12} = -1$ or $+1$ or 0 depending on event choice $\tilde{A}_1, \tilde{A}_2$.

The probability of unification of N events $\tilde{A}_k$ is given by formula that generalizes equation (27):

$$P\big(\bigcup_{k=1}^{N}\tilde{A}_k\big) = \sum_{k=1}^{N} P(A_k) + 2\sum_{k<l=1}^{N} g_{kl}P(A_k\cap A_l) +$$
$$+4\sum_{k<l<m=1}^{N} g_{klm}P(A_k\cap A_l\cap A_m) + \ldots, \tag{28}$$

where g_{kl}, g_{klm} and other multipliers take one of the values 0, $+1$, -1 depending on the selected events $\tilde{A}_k, \tilde{A}_l, \tilde{A}_m, \ldots$.

Markovian process in such a physical system will be described via equation that gives the probability of event B_j at $t>0$ if one of the events $\tilde{A}_k$, $k=1,2,\ldots,N$ took place at $t=0$

$$P^t(B_j) = P^t\big(B_j\cap\big(\bigcup_{k=1}^{N}\tilde{A}_k\big)\big). \tag{29}$$

With using (28) the equation (29) for Markovian process can be written as

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j\cap A_k) + 2\sum_{k<l=1}^{N} g_{jkl}P^t(B_j\cap(A_k\cap A_l)) +$$
$$+4\sum_{k<l<m}^{N} g_{jklm}P^t(B_j\cap(A_k\cap A_l\cap A_m)) + \ldots. \tag{30}$$

In order to simplify equations (30) we introduce probabilities of transitions from events A_k at $t=0$ to events B_j at $t>0$ (9) and probabilities of events A_k at initial time moment $t=0$ ($P^0(A_k)$, $P^0(A_kA_l),\ldots$).

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j|A_k)P^0(A_k) + 2\sum_{k<l=1}^{N} g_{jkl}P^t(B_j|A_kA_l)P^0(A_kA_l) +$$
$$+4\sum_{k<l<m}^{N} g_{jklm}P^t(B_j|A_kA_lA_m)P^0(A_kA_lA_m) + \ldots. \tag{31}$$

6. Equations (8), (16), (25), (31) describe Markovian processes in compatible events space, representing a certain physical system. The equations enable us to find to find $P^t(B_j)$ – the probability of realization of event B_j at t. As it is provided by the model of investigated physical system there should be initially defined the probabilities $P^0(A_k)$, $P^0(A_kA_l),\ldots$ of compatible events A_k at $t=0$ and probabilities of transitions from A_k to B_j at time interval t ($P^t(B_j|A_k)$, $P^t(B_j|A_kA_l),\ldots$) combined with coefficients g_{jkl} $g_{jklm},\ldots$.

The conformity principle is fulfilled for equations (8), (16), (25), (31) when events are incompatible and the following conditions are true

$$P^0(A_kA_l) = \ldots = P^0(A_1\ldots A_N) = 0, \tag{32}$$

and equations take form (4) of Markovian chains for incompatible states.

If events are compatible in pairs, i.e.

$$P^0(A_k)\neq 0, \quad P^0(A_kA_l)\neq 0 \tag{33}$$

$$P^t(B_j|A_k) \neq 0, \quad P^t(B_j|A_kA_l) \neq 0 \tag{34}$$

and simultaneously the conditons

$$P^0(A_kA_lA_m) = 0, \quad \ldots, \quad P^0(A_1,\ldots,A_N) = 0 \tag{35}$$

$$\begin{aligned} &P^t(B_j|A_kA_lA_m) = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &P^t(B_j|A_1\ldots A_N) = 0 \end{aligned} \tag{36}$$

take place then the equations (28), (30), (31) are simplified und take the form

$$P\big(\bigcup_{k=1}^{N} \tilde{A}_k\big) = \sum_{k=1}^{N} P(A_k) + 2\sum_{k<l=1}^{N} g_{jkl}P(A_k \cap A_l), \tag{37}$$

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j \cap A_k) + 2\sum_{k<l=1}^{N} g_{jkl}P^t(B_j \cap (A_k \cap A_l)), \tag{38}$$

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j|A_k)P^0(A_k) + 2\sum_{k<l=1}^{N} g_{jkl}P^t(B_j|A_kA_l)P^0(A_kA_l). \tag{39}$$

7. The introduced equations (8), (16), (25), (31), are supposed to describe stochastic processes in certain real physical systems. Equations (37) - (39), which describe processes in the space of random mutually compatible events, are specifically equivalent to quantum mechanics equations. In this case we have a base to investigate the possibility of using the equations (31) to describe processes in the physics of elementary particles.

References

[1] **Accardi, L.** The pobabilistic roots of the quantum mechanical paradoxes. The wave – particle dualism. A tribute to Lonis de Broglie on his 90th Birthday, Edited by S. Diner, D. Fargue, G. Lochack and F. Selleri. – Dordrecht: D. Reidel Publ. Company, 1970. – 47 - 55 p.

[2] **Popov, N.N.** The elements of quantum probabilities theory / N.N. Popov. – M.: Computing center of RAS, 1996. – 206 p.

[3] **Holevo, A.S.** Statistical structure of quantum theory / A.S. Holevo. – Berlin, Heidelberg, New York, London, Paris, Tokyo, Hong Kong, Barselona, Budapest: Springer – Verlag. – 2001. – 150 p.

[4] **Holevo, A.S.** Probabilistic and statictic aspects of quantum theory / A.S. Holevo. – M.: The Institute of space research, 2003. – 410 p.

[5] **Hrennikov, A.U.** Nonkolmogorov probabilities theories and quantum physics / A.U. Hrennikov. – M.: Physmatlit, 2003. – 208 p.

[6] **Kolmogorov, A.N.** The base concepts of probability theory / A.N. Kolmogorov. – M.: Science, 1974. – 120 p.

[7] **Rozanov, U.A.** Lections on probability theory / U.A. Rozanov. – M.: Science, 1968. – 120 p.

[8] **Shiryaev, A.N.** The probbability / A.N. Shiryaev. – M.: Science, 1989. – 640 p.

[9] **Biryukov, A.A.** Markovian chaines for combined states and quantum mecanical equations / A.A. Biryukov // Theoretical and mathematical physics. – 1971. – V.7, iss. 1. – P. 56 - 60.

PoS(QFTHEP2010)085