Equations of Markovian processes for compatible events and quantum physics.

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The equations for a number of Markovian process models in combined event space are built. It is shown that equations of the pair - compatible events model are equivalent to quantum mechanics equations.
1. The development of innovative technologies particularly in the questions of information processing and transmission stimulates further research of probabilistic model of physical systems and stochastic processes in them. The research of the mathematical structure of the quantum theory from the viewpoint of the probability theory provokes a big interest. A number of works can be offered where original results and surveys of this sphere are presented, for example [1] – [5].

At present, markovian processes are widely used for random process description [6] – [8]. Consider a certain physical system to construct the equations of Markovian processes, the state of which state at each moment of time \( t \) is characterized by a set \( N \) of random events, which can be realized with the certain probabilities. Let’s designate these events at the moment \( t = 0 \) as \( A_k \), \( k = 1, \ldots, N \) and at the moment \( t > 0 \) as \( B_j \), \( j = 1, 2, \ldots, N \).

The first rule of the Markovain process theory is the statement that the probability \( P_t(B_j) \) of the event occurrence \( B_j \) at the moment \( t \) is defined by the realization of one of the events \( A_k \) at the previous moment \( t = 0 \), that is

\[
P_t(B_j) = P_t(B_j \cap (\bigcup_{k=1}^{N} A_k)). \tag{1}
\]

The second rule is the condition, that the events \( A_k \), \( k = 1, 2, \ldots, N \) are incompatible with each other, that is

\[
P(\bigcup_{k=1}^{N} A_k) = \sum_{k=1}^{N} P(A_k). \tag{2}
\]

Taking into account (2) we’ll write down the equation (1) as

\[
P_t(B_j) = \sum_{k=1}^{N} P_t(B_j \cap A_k) \tag{3}
\]

The formula (3) is the equation of Markovian process for incompatible state determined by events \( A_k, B_j \). It can be conveniently presented as

\[
P_t(B_j) = \sum_{k=1}^{N} P_t(B_j | A_k) P^0(A_k). \tag{4}
\]

Here \( P^0(A_k) \) is the probability of event occurrence \( A_k \) at the moment \( t = 0 \), where

\[
P_t(B_j | A_k) = \frac{P_t(B_j \cap A_k)}{P^0(A_k)} \tag{5}
\]

is the conditional probability of event realization \( B_j \) at the moment \( t \) under the condition of event realization \( A_k \) at the moment \( t = 0 \) (the probability of transition from event \( A_k \) to event \( B_j \) for an interval of time \( t \)).

In this work the models of stochastic processes in space of random events are offered, in which the Markovian condition (1) is preserved and the condition (2) of events incompatibility with each other at each moment of time of system supervision is removed.

We’ll show, that the equations for one of the presented models are compatible with the equations of the quantum mechanics. The construction of Markovian chains equations for compatible
In accordance with definition of conditional probabilities it is convenient to write equation (7) in the form

\[
P^\prime(B_j) = \sum_{k=1}^{N} P'(B_j | A_k) P^0(A_k) - \sum_{k<l=1}^{N} P'(B_j | A_k A_l) P^0(A_k A_l) + \]

\[
+ \sum_{k<l<m=1}^{N} P'(B_j | A_k A_l A_m) P^0(A_k A_l A_m) - \ldots (-1)^{N-1} P'(B_j | A_1 A_2 \ldots A_N) P^0(A_1 A_2 \ldots A_N),
\]

where \( P^0(A_k), P^0(A_k A_l), P^0(A_k A_l A_m \ldots), P^0(A_1 \ldots A_N) \) are the probabilities of events \( A_k, A_k A_l, A_k A_l A_m \ldots, A_1 \ldots A_N \) correspondingly at initial time moment \( t = 0; \)

\[
P'(B_j | A_k) = \frac{P'(B_j \cap A_k)}{P^0(A_k)}
\]

\[
P'(B_j | A_k A_l) = \frac{P'(B_j \cap (A_k \cap A_l))}{P^0(A_k \cap A_l)}
\]

\[
P'(B_j | A_1 \ldots A_N) = \frac{P'(B_j \cap (A_1 \ldots A_N))}{P^0(A_1 \cap A_2 \ldots A_N)}
\]

– conditional probability of event \( B_j \) at \( t > 0 \) when events \( A_k, A_k \cap A_l, \ldots \) are correspondingly realized at \( t = 0 \) (probability of transition from events \( A_k, A_k \cap A_l, \ldots \) to \( B_j \) during time interval \( t \)).
3. Let’s consider physical system at any time moment, which similarly to the system from 12, is characterized by compatible random events \( A_k, k = 1, 2, \ldots, N \). But there are conditions for events \( A_k, k = 1, 2, \ldots, N \) which are realized in the system. The events \( A_k \) are

\[
A_k = A_k / \left( \bigcup_{i=1}^{N} (A_k \cap A_i) i \neq k \right), \quad k = 1, 2, \ldots, N.
\]

For the case of \( N = 2 \) we have

\[
A_1 = A_1 / (A_1 \cap A_2) = A_1 / A_2
\]

\[
A_2 = A_2 / (A_2 \cap A_1) = A_2 / A_1
\]

The probability of events unification (11) is determined by equation [8]

\[
P(A_1 \cup A_2) = P(A_1) + P(A_2) - 2P(A_1 \cap A_2).
\]

With mathematical induction method the probability of unification of \( N \) events can be proved to be presented like

\[
P\left( \bigcup_{k=1}^{N} A_k \right) = \sum_{k=1}^{N} P(A_k) - 2 \sum_{k<l}^{N} P(A_k \cap A_l) +

+ 4 \sum_{k<l<m}^{N} P(A_k \cap A_l \cap A_m) - \ldots - (-2)^{N-1} P\left( \bigcap_{k=1}^{N} A_k \right)
\]

Consider Markovian stochastic process in this system, going under the next conditions. Random events \( B_j \) occur at \( t > 0 \), if one of the events \( A_k \) took place at \( t = 0 \). In accordance with the former condition the probability of \( B_j \) is defined as

\[
P' (B_j) = P' (B_j \cap \left( \bigcup_{k=1}^{N} A_k \right)).
\]

Using equation (13), the equation (14) can be written in the form

\[
P' (B_j) = \sum_{k=1}^{N} P' (B_j \cap A_k) - 2 \sum_{k<l}^{N} P' (B_j \cap (A_k \cap A_l)) +

+ 4 \sum_{k<l<m}^{N} P' (B_j \cap (A_k \cap A_l \cap A_m)) + \ldots +

+ (-2)^{N-1} P' (B_j \cap (A_1 \cap \ldots \cap A_N)).
\]

Substituting transition probabilities (9) into equation (15) we derive

\[
P'(B_j) = \sum_{k=1}^{N} P'(B_j|A_k)P^0(A_k) - 2 \sum_{k<l}^{N} P'(B_j|A_kA_l)P^0(A_kA_l) +

+ 4 \sum_{k<l<m}^{N} P'(B_j|A_kA_lA_m)P^0(A_kA_lA_m) + \ldots +
\]
- The probabilities of events hold for their union; the events take place simultaneously; without the limits of Kolmogorov’s axiomatics. Let us define the events explained in the framework of the Kolmogorov’s axiomatics. This fact must be taken as a new

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\[ \sum_{k=1}^{N} P(A_k) = \sum_{k<j=1}^{N} P(A_k \cap A_j) + \cdots + 2P(A_1 \cap A_2) \]

\[ P(A_k^+ \cup A_j^+) = P(A_k) + P(A_j) + 2P(A_k \cap A_j) \].

4. The model of the process, describing the system evolution, where both compatible events \( A_k, k = 1, 2, \ldots, N \) and events \( A_k^+ \) are realized, is an alternative to the model of Markovian process that is given in 13.

Let us consider the system, where there are only two compatible events \( A_1 \) and \( A_2 \) and two events \( A_1^+, A_2^+ \) correspondingly, and specify the properties of introduced events. The probability of events unification \( A_k^+ \cap A_l^+ \) is determined as

\[ P(A_k^+ \cup A_l^+) = P(A_k) + P(A_l) + 2P(A_k \cap A_l) \]

On the basis of expression (17) formula for probability of unification of \( N \) events \( A_k^+ \) is found by the mathematical induction method

\[ P(\bigcup_{i=1}^{N} A_i^+) = \sum_{k=1}^{N} P(A_k) + 2 \sum_{k<j=1}^{N} P(A_k \cap A_j) + \cdots + 4 \sum_{k<j<l=m=1}^{N} P(A_k \cap A_l \cap A_m) + \cdots + (2)^{N-1} P(\bigcap_{i=1}^{N} A_i). \]

The existence of the events \( A_k^+ \) and the expression (17) for the probability of their union cannot be explained in the framework of the Kolmogorov’s axiomatics. This fact must be taken as a new statement.

The equations (17), (18) may be derived taken into account the structure of events \( A_k^+ \) model without the limits of Kolmogorov’s axiomatics. Let us define the events \( A_k^+ \)

\[ A_k^+ = A_k \cup \left( \bigcup_{i=1}^{N} (A_k \cap A_i), l \neq k \right), \quad k = 1, 2, \ldots, N, \]

under the conditions:
- The events \( A_k \) and \( \bigcup_{i=1}^{N} (A_k \cap A_i), l \neq k \), \( k = 1, 2, \ldots, N \), are not compatible, i.e. the equations

\[ A_k^+ \neq A_k, \quad (A_k \cap A_l) \neq 0, \quad k \neq l = 1, 2, \ldots, N \]

\[ (A_k \cap A_l) \cup (A_l \cap A_k) \neq A_k \cap A_l, \quad (A_k \cap A_l) \cup (A_l \cap A_k) \neq (A_l \cap A_k), \quad k \neq l = 1, 2, \ldots, N \]

\[ P(A_k \cap A_l) = P(A_l \cap A_k), \quad k, l = 1, 2, \ldots, N. \]
One thing is clear, this definition of events $A^+_k$ should be considered as additional postulate in Kolmogorov axiomatics. Using the definition (19) and conditions $(20) - (22)$ one can easily prove the formula (17) for $N = 2$.

Let us consider in this model the Markovian process, where probability of event $B_j$ at $t > 0$ is realized if one or more events $A^+_i$, $i = 1, 2, \ldots, N$ took place at $t = 0$, i.e.

$$P^t(B_j) = P^t(B_j \cap \bigcup_{i=1}^{N} A^+_i).$$

(23)

With using (18) the equation (23) can be written in the form

$$P^t(B_j) = \sum_{k=1}^{N} P^t(B_j \cap A_k) + 2 \sum_{k<j}^{N} P^t(B_j \cap (A_k \cap A_j)) +$$

$$+ 4 \sum_{k<j<m}^{N} P^t(B_j \cap (A_k \cap A_j \cap A_m)) + \ldots + 2^{N-1} P^t(B_j \cap (A_1 \cap \ldots \cap A_N)).$$

Equation (24) (similarly to equation (15)) can be transformed by adding the event possibilities at the initial time and transition possibilities

$$P^t(B_j) = \sum_{i=1}^{N} P^t(B_j|A_i)P^0(A_i) + 2 \sum_{i<j}^{N} P^t(B_j|(A_iA_j))P^0(A_iA_j) +$$

$$+ 4 \sum_{i<j<k}^{N} P^t(B_j|(A_iA_jA_k))P^0(A_iA_jA_k) + \ldots +$$

$$+ 2^{N-1} P^t(B_j|(A_1 \ldots A_N))P^0(A_1 \ldots A_N).$$

(25)

Equations (24), (25) of Markovian processes for compatible events in this model of physical system are of the same structure as equations (15), (16) under the condition that all terms are positive.

5. Let us derive the equation of Markovian process in compatible event space for a physical system, associating the models mentioned before in 13 and 14.

Consider both compatible events $A_k$, $k = 1, 2, \ldots, N$ and events $A^-_k$, $k = 1, 2, \ldots, N_1$ and $A^+_k$, $k = 1, 2, \ldots, N_2$ ($N_1 + N_2 = N$) in order to describe a physical system state at every time moment.

For the case of $N = 2$ the possibilities of events unification $P(A^-_1 \cup A^+_2)$, $P(A^+_1 \cup A^-_2)$ are defined by expressions (12), (17). It is provable that probability of events $A^-_1$ and $A^+_2$ unification ($P(A^-_1 \cup A^+_2)$) is given by the equation

$$P(A^-_1 \cup A^+_2) = P(A^+_1 \cup A^-_2) = P(A_1) + P(A_2).$$

(26)

Denote $\bar{A}_k$ the event, which can be either event $A^-_k$ or event $A^+_k$. Using this denotation formulae (12), (17), (26) can be written as

$$P(\bar{A}_1 \cup \bar{A}_2) = P(A_1) + P(A_2) + 2g_{12}P(A_1 \cap A_2),$$

(27)

where $g_{12} = -1$ or $+1$ or $0$ depending on event choice $\bar{A}_1$, $\bar{A}_2$. 

6.
The probability of unification of $N$ events $\bar{A}_k$ is given by formula that generalizes equation (27):

$$P\left(\bigcup_{k=1}^{N} \bar{A}_k\right) = \sum_{k=1}^{N} P(A_k) + 2 \sum_{k<l=1}^{N} g_{kl}P(A_k \cap A_l) +$$

$$+ 4 \sum_{k<l<m=1}^{N} g_{klm}P(A_k \cap A_l \cap A_m) + \ldots, \quad (28)$$

where $g_{kl}$, $g_{klm}$ and other multipliers take one of the values $0$, $+1$, $-1$ depending on the selected events $\bar{A}_k$, $\bar{A}_l$, $\bar{A}_m$, ....

Markovian process in such a physical system will be described via equation that gives the probability of event $B_j$ at $t > 0$ if one of the events $\bar{A}_k$, $k = 1, 2, \ldots, N$ took place at $t = 0$

$$P'(B_j) = P'\left(B_j \cap \left(\bigcup_{k=1}^{N} \bar{A}_k\right)\right). \quad (29)$$

With using (28) the equation (29) for Markovian process can be written as

$$P'(B_j) = \sum_{k=1}^{N} P'(B_j \cap A_k) + 2 \sum_{k<l=1}^{N} g_{kl}P'(B_j \cap (A_k \cap A_l)) +$$

$$+ 4 \sum_{k<l<m=1}^{N} g_{klm}P'(B_j \cap (A_k \cap A_l \cap A_m)) + \ldots. \quad (30)$$

In order to simplify equations (30) we introduce probabilities of transitions from events $A_k$ at $t = 0$ to events $B_j$ at $t > 0$ (9) and probabilities of events $A_k$ at initial time moment $t = 0$ ($P^0(A_k)$, $P^0(A_kA_l), \ldots$).

$$P'(B_j) = \sum_{k=1}^{N} P'(B_j|A_k)P^0(A_k) + 2 \sum_{k<l=1}^{N} g_{kl}P'(B_j|A_kA_l)P^0(A_kA_l) +$$

$$+ 4 \sum_{k<l<m=1}^{N} g_{klm}P'(B_j|A_kA_lA_m)P^0(A_kA_lA_m) + \ldots. \quad (31)$$

6. Equations (8), (16), (25), (31) describe Markovian processes in compatible events space, representing a certain physical system. The equations enable us to find $P'(B_j)$ – the probability of realization of event $B_j$ at $t$. As it is provided by the model of investigated physical system there should be initially defined the probabilities $P^0(A_k)$, $P^0(A_kA_l), \ldots$ of compatible events $A_k$ at $t = 0$ and probabilities of transitions from $A_k$ to $B_j$ at time interval $t$ ($P^0(B_j|A_k)$, $P^0(B_j|A_kA_l)$, ...) combined with coefficients $g_{kl}g_{klm}, \ldots$.

The conformity principle is fulfilled for equations (8), (16), (25), (31) when events are incompatible and the following conditions are true

$$P^0(A_kA_l) = \ldots = P^0(A_1 \ldots A_N) = 0, \quad (32)$$

and equations take form (4) of Markovian chains for incompatible states.

If events are compatible in pairs, i.e.

$$P^0(A_k) \neq 0, \quad P^0(A_kA_l) \neq 0 \quad (33)$$
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\[ P'(B_j | A_k) \neq 0, \quad P'(B_j | A_k A_l) \neq 0 \]  \hspace{1cm} (34)

and simultaneously the conditions

\[ P^0(A_k A_l A_m) = 0, \quad \ldots, \quad P^0(A_1, \ldots, A_N) = 0 \]  \hspace{1cm} (35)

\[ P'(B_j | A_k A_l A_m) = 0 \]  \hspace{1cm} (36)

\[ P'(B_j | A_1 \ldots A_N) = 0 \]  \hspace{1cm} (37)

\[ P'(B_j) = \sum_{k=1}^{N} P'(B_j | A_k) + 2 \sum_{k<l=1}^{N} g_{jkl} P'(B_j | A_k A_l) \]  \hspace{1cm} (38)

\[ P'(B_j) = \sum_{k=1}^{N} P'(B_j | A_k) P^0(A_k) + 2 \sum_{k<l=1}^{N} g_{jkl} P'(B_j | A_k A_l) P^0(A_k A_l) \]  \hspace{1cm} (39)

7. The introduced equations (8), (16), (25), (31), are supposed to describe stochastic processes in certain real physical systems. Equations (37) - (39), which describe processes in the space of random mutually compatible events, are specifically equivalent to quantum mechanics equations. In this case we have a base to investigate the possibility of using the equations (31) to describe processes in the physics of elementary particles.

References


