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Light hadron spectroscopy and pseudoscalar decay constants

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I review recent lattice QCD results on light hadron spectroscopy and pseudoscalar decay constants.

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Since nearly four decades QCD is believed to be the fundamental theory of the strong interaction. Its fundamental degrees of freedom are quarks and gluons, none of which are experimentally observed. Instead, one observes a large number of bound state hadrons that are believed to be the low energy degrees of freedom of QCD. These objects are inherently nonperturbative. Computing their properties - especially their masses - therefore constitutes one of the most fundamental challenges for lattice QCD as the prime method of treating QCD in the nonperturbative regime.

During the last couple of years there has been enormous progress towards reaching this goal. Driven by conceptual and algorithmic advances as well as advances in computer technology, we are currently able to compute ground state light hadron masses to within a total accuracy of a few percent in $2+1$ flavor QCD. Our understanding of excited states is not as detailed as for the ground states, but huge qualitative progress has been made.

In the current review, I will summarize the state of the art in light hadron (i.e. hadrons composed of u, d and s valence quarks) spectroscopy. I start with a discussion of systematic uncertainties (sect. 1) followed by a brief overview of the currently available data sets (sect. 2). I then review ground state spectroscopy (sect. 3) including recent attempts to treat electromagnetic effects (sect. 4) and excited state spectroscopy (sect. 5). I close with a section on the closely related light pseudoscalar decay constants (sect. 6).

A very similar mix of topics has been reviewed at the lattice 2009 conference [1]. A more detailed recent review about excited state spectroscopy can be found in [2]. A review of recent ensembles generated by various collaborations is given in [3].

1. Systematic errors

Every lattice calculation of the light hadron spectrum generically contains a number of extrapolations. Typically the three most important ones are continuum, infinite volume and chiral extrapolations. While the continuum extrapolation is unavoidable, the chiral extrapolation can be circumvented by directly going to or reweighting to the physical point. Finite volume effects for stable particles are exponentially suppressed in the lattice size and can therefore in principle be made very small.

In addition current lattice calculations of the light hadron spectrum typically contain a number of controlled approximations that can systematically be corrected for by non-lattice methods. For the ground state hadron spectrum and the pseudoscalar decay constants the two most important ones are the isospin limit and the treatment of QED effects. The size of these effects is given by the $\pi^0 - \pi^\pm$ mass splitting. Although it is less common today, one may additionally have uncontrolled approximations such as the quenched approximation.

Furthermore, depending on the action used and the specific observable, one may have to account for additional systematic effects such as flavor/isospin breaking or level shifts of resonant states. I will briefly summarize here, how the three main sources of systematic errors affect todays light hadron spectrum and decay constant calculations.

1.1 Continuum extrapolation

A final continuum extrapolation is an unavoidable part of any lattice calculation that wants to make a statement about the underlying fundamental continuum theory. The severity of the contin-

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uum extrapolation however depends very strongly on both the action used and the combination of scale setting observable and measured observable.

While gluonic actions used today are generically a variant of $O(a^2)$ Symanzik-improved actions and show a continuum scaling of at least $O(\alpha_s a^2)$, the scaling behavior of the various fermion actions is typically not as good. Formally, staggered fermions and twisted mass fermions at maximal twist as well as exactly chiral fermions show $O(a^2)$ continuum scaling while Wilson type fermions generically start with $O(a)$ scaling. Only improved staggered fermions such as asqtad have a leading scaling behavior of $O(\alpha_s a^2)$. There are several caveats to this statement however.

For twisted mass fermions, $O(a^2)$ scaling is only strictly realized at maximal renormalized twist [4, 5], which requires the tuning of one additional parameter. This tuning is routinely done as part of any twisted mass calculation (see e.g. [6]). Since this tuning has a typical accuracy on the few percent level, it is expected that the $O(a^2)$ terms - though formally subleading - are in fact numerically dominant. In addition, twisted mass calculations often employ a doublet of valence fermions with opposite Wilson parameter to cancel remnant $O(a)$ effects.

Similarly $O(a^2)$ scaling is only strictly realized for chirally symmetric fermions if the chiral symmetry is exact. Fermion formulations that incorporate an inexact chiral symmetry, such as domain wall fermions, do formally have a remaining $O(\alpha_s^n a)$ scaling behavior. The smallness of the residual mass and other numerical evidence [7] however suggest that, similar to the twisted mass case, the $O(a^2)$ term is dominant although it is formally subleading.

Wilson type fermions on the other hand are typically Symanzik improved by the addition of a Sheikoleslami-Wohlert (clover) term. At tree level ($c_{SW} = 1$), this results in an $O(\alpha_s a)$ scaling of on-shell observables while with a suitable nonperturbative tuning one can in principle obtain $O(a^2)$ scaling. In addition, there is numerical evidence [8 – 11] that the scaling behavior of clover fermions is substantially improved by gauge link smearing, which is commonly used today.

Apart from the action used, the continuum scaling is also largely dependent on the observables considered. In lattice QCD all observables (as well as all input parameters) are dimensionless quantities. Therefore, if one is interested in dimensionful quantities such as the baryon masses, the scaling might be quite different depending on the scale setting variable used. Ratios of baryon masses generally have shown a very mild scaling behavior and consequently setting the scale by one baryon mass [12, 13] or a combination of baryon masses [14, 15] is a typical procedure in calculating light baryon spectra and related quantities [16, 17]. One can even go further and use dimensionless mass ratios to a scale setting mass throughout the analysis [12]. A slightly different approach is the scale setting by the MILC collaboration which uses r_1 [18].

Some care has to be taken about the size of the scaling window. While generally scaling is not expected to set in for lattice spacings coarser than $a \sim 0.1 - 0.15$ fm, it has been observed [19 – 22] that for fine lattices the autocorrelation time of the topological charge is rapidly growing. It therefore seems to be prohibitively expensive with current algorithms to obtain a sufficiently large and statistically independent ensemble of configurations for lattice spacings finer than $a \sim 0.05$ fm.

Generally, for the observables considered in this review continuum scaling is rather mild and not a leading source of systematic error.

1.2 Reaching the physical point

Historically, extrapolating lattice results to physical pion masses has been one of the leading

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sources of systematic error in lattice calculations. While it is often referred to as chiral extrapolation, the term does not accurately reflect the current state of the art anymore. Physical pion masses have been reached both by reweighting techniques [23] and by direct simulation [11][1]. Other results have been obtained with pion masses close to the physical one such that the remaining extrapolation is tiny and does not substantially contribute to the systematic error anymore [12, 30, 31].

In general, there are two strategies of obtaining results at the physical point: On the one hand, one can extrapolate or interpolate to the physical point using either a chiral expansion around the singular point $m = 0$ or a regular Taylor expansion around a finite mass. Alternatively, one may tune the parameters or reweight existing ensembles to reach the physical point exactly. The latter strategy has been followed by the PACS-CS collaboration [23] and, to a lesser extent, by the RBC-UKQCD collaboration [7] who reweighted their ensembles to the physical strange quark mass, performing a subsequent extrapolation to the physical pion mass. All other results discussed here use a form of extrapolation or interpolation to obtain results at the physical point.

For pseudoscalar decay constants, chiral perturbation theory (χPT) [32 – 34] presents a natural framework to describe lattice data. It is an effective field theory description based on the chiral symmetry properties of QCD and treats pseudoscalar mesons as fundamental fields. It represents an asymptotic series expansion around either the SU(2) or SU(3) chiral point. A variant where the kaon is treated as a heavy rather than a chiral particle is known as heavy kaon χPT [35]. Refinements of χPT take discretization terms of specific lattice actions [36 – 38] and (partial) quenching [39 – 42] into account explicitly.

As an effective theory, each order of the chiral expansion carries with it a number of expansion parameters (low energy constants or LECs). Explicit expressions for meson masses and pseudoscalar decay constants up to NNLO are available in the continuum [43 – 46] while for lattice formulations only NLO expressions are available.

In order to fix the LECs different strategies are employed. One may either use the full information available on the required LECs to constrain the fit, use partial information or let the fit determine the LECs and then optionally check for the consistency of the result with other determinations. In fits where LECs are fully constrained the chiral description of the data is sometimes not satisfactory. In such cases “analytic” higher order terms are often used. These are ad-hoc polynomial higher order terms introduced for the sole purpose of obtaining a reasonable fit. These functional forms may be viewed as a hybrid between a chiral and a regular Taylor expansion that will be discussed below.

For masses other than the pseudoscalars, χPT generically predicts a leading nonanalytic term of the form M_{PS}^3 [47]. More formally, one can include baryons in χPT [48] but the resulting series is only slowly converging. An alternative formulation with better convergence properties is heavy baryon χPT [49, 50] which treats baryons as nonrelativistic particles and currently is most commonly used to fit lattice baryon data. Recently, the covariant approach [51] that promises better convergence behavior for heavier pion masses has been revived [52, 53] and used for chiral fits of the baryon octet.

An alternative to the chiral expansion for describing the pion mass dependence of any hadronic

[1]With staggered fermions, the physical point has also been reached in the sense that the lowest pseudoscalar mass is at the physical pion mass [24 – 29]

observable is a Taylor expansion around a finite pion mass. In contrast to the chiral expansion, the Taylor expansion is performed around a nonsingular point and consequently has a finite radius of convergence. Typically this convergence radius is given by the distance of the expansion point to the chiral limit. Usually, the expansion is performed in powers of the pseudoscalar mass square $M_{\mathrm{PS}}^2 - M_{\mathrm{PS}_0}^2$, but the chiral behavior of baryon masses has also been successfully described by a linear expansion in M_{PS} [54]. Optimal convergence is achieved in principle by placing the expansion point at the middle of the intervall spanned by all simulation points and the physical point (for further discussion of this point see [12, 55]). Note that from a practical perspective the choice of the expansion point $M_{\mathrm{PS}_0}^2$ does not play a role in the fit itself as a redefinition of $M_{\mathrm{PS}_0}^2$ may be absorbed by redefining the lower order fit coefficients.

It is also worth noting that one may try to fit ratios [12] or differences [14, 15] of baryon masses in order to cancel common contributions and obtain a more regular chiral behavior.

Recently, SU(3) breaking effects in baryon multiplets have also been studied in the $1/N_c$ expansion [56] offering an alternative way of describing the chiral behavior of baryon mass multiplets.

1.3 Finite volume effects

The third common source of error in a lattice calculation is the finiteness of the volume of the simulated box. As is the case for the continuum limit, the infinite volume limit can never be reached and an extrapolation in the volume is in principle unavoidable. For most observables however the leading finite volume corrections are exponentially small in the box size and not polynomially and can therefore be made arbitrarily small by increasing the volume [57].

The origin of these corrections are particles propagating nontrivially around a spatial dimension of the lattice. The leading contribution obviously originates from the lightest particles, the pions. One can more systematically treat their effect in finite volume chiral perturbation theory [58 – 60]. The χPT treatment has also been combined with the approach of [57] in [61, 62]. A similar expansion for baryons has also been pioneered [63]. As one can see from fig. 1, current lattices are typically large enough to have percent level or smaller finite volume corrections on the pion mass. Note however, that corrections to baryon masses can be substantially larger [63].

A different sort of finite volume effect can be observed for resonant states. In the continuum, resonances are embedded in a continuous spectrum of scattering states. In a finite volume, the spectrum becomes discrete and the energy of the possible scattering states increases with shrinking volume due to the increasing magnitude of the discrete lattice momenta. Ultimately, for small enough volumes, the resonances end up dominating the ground state. A systematic treatment of these finite volume energy shifts was developed in [64 – 66]. Recently it has been suggested to use this volume, mass and momentum dependence of the energy levels on a statistical basis and identify resonances via eigenstate densities [67 – 69]. An alternative method based on finite time correlators has also recently been suggested in [70].

Although conceptually clear, the treatment of resonant states in a region where they are not the ground state faces the huge challenge of reliably extracting the ground state as well as a number of excited states. One therefore often uses the assumption that an operator which does not mirror the valence quark structure of a scattering state will almost exclusively couple to the resonance for extracting directly the desired resonance level. The validity of this assumption is confirmed by recent studies [71, 72].

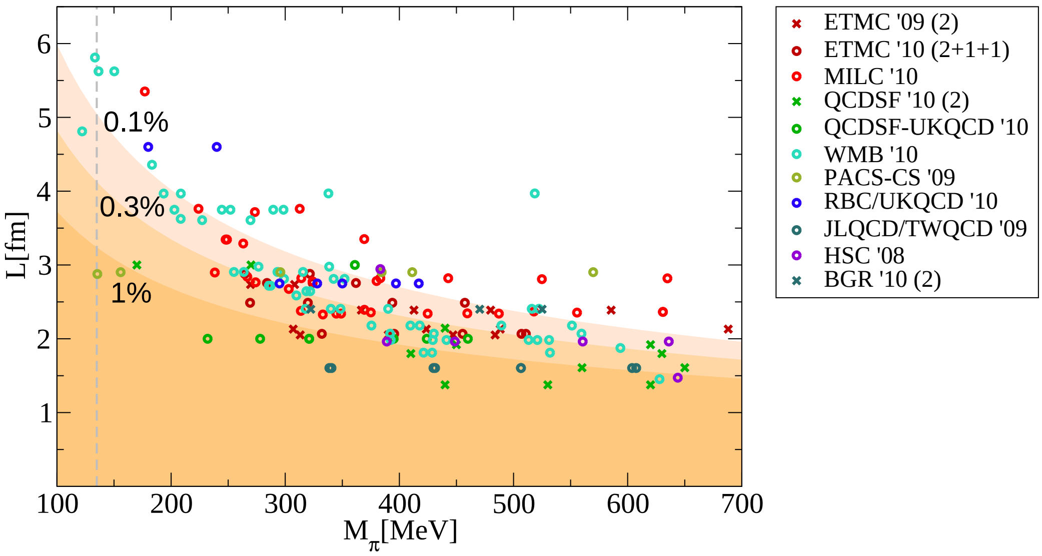

Figure 1: The landscape of recent dynamical fermion simulations projected to the L vs. M_π plane. The borders of the shaded regions are placed where the expected relative error of the pion mass is 1%, 0.3% resp. 0.1% according to [62]. Data points are taken from the following references: ETMC'09(2) [75], ETMC'10(2+1+1) [6], MILC'10 [18], QCDSF'10(2) [76], QCDSF-UKQCD'10 [15], WMB'10 [11, 12], PACS-CS'09 [23, 77], RBC-UKQCD'10 [7, 78], JLQCD/TWQCD'09 [79], HSC'10 [71] and BGR'10(2) [72]. All ensembles are from $N_f = 2+1$ simulations except explicitly noted otherwise. For staggered resp. twisted mass ensembles, the Goldstone resp. charged pion masses are plotted.

Fixing the global topological charge in QCD is a restriction that becomes irrelevant in the infinite volume limit. For this reason fixing the topological charge in lattice QCD calculations may be viewed as introducing an additional third type of finite volume corrections [73, 74].

2. Ensemble overview

In order to assess currently available lattice ensembles with respect to the three main sources of systematic error discussed in the previous section, it is instructive to look at their position in a landscape with respect to the four quantities: light and strange quark masses (physical point), lattice spacing (continuum) and volume. Because light and strange quark masses are scheme and scale dependent quantities, it is easier to use the quantities M_π and $\sqrt{2M_K^2 - M_\pi^2}$ instead that are proportional to the square root of the sum of light quark masses resp. the strange quark mass to leading order.

In figs. 2-3 three projections of this landscape are plotted. The first one, fig. 2, displays the position of current ensembles in the $\sqrt{2M_K^2 - M_\pi^2}$ vs. M_π plane. As one can see, the physical point has already been reached. In fig. 1 the landscape is projected to the L vs. M_π plane. One observes that the bulk of current day lattice ensembles lies in a region where the pion mass is expected to be

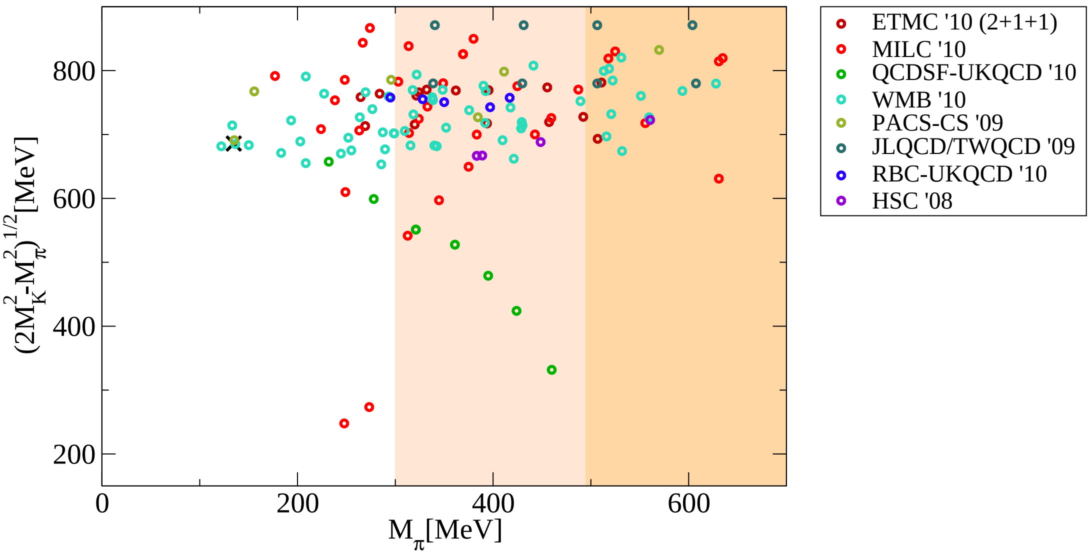

Figure 2: The landscape of recent dynamical fermion simulations projected to the $\sqrt{2M_K^2 - M_\pi^2}$ vs. M_π plane. The cross marks the physical point while shaded areas with increasingly light shade indicate physically more desirable regions of parameter space. Data points are taken from the following references: ETMC'10(2+1+1) [6], MILC'10 [18], QCDSF-UKQCD'10 [15], WMB'10 [11, 12], PACS-CS'09 [23, 77], RBC-UKQCD'10 [7, 78], JLQCD/TWQCD'09 [79], HSC'10 [71] and all ensembles are from $N_f = 2+1$ simulations except explicitly noted otherwise. For staggered resp. twisted mass ensembles, the Goldstone resp. charged pion masses are plotted.

affected by finite volume corrections by less than one per cent. Finally, fig. 3 displays a projection to the M_π vs. a plane. Also here one can see that present day simulations start populating the interesting regime of physical or near physical pion masses at a range of relatively small lattice spacings.

The important point is of course to have ensembles that simultaneously lie in the desirable regions with respect to all coordinates of the landscape, i.e. that are at or close to the physical point at large volumes and a range of relatively small lattice spacings.

3. Ground states

We begin our overview of calculations of the light hadron spectrum by results of the ETM collaboration for $N_f = 2$ twisted mass fermions [13]. The ETM collaboration used $N_f = 2$ twisted mass fermions at maximal renormalized twist on a tree level Symanzik improved gauge action. Two lattice spacings ($a \sim 0.07$ fm and $a \sim 0.09$ fm) were used with charged pion masses in the range 270 to 500 MeV.[2] The lattice spacing was set via the nucleon mass and chiral extrapolations were performed with a variety of different ansätze. The valence strange quark mass is set by tuning

[2]The isospin splitting of the pions is $M_\pi^{\pm 2} - M_\pi^{0 2} \sim (150-220\text{ MeV})^2$ [80]

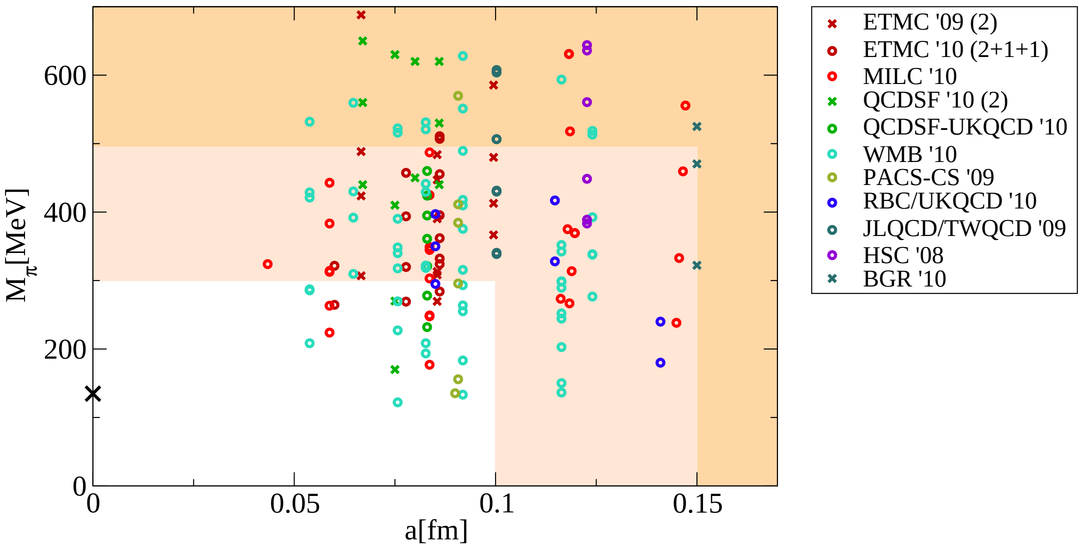

Figure 3: The landscape of recent dynamical fermion simulations projected to the M_π vs. a plane. The cross marks the physical point while shaded areas with increasingly light shade indicate physically more desirable regions of parameter space. Data points are taken from the following references: ETMC'09(2) [75], ETMC'10(2+1+1) [6], MILC'10 [18], QCDSF'10(2) [76], QCDSF-UKQCD'10 [15], WMB'10 [11, 12], PACS-CS'09 [23, 77], RBC-UKQCD'10 [7, 78], JLQCD/TWQCD'09 [79], HSC'10 [71] and BGR'10(2) [72]. All ensembles are from $N_f = 2+1$ simulations except explicitly noted otherwise. For staggered resp. twisted mass ensembles, the Goldstone resp. charged pion masses are plotted.

the kaon mass to its physical value. The final result, displayed in fig. 4, employs $O(p^3)$ resp. NLO SU(2) heavy barion χPT chiral extrapolation for baryons without resp. with valence strange quarks. The continuum extrapolation was performed using a constant which was demonstrated to be sufficient at the given level of accuracy. Exponential finite volume corrections were taken into account in the final fit form. Resonant state finite volume corrections were not performed but are believed to be irrelevant in the region of parameter space covered by the simulations. Effects of the twisted mass isospin breaking were observed to be negligible except in the case of the Ξ where they amounted to a 6% correction.

The current results on light hadron masses from the MILC collaboration are summarized in [18]. For the baryon mass analysis ensembles at 3 of the currently available 6 lattice spacings $a \sim 0.06$ fm, $a \sim 0.09$ fm and $a \sim 0.12$ fm are used. The smallest Goldstone pion mass is ~ 180 MeV corresponding to an RMS pion mass of ~ 250 MeV. Both gauge and fermion (asqtad) actions have $O(\alpha_s a^2)$ scaling behavior and the continuum extrapolation is done linearly in this quantity. Depending on the specific observable, chiral or polynomial fit forms are found to best describe the data and the scale is set via r_1. The final results, presented in [18] are in good agreement with the observed hadron spectrum.

A subset of the MILC ensembles with $a \sim 0.12$ fm and a smallest pion mass of ~ 290 MeV

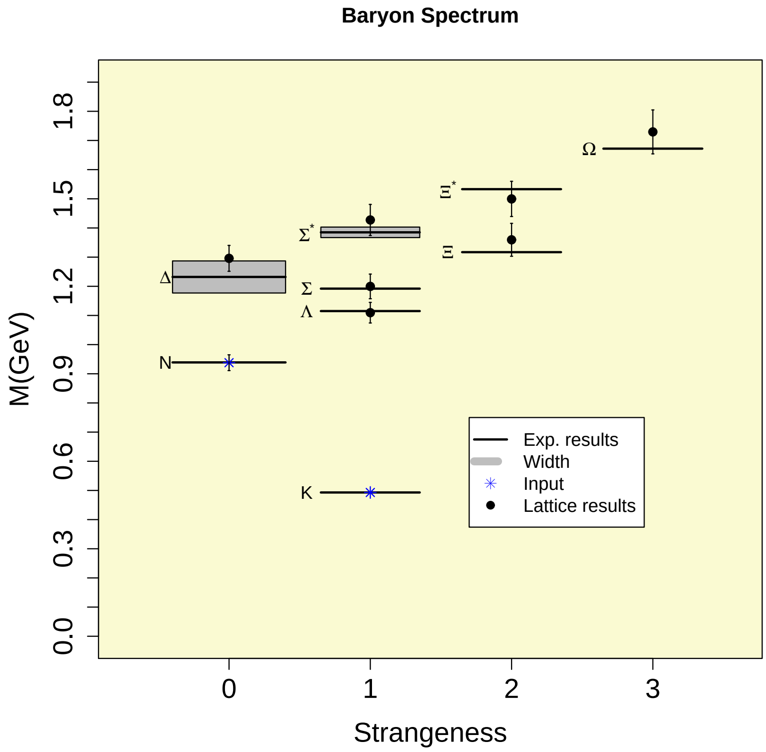

Figure 4: Baryon spectrum obtained by the ETM collaboration with $N_f = 2$ twisted mass fermions. The plot is reproduced from [13] with friendly permission of the ETM collaboration.

has been studied in [54] in a mixed action setup with domain wall valence quarks. Comparing different chiral fit forms for the nucleon mass it was demonstrated that a simple linear fit in M_π gives the best description of the data and extrapolates to the correct value at the physical point. In the same paper, this feature has also been found in other collaborations data.

The QCDSF-UKQCD collaboration has recently proposed a different approach to the physical point starting from an SU(3) symmetric theory and systematically expanding in the SU(3) breaking parameter while keeping $2M_K^2 + M_\pi^2$ constant [14, 15]. Preliminary results at a single lattice spacing show a linear dependence of the octet and decuplet masses considered and a good agreement with the experimentally observed hadron spectrum. An $N_f = 2+1$ nonperturbatively improved single step stout smeared clover action on a tree level Symanzik improved gauge action was used for this study. Finite size corrections are not yet applied.

The light hadron spectrum calculation of the Wuppertal-Marseille-Budapest (WMB) collaboration [12] was performed with tree level improved 6-step stout smeared $N_f = 2+1$ clover fermions on a tree level Symanzik improved gauge action. Pion masses down to 190 MeV and three lattice spacings $a \sim 0.06$ fm, $a \sim 0.8$ fm and $a \sim 0.12$ fm were used. Finite volume corrections were applied including energy shifts for resonant states. The continuum extrapolation was performed with a term linear in a or a^2 and chiral fits were done with both Taylor and NLO heavy baryon χPT with a free coefficient. The systematic error was estimated by the spread of the results of all analyses weighted by the fit quality. Good agreement with the experimentally observed light

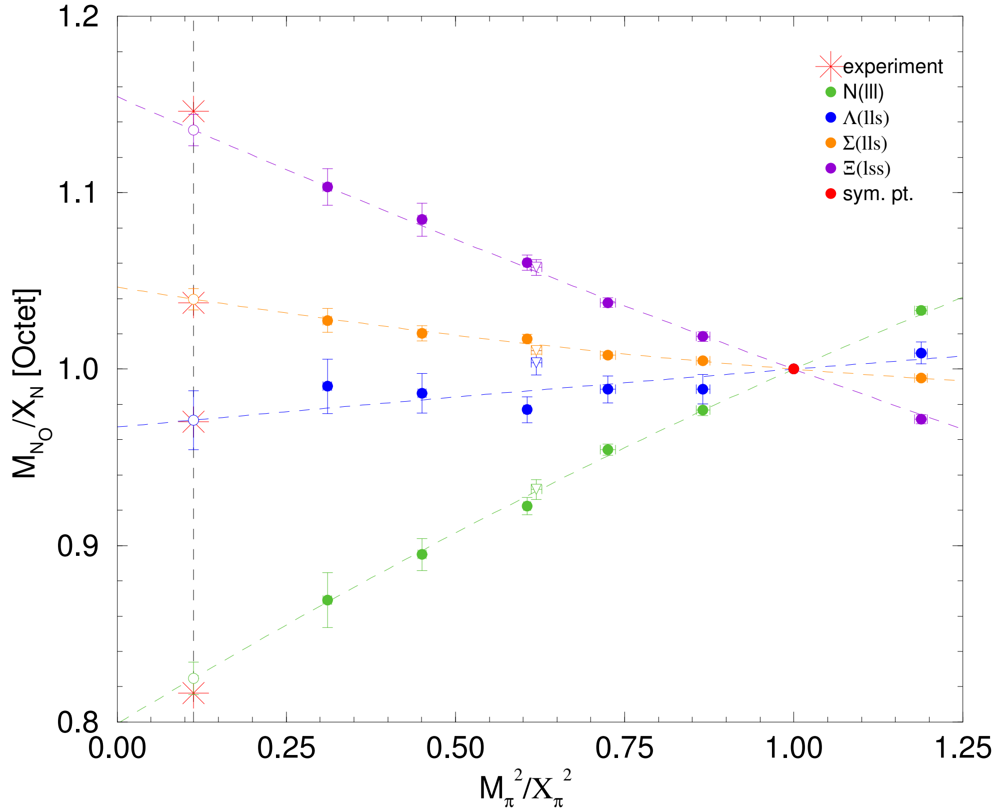

Figure 5: Chiral behavior of the ratio of individual octet masses over the average octet mass $X_N = \frac{1}{3}(M_N + M_\Sigma + M_\Xi)$ vs. the ratio of the square of the pion mass over the square average of pseudoscalar meson masses $X_\pi^2 = \frac{1}{3}\left(2M_K^2 + M_\pi^2\right)$ as obtained by the QCDSF-UKQCD collaboration. The plot is reproduced from [15] with friendly permission of the QCDSF-UKQCD collaboration.

hadron spectrum was found.

The PACS-CS collaboration has published results for the light hadron spectrum using both a chiral extrapolation [77] and a direct reweighting to the physical point [23]. In both cases $N_f = 2+1$ nonperturbatively $O(a)$ improved cover fermions on an Iwasaki gauge action were used at a single lattice spacing $a \sim 0.09$ fm. Pion masses down to ~ 150 MeV were directly simulated and a reweighting to the physical point was carried out with the lightest ensemble. In the extrapolated ensemble finite size effects on the pseudoscalar masses were corrected using SU(2) χPT at NLO. The tiny chiral extrapolation was performed linearly in the light quark mass and M_Ω was used to set the scale. More involved chiral forms were subsequently investigated in [81]. Similarly in the reweighted ensemble the masses of the π, K and Ω were used to tune to the physical point. The resulting vector meson and baryon spectra are in good agreement with experiment (see fig. 6).

The RBC-UKQCD collaboration has recently performed a pioneering calculation of the η and η' masses using $N_f = 2+1$ flavor domain wall ensembles on an Iwasaki gauge action [82]. Three pion masses in the range $400-700$ MeV on a single lattice spacing $a \sim 0.11$ fm were used. A two operator basis with gauge fixed wall sources was used to extract the correlation functions. A mixing angle of $\Theta = -9.2(4.7)^\circ$ and masses $M_\eta = 583(15)$ MeV and $M_{\eta'} = 853(123)$ MeV were

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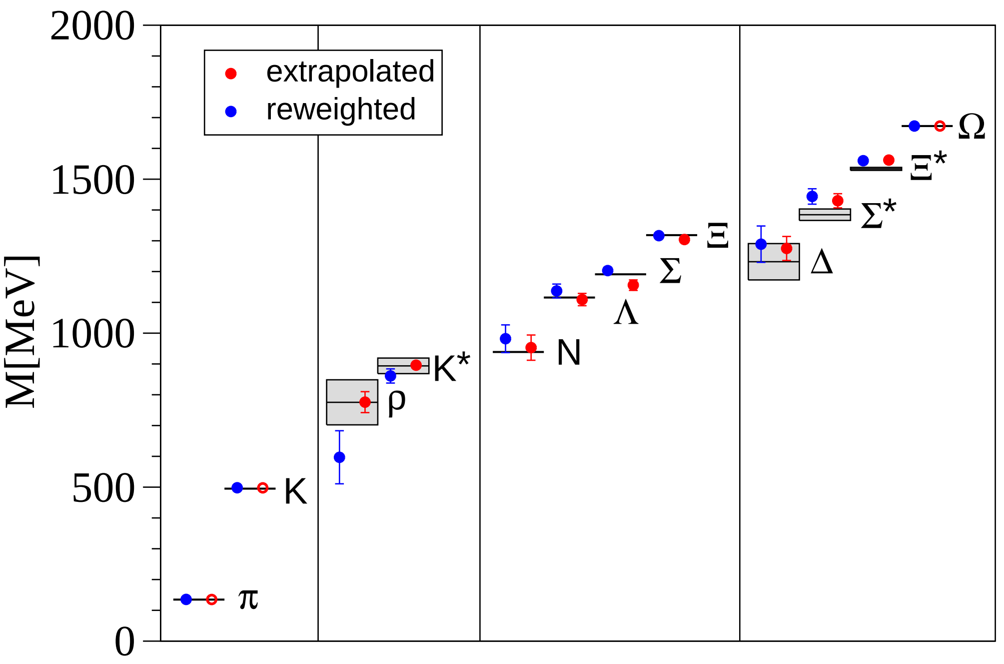

Figure 6: Comparison of the light hadron spectrum results from PACS-CS extrapolated [77] and reweighted [23] ensembles with experiment.

	$(M_{\pi^+} - M_{\pi^0})_{\mathrm{QED}}$	$(M_{K^+} - M_{K^0})_{\mathrm{QED}}$	$(M_n - M_p)_{\mathrm{QED}}$
[83]	4.50(23) MeV	1.33(4) MeV	-0.38(7) MeV
[84]	5.1(1.1) MeV	2.2(0.2) MeV	-

Table 1: Comparison of quenched QED results for the electromagnetic splitting of hadron masses from [83] and [84]. All errors are statistical only.

found.

4. Electromagnetic effects

As the precision in lattice hadron spectroscopy is increasing, subleading effects such as isospin splitting and QED corrections start to become relevant for certain observables.

Electromagnetic splitting in the hadron spectrum was recently investigated in quenched, non-compact QED [83] on RBC/UKQCD $N_f = 2+1$ domain wall ensembles with Iwasaki gauge action at $a \sim 0.11$ fm. In a partially quenched setup valence pions down to a mass of ~ 250 MeV were used. Chiral fits were performed using SU(2) and SU(3) χPT at NLO. Systematic error estimates due to the chiral extrapolation, finite lattice spacing, finite volume and QED quenching were presented. The electromagnetic hadron mass splittings found are summarized in table 1.

At this conference, the WMB collaboration has also presented preliminary results of electromagnetic meson mass splitting [84] using quenched, noncompact QED on a subset of their $N_f = 2+1$, 2 step HEX smeared clover ensembles on a tree level Symanzik improved gauge action. Four ensembles with pion masses in the range $200-450$ MeV were analyzed and an extra additive mass renormalization was applied to light valence quarks in order to retune the theory to an almost unitary point with degenerate u and d masses. In addition a constant electromagnetic background field was introduced to cancel leading finite volume effects. First preliminary results are given in table 1.

Progress has also been reported at this conference regarding the inclusion of quenched QED effects into rooted staggered χPT [85]. This calculation will complement forthcoming MILC simulations including quenched QED that are currently in progress.

5. Excited states

Hadron spectroscopy of excited states is a substantially more difficult task than it is for ground states. While the extraction of ground state masses (at least in the case of non-resonant states with no disconnected contributions) is relatively straightforward, the extraction of higher excited states is substantially more challenging. Excited state contributions to the propagator are exponentially suppressed at large Euclidean times and the relevant energy levels may be hidden between a number of scattering states. Accordingly, finite volume corrections or a continuum extrapolation that are common in ground state spectroscopy are currently beyond the level of precision obtainable in excited state spectroscopy. In some cases even a controlled chiral extrapolation or $N_f = 2+1$ dynamical quarks are currently beyond reach.

The hadron spectrum collaboration is using anisotropic lattices in order to obtain a fine time resolution of the propagators. The lattice spacing in time direction is tuned to be smaller by a factor of $\xi \sim 3.5$ than the lattice spacing in the spatial directions [86]. In their excited state spectroscopy studies [71, 87, 88] they employ $N_f = 2+1$ anisotropic clover fermions on a tree level tadpole improved Symanzik gauge action. A single spatial lattice spacing $a_s \sim 0.12$ fm and three pion masses in the range $390-530$ MeV are used. The scale is set with M_Ω. A variational method based on a large number (6-10) of specifically tailored interpolating operators are used to extract the tower of excited states in the different channels. Results are reported at three different pion masses and show a nice overall qualitative agreement with the experimentally observed excited hadron spectrum (see fig. 7). The authors emphasize the need for multi hadron interpolating operators in order to reliably identify scattering states.

On the more conceptual side, progress in constructing optimized single hadron operators via a combination of Laplacian-Heavyside (LapH) smeared sources with stochastic estimator techniques was presented in [89] and first results of this technique on anisotropic HSC lattices have been reported in [90].

The BGR collaboration has computed ground and excited state hadron spectra using $N_f = 2$ single step stout smeared chirally improved fermions on a tadpole improved Lüscher-Weisz gauge action at a singe lattice spacing $a \sim 0.15$ fm [72, 91, 92]. Three pion masses in the range $320-530$ MeV were used and the scale was set with r_0 . Gaussian smeared quark sources were used in combination with a variational method based on three interpolating operators to extract the energy levels. A chiral extrapolation linear in M_π was performed and the strange quark was introduced in a partially quenched setup. A good signal for the ground state was found but excited and scattering state signals were generally weak.

A long standing puzzle in excited state baryon spectroscopy is the correct ordering of the negative parity ground state $N^{\frac{1}{2}-}$ and the first positive parity excitation of the nucleon (the Roper

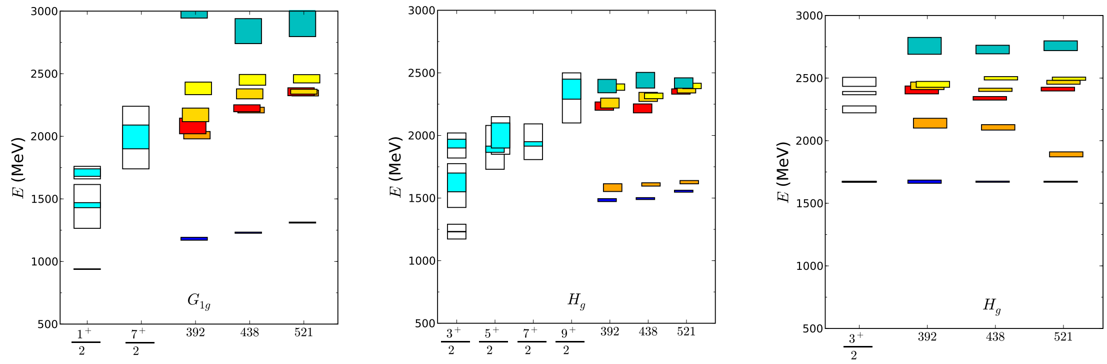

Figure 7: Comparison of part of the excited state spectrum of nucleon (left) Δ (middle) and Ω (right) type baryons as computed by the hadron spectrum collaboration at three different pion masses with experiment. More details can be found in the original paper. The plot is reproduced from [87] with friendly permission of the hadron spectrum collaboration.

resonance). Experimentally, the Roper state has a mass of 1440 MeV while the mass of the $N\frac{1}{2}^-$ is 1553 MeV. On the lattice however, the Roper is usually found to be heavier than the $N\frac{1}{2}^-$.

The CSSM collaboration has recently proposed a method of analyzing eigenstate projected correlation functions of a full multi-source propagator matrix [93]. The method was applied to investigate the level ordering in the excited nucleon system both on quenched [94–96] and $N_f = 2+1$ dynamical [97, 98] configurations. Fat link irrelevant clover (FLIC) valence fermions were used on either a quenched DBW2 gauge action at a single lattice spacing $a \sim 0.13$ fm that was determined by r_0 or the PACS-CS dynamical ensembles discussed in sect. 3. Large operator bases of up to 8 were used and signals for up to 3 excited states were identified. The chiral behavior of both positive and negative nucleon excitations was studied and some evidence was found for the correct ordering of the negative parity ground state and the Roper resonance as one approaches physical pion masses.

6. Decay constants f_π and f_K

We now turn our attention to recent results for the pseudoscalar decay constants f_π and f_K.

The ETM collaboration has performed an analysis of decay constants in $N_f = 2$ twisted mass QCD [75, 80] where f_π was used to set the scale. The analysis in [75] is based on an extension of the ensembles used in the spectrum calculation [13] with a third, coarser lattice spacing $a \sim 0.10$ fm. The strange quark is introduced in a partially quenched setup. A combined chiral and continuum extrapolation is performed and finite size effects have been corrected for. The continuum extrapolation has been performed with $O(a^2)$ and $O(a^2\mu)$ terms (where μ is the twisted light mass) while the chiral fit was done using SU(2) χPT at NLO with an SU(3) fit providing a crosscheck. The final result is given as $f_K/f_\pi = 1.210(18)$.

At this conference, the ETM collaboration has also presented first preliminary results for the decay constants in $N_f = 2+1+1$ twisted mass QCD [6, 99]. They used Iwasaki gauge and a mixed action setup with Osterwalder-Seiler valence quarks. A crosscheck was performed in a unitary setup and unitarity violating effects were found to be within the statistical error. The two

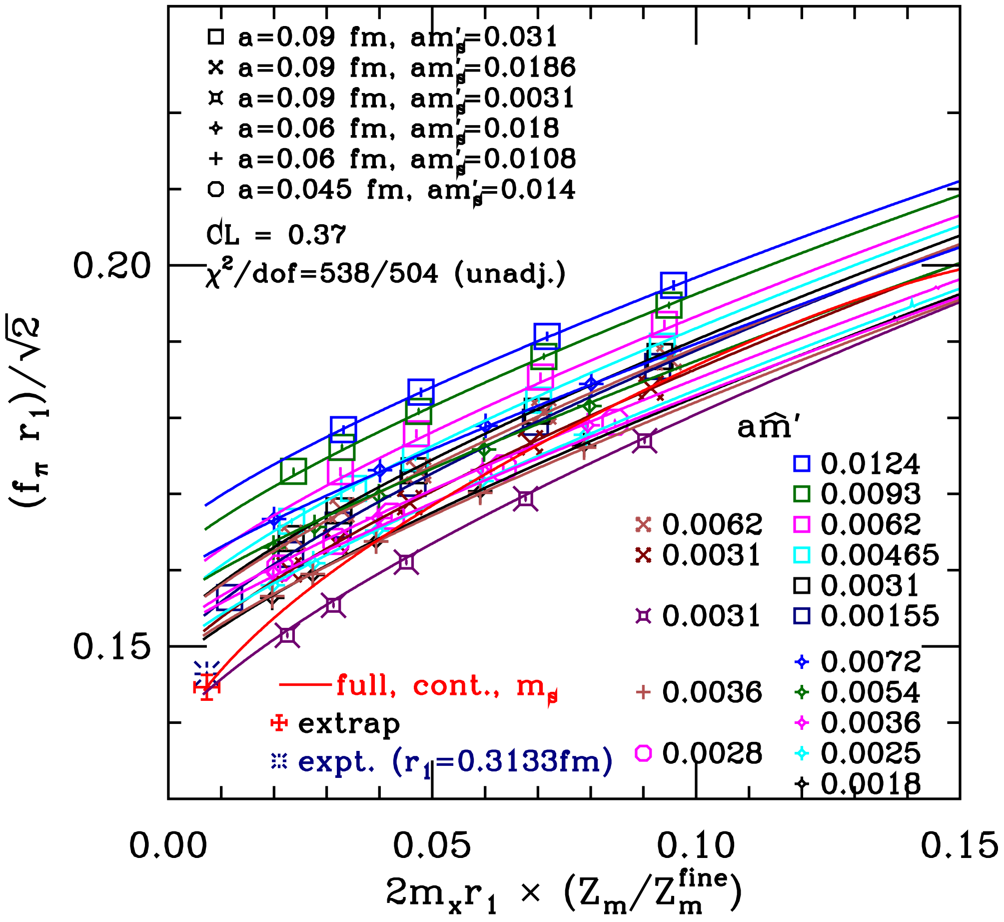

Figure 8: The MILC collaborations SU(3) chiral fits to partially quenched lattice data of the pseudoscalar decay constant in terms of the scale setting parameter r_1. The figure displays part of a combined fit to decay constants and quark masses. Reproduced from [31] with friendly permission of the MILC collaboration.

finest of the available three lattice spacings $a \sim 0.08$ fm and $a \sim 0.06$ fm were used and a constant continuum extrapolation was performed. Finite size effects were corrected using χPT and f_π was again used to set the scale. The preliminary result with statistical errors only is $f_K/f_\pi = 1.224(13)$. Results for individual decay constants have not been given but preliminary results on the axial renormalization constants are available [100].

At this conference, the MILC collaboration has presented new preliminary results on pseudoscalar decay constants using their 3 finest sets of partially quenched $N_f = 2+1$ asqtad rooted staggered fermions [31] updating a previous analysis [101, 102]. They use an elaborate combined fit function for both pseudoscalar decay constants and pseudoscalar masses incorporating full NLO staggered χPT and the full NNLO continuum expression. The LECs of this SU(3) chiral form are first fit in a region of artificially small strange quark mass to avoid convergence issues with the SU(3) chiral expansion. Finally, N^{3}LO and N^{4}LO analytic terms are added and a fit to the large ensemble with approximately physical strange quark mass is performed. This final fit has 504 degrees of freedom and its decay constant part is displayed in fig. 8. Using the HPQCD value of r_1 [103] one obtains $f_\pi = 129.2(0.4)(1.4)$ MeV, which is in nice agreement with the value obtained by an SU(2) chiral fit $f_\pi = 128.3(9)\left(^{+2.0}_{-8}\right)$ MeV [104]. With physical f_π input they further find $f_K/f_\pi = 1.197(2)\left(^{+3}_{-7}\right)$.

Due to an updated scale determination, the HPQCD collaboration has given an update [105] on their older values of f_K and f_π from [106]. The new results are $f_\pi = 132(2)$ MeV $f_K = 159(2)$ MeV. The decay constant ratio has not been updated and remains at $f_K/f_\pi = 1.189(2)(7)$.

The JLQCD/TWQCD collaboration has published a first preliminary value of f_K/f_π with $N_f = 2+1$ dynamical overlap quarks on Iwasaki gauge action in a fixed topological sector [79]. Ensembles with a combination of 5 different pion masses in the range $340-870$ MeV and two different strange quark masse were produced at the single lattice spacing $a \sim 0.1$ fm. Chiral fits were performed using NNLO SU(3) χPT and the scale was set with the physical value of f_π. The results were corrected for conventional finite size effects as well as those associated with fixing the topology. The preliminary result obtained is $f_K/f_\pi = 1.210(12)$ where the errors are statistical only.

The QCDSF-UKQCD collaboration has presented at this conference a first determination of the pseudoscalar decay constant ratio on their $N_f = 2+1$ ensembles [107]. The preliminary result is $f_K/f_\pi = 1.221(15)$.

Using the same ensembles as for the light hadron spectrum[12], the WMB collaboration has recently determined the pseudoscalar decay constant ratio [108]. Finite volume corrections were applied and the continuum extrapolation was done using either a or a^2 terms. In addition three different chiral fit forms (SU(2) resp. SU(3) χPT at NLO and a Taylor series expansion) were used and found to be in perfect agreement as shown in fig. 9. The systematic error was estimated by the spread of the results yielding the final result $f_K/f_\pi = 1.192(7)(6)$.

The PACS-CS collaboration has published preliminary values of the pseudoscalar decay constants both on their $a \sim 0.09$ fm unreweighted [77] and reweighted [23] ensembles. For the extrapolated ensemble, SU(2) and SU(3) χPT chiral forms were used and finite size effects were treated. For the extrapolated ensemble, the renormalization constant Z_A was computed in 1-loop perturbation theory. For the reweighted ensemble Z_A was determined nonperturbatively in a Schrödinger functional method [109]. In both cases, the scale was set by M_Ω. The results are $f_\pi = 134.0(4.2)$ MeV, $f_K = 159.4(3.1)$ MeV and $f_K/f_\pi = 1.189(20)$ for the extrapolated and $f_\pi = 124.1(8.5)(0.8)$ MeV, $f_K = 165.5(3.4)(1.0)$ MeV and $f_K/f_\pi = 1.333(72)$ for the reweighted ensemble where the first error is statistical and the second due to renormalization.

The RBC-UKQCD collaboration has recently published final results of pseudoscalar decay constants using $N_f = 2+1$ partially quenched domain wall quarks on an Iwasaki gauge action [7]. Ensembles were produced at two lattice spacings $a \sim 0.08$ fm and $a \sim 0.11$ fm and a continuum extrapolation linear in a^2 was performed. The scale was set using M_Ω and ensembles were reweighted to the physical strange mass. Analytic and NLO SU(2) χPT forms were used to describe the chiral behavior with finite volume corrections were taken into account. The renormalization constant Z_A was computed nonperturbatively using ratios of local and conserved current matrix elements. The final results obtained are $f_\pi = 124(2)(5)$ MeV, $f_K = 149(2)(3)$ MeV and $f_K/f_\pi = 1.204(7)(25)$. The authors report a marked difference in the continuum extrapolated value of f_π depending on the fit form used with NLO SU(2) χPT giving significantly lower results than a leading order Taylor fit.

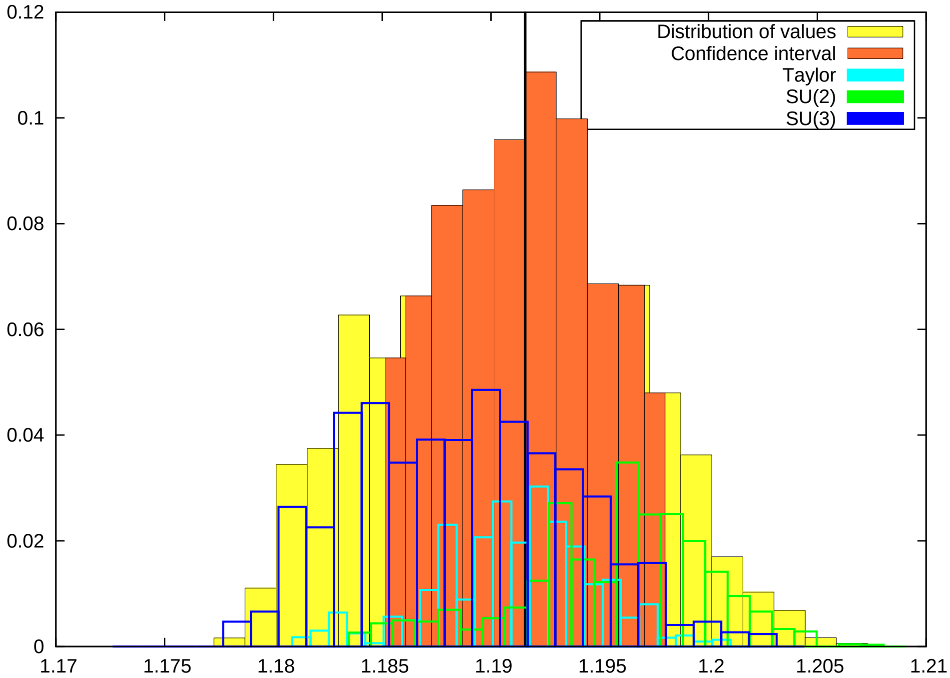

Figure 9: The histogram of results for f_K/f_π from [108] over different analysis procedures weighted by their respective fit quality. Subhistograms for the three different chiral fit forms are also plotted. One can see that their spread is smaller than their width and their integrated fit quality is approximately equal.

Fig. 10 summarizes recent lattice determinations of the decay constant ratio f_K/f_π. There is a remarkable overall consistency both among the different determinations and between the direct lattice determinations and the standard model expectation.

7. Concluding remarks

It is fair to say that in recent years lattice QCD has fulfilled one of its original promises - the post-diction of the ground state light hadron spectrum. There is a remarkable convergence of results from different groups using a wide range of lattice actions and analysis techniques which demonstrates that a large part of the experimentally observed ground state hadron spectrum can be reproduced within a few percent accuracy. The most important message from the lattice to the general particle physics community is that our techniques are in place and our sources of systematic errors are understood and in some relevant cases under control. Both central values and total error estimates of phenomenologically relevant lattice predictions such as those of the pseudoscalar decay constants can be trusted and contain valuable physics information. In these areas lattice calculations are preparing to enter the sub-percent level precision regime with inclusion of subleading effects such as QED corrections.

In contrast, the current status of excited state hadron spectroscopy is a reminder that precision calculations are currently possible only for a small subset of observables that are relatively easy to compute. Although progress has been impressive, it still remains a challenging task to obtain a full quantitative understanding of the excited hadron spectrum.

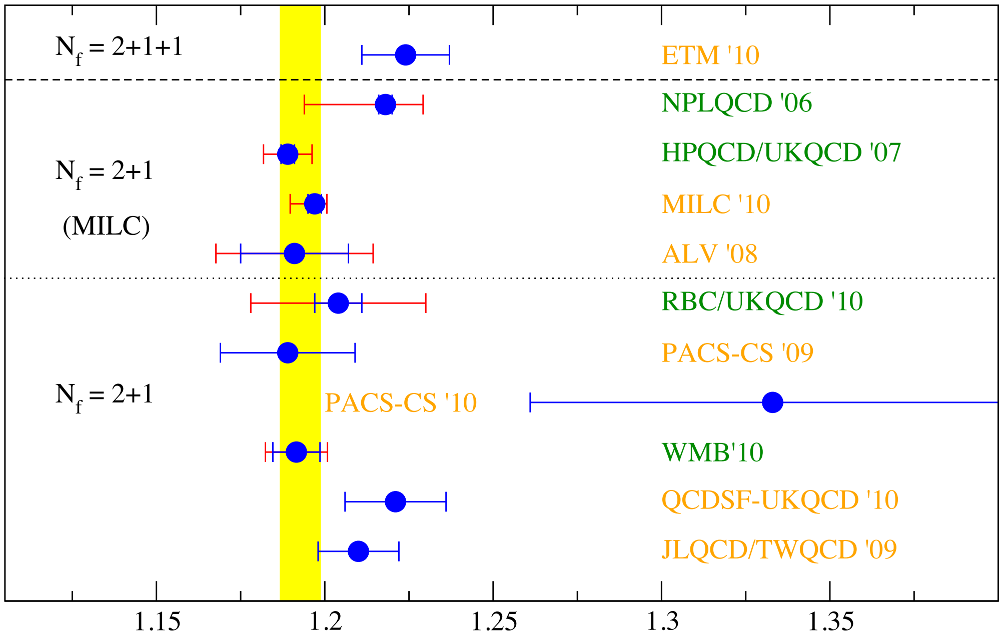

Figure 10: Comparison of recent results for the ratio of decay constants f_K/f_π in $N_f = 2+1$ and $N_f = 2+1+1$ QCD. Green entries correspond to final results while orange ones are preliminary. The four determinations based on the MILC ensembles are separated in the plot. Statistical (blue) and full (red) error bars are given where available. The yellow band corresponds to an indirect determination assuming first row CKM unitarity and using the values $|V_{ud}| = 0.97425(22)$ from [110], $|V_{ub}| = 0.00389(44)$ from [111] and $\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27599(59)$ from [112]. Lattice results are taken from ETM'10 [99], NPLQCD'06 [113], HPQCD/UKQCD'07 [106], MILC'10 [31], ALV'08 [114], RBC-UKQCD'10 [7], PACS-CS'09 [77], PACS-CS'10 [23], WMB'10 [108], QCDSF-UKQCD'10 [107] and JLQCD/TWQCD'09 [79].

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