PROCEEDINGS OF SCIENCE

Center Symmetry Restoration with 2 Flavor Large N Yang-Mills in the Adjoint Representation

Simon Catterall
Syracuse University
E-mail: smc@physics.syr.edu

Richard Galvez*
Syracuse University
E-mail: ragalvez@syr.edu

We report on the restoration of center symmetry in two flavor large N Yang Mills lattice field theory with dynamical fermions in the adjoint representation. Numerical evidence is given to show correlators of $|P_\mu|$ tend to zero in the large N limit. Wilson fermions were employed on a 2^4 sized lattice for a variety of bare quark masses and coupling strength. We argue that this model may offer an alternative route to understanding the conformal window of Yang Mills with dynamical adjoint fermions.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010
June 14-19, 2010
Villasimius, Italy

*Speaker.

PoS(Lattice 2010)050

1. Physics Motivation– Technicolor

There has been much recent interest in theories of strong dynamics, which may serve to break EW symmetry [1, 2]. In such theories, the Higgs appears as a composite particle of "techniquarks" – new fermions with a new gauge interaction, typically SU(N). In these models the W,Z bosons acquire their masses by eating pseudo Goldstone bosons associated with the spontaneous breaking of a new global chiral symmetry. However, due to their strong interaction dynamics, Technicolor theories require non-perturbative techniques to make predictions about quantities of physical interest such as the chiral condensate, technihadron spectrum, mass gap, string tension, etc. This is where the lattice community has contributed most recently, supplying non-perturbative computations [3, 4, 5, 6].

Due to possible conflicts with EW precision experiments, Technicolor dynamics cannot be exactly as scaled down QCD. One possible scenario is for the theory to be near conformal or *walking* [7]. Minimal key ingredients towards walking dynamics have been explored using an SU(2) gauge group with $N_f = 2,3$ by various groups in the lattice community [8, 9].

One thing that has become obvious thus far is that simulations of walking technicolor can become computationally demanding fairly quickly. The issue is that near conformality implies that finite volume effects become relevant, since since the theory would be quasi scale independent and correlation lengths grow without bound. It would seem that we need larger and larger lattices to simulate this theory and also clever ways to extrapolate the infinite volume limit. This has been investigated by multiple groups using various volume extrapolation techniques. Most of the work can be found here [10, 4, 5].

We however prepose a different way of investigating near conformal behavior using Eguchi-Kawai (EK) reduction [11]. For the most part, this approach has been discontinued by the Lattice community until recently when various groups have found that center symmetry is realized with fermions in the adjoint representation [12, 13, 14, 15]. Traditionally one could always circumvent the center symmetry problem with fundamental fermions by working in two dimensions [16].

Essentially, EK reduction provides a means of measuring infinite volume correlators using finite volume simulations provided that the number of colors is taken to infinity. The primary barrier to using EK reduction is the spontaneous (or explicit) breaking of center symmetry since without it the formal proof of EK reduction fails. For the proof of center symmetry breaking in pure gauge theory, see the seminal paper by Bhanot Heller and Neuberger [17].

With center restored in adjoint gauge theory, the attention is thus shifted from a large volume extrapolation towards a large N one. Since there are many simplifications available to us at large N, we consider that this extrapolation would yield more intuition towards the understanding of the conformal window in ETC theories.

Another important point one should consider is that the transitional value of N_f that gives one a conformal field theory (N_f*) is independent of N_c at leading order in N [8]. Thus the question of the conformality or otherwise of theories involving adjoint fermions could in principle be decided on the basis of large N, small volume simulations. This is the approach we pursue. The simulations reported in this talk constitute a numerical proof that this approach is valid – that is we investigate the question of center symmetry breaking in 2 flavor adjoint $SU(N)$ gauge theories with Wilson fermions on small lattices. We show center symmetry is restored in the two flavor case.

1.1 Back of the Envelope Proof

As a quick sketch of a proof for EK reduction, consider the following. First we know that translational symmetry must hold since one point is essentially identified with the next. Center symmetry for each compactified spatial direction must also hold for reasons to be clear in a moment.

In general, Eguchi and Kawai showed that if $\mathcal{O}$ is some operator in some SU(N) gauge theory, $< \mathcal{O}_f >= \Gamma$, while $< \mathcal{O}_r >= \Gamma +$ some extra terms proportional to Wilson lines, like $< \mathrm{TrU}_\mu \mathrm{TrU}_\nu \ldots >$. Now we can use factorization in N: $< \mathrm{TrU}_\mu \mathrm{TrU}_\nu >=< \mathrm{TrU}_\mu >< \mathrm{TrU}_\nu > + \frac{1}{\mathrm{N}}$(something finite..). Applying a center symmetry transformation on all the fields: $U_\mu \to zU_\mu$ for $z \in$ SU(N), we find that $< \mathrm{TrU}_\mu >$ must $\to 0$, as it is a non-symmetric product in the center-symmetric theory. Therefore, without too much analysis, we find that expectation values in the reduced theory and in the full theory must agree, but only strictly at infinite N.

Polyakov (or Wilson) lines thus become relevant order parameters. For SU(N), center symmetry is Z_N, i.e. $U_\mu \to e^{\frac{i2\pi k}{N}} U_\mu$ for integer values of k. But what about adjoint fermions? One-loop analysis shows that center symmetry is restored with the addition of adjoint quarks endowed with periodic boundary conditions and arbitrary N_f [18]. This needed to be checked non-perturbatively and in particular, finite coupling. One flavor was checked numerically by [13] and 1/2 flavor [15]. We address two flavors in this paper with the foresight of later applying our understanding to the conformal window problem.

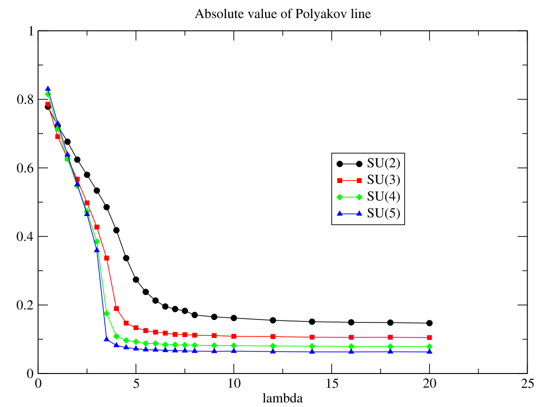
2. Numerical Results

Simulations were carried out on a small volume of 2^4 with N ranging from 2 - 7 and t' Hooft coupling $1/\beta = \lambda = g^2 N$ values of $0.5, 1, 2, 5$. We used Wilson dynamical adjoint fermions with bare quark masses between -2 - 8 and found the critical line to be near mass = -1. The standard HMC algorithm was used, with in the order of a 1000 measurements. As one can see in the attached figures, we were able to reproduce the known result that center symmetry spontaneously breaks with fundamental fermions. However, when we simulated with adjoint fermions we found that center symmetry is maintained.

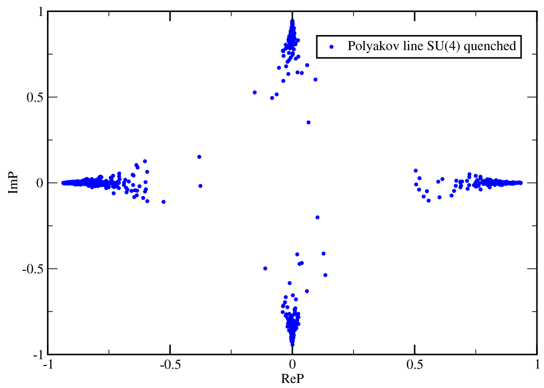
3. Conclusions

We find finally that $| < P_\mu > |$ is consistent with zero as λ goes to 0. More exotic order parameters like $< P_1 P_2^\dagger >$ also $\to 0$. Hence we now have non-perturbative reason to believe EK reduction holds with adjoint fermions. Main conclusion colaborators say it best (M. Ünsal and L. Yaffe) [19]: "The 1/N suppression of finite volume effects in large N center symmetric theories allows one to trade a large volume extrapolation for a large N extrapolation, and should be helpful for studies of conformal windows in the large N theories." We may now hence determine N_f* by simulating the SU(N) theory at small volumes with large N.

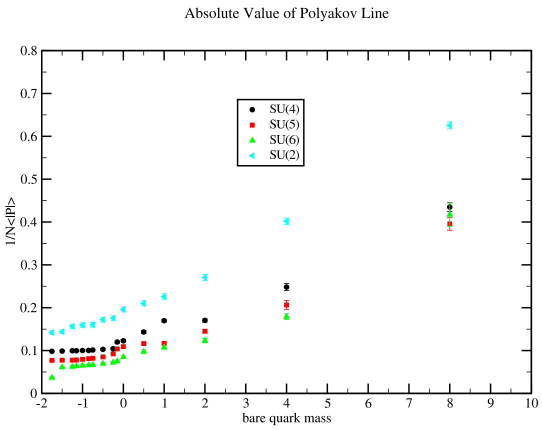
We are presently working on measurements of the meson propagators. The large N scaling of the meson masses should allow us to determine whether or not SU(∞) $N_f = 2$ for vanishing quark masses is conformal, near conformal, or no where near.



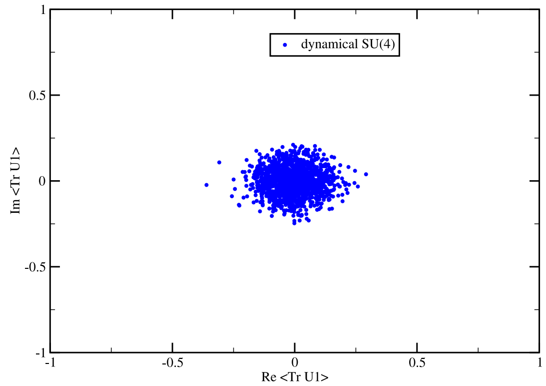
(a) Pure gauge breaks center ($m = \infty$), as expected.



(b) Pure gauge breaks center ($m = \infty$), as expected.



(c) Inclusion of adjoint fermions restores center.



(d) Inclusion of adjoint fermions restores center ($m_{bq} = -1.$)

Figure 1: In subfigures (a) and (b) we see that pure gauge breaks center symmetry as expected, while center is restored when adjoint fermions are included as can be evidenced by subfigures (c) and (d)

References

[1] F. Sannino and K. Tuominen, Phys. Rev. D **71**, 051901 (2005) [arXiv:hep-ph/0405209].

[2] H. S. Fukano and F. Sannino, Phys. Rev. D **82**, 035021 (2010) [arXiv:1005.3340 [hep-ph]].

[3] S. Catterall, L. Del Debbio, J. Giedt and L. Keegan, arXiv:1010.5909 [hep-ph].

[4] S. Catterall, J. Giedt, F. Sannino and J. Schneible, arXiv:0910.4387 [hep-lat].

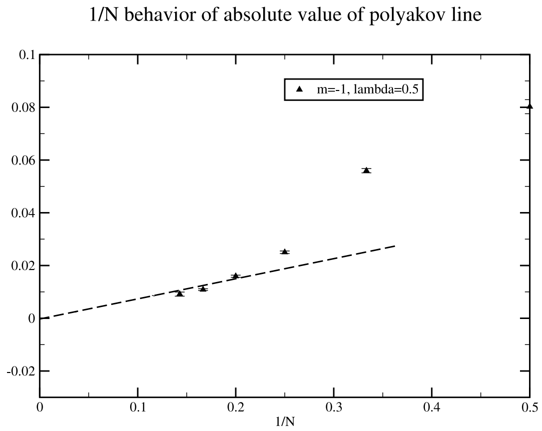
[5] A. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, Nucl. Phys. A **820**, 191C (2009).

[6] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, Phys. Rev. D **82**, 014510 (2010) [arXiv:1004.3206 [hep-lat]].

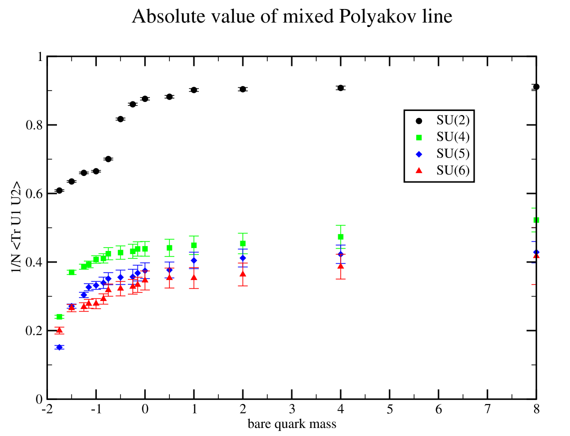
[7] N. Evans and F. Sannino, arXiv:hep-ph/0512080.

[8] D. D. Dietrich and F. Sannino, Phys. Rev. D **75**, 085018 (2007) [arXiv:hep-ph/0611341].

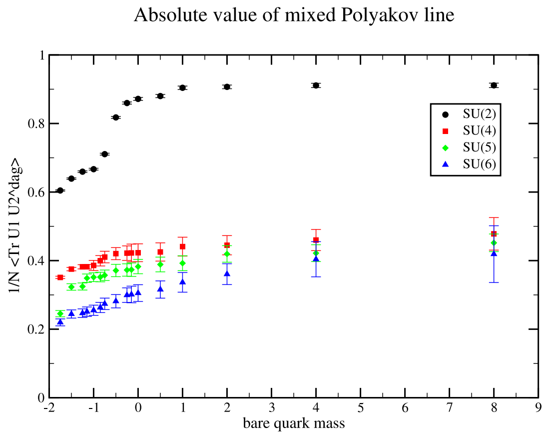
[9] A. J. Hietanen, K. Rummukainen and K. Tuominen, Phys. Rev. D **80**, 094504 (2009) [arXiv:0904.0864 [hep-lat]].



(a) 1/N dependence of $|< P_1 >|$.



(b) More exotic products may break center, but they do not.



(c) More exotic products may break center, but they do not.

Figure 2: In subfigure (a) we fix $\lambda = 0.5, m_{bq} = -1$ and examine 1/N dependence. We note that, excluding finite N effects, the y-intercept of $|< P_1 >|$ is consistent with zero. In subfigures (b) and (c) we calculate other exotic correlators that indeed also tend to zero as 1/N.

[10] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, Phys. Rev. D **80**, 074507 (2009) [arXiv:0907.3896 [hep-lat]].

[11] T. Eguchi and H. Kawai, Phys. Rev. Lett. **48**, 1063 (1982).

[12] S. Catterall, R. Galvez and M. Unsal, JHEP **1008**, 010 (2010) [arXiv:1006.2469 [hep-lat]].

[13] B. Bringoltz, JHEP **1001**, 069 (2010) [arXiv:0911.0352 [hep-lat]].

[14] A. Hietanen and R. Narayanan, arXiv:1001.2856 [hep-lat].

[15] A. Hietanen and R. Narayanan, arXiv:0911.2449 [hep-lat].

[16] R. Galvez, A. Hietanen and R. Narayanan, Phys. Lett. B **672**, 376 (2009) [arXiv:0812.3449 [hep-lat]].

[17] G. Bhanot, U. M. Heller and H. Neuberger, Phys. Lett. B **113**, 47 (1982).

[18] P. Kovtun, M. Unsal and L. G. Yaffe, JHEP **0706**, 019 (2007) [arXiv:hep-th/0702021].

[19] M. Unsal and L. G. Yaffe, JHEP **1008**, 030 (2010) [arXiv:1006.2101 [hep-th]].