PROCEEDINGS OF SCIENCE

Charmonium spectroscopy from $N_f = 2+1$ dynamical anisotropic lattices

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Preliminary results for the charmonium spectrum are presented. The calculation uses distilled quark propagators measured on the $N_f = 2+1$ dynamical anisotropic lattices of the Hadron Spectrum Collaboration and includes an exploration of disconnected diagrams and decays relevant for threshold effects.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010
June 14-19, 2010
Villasimius, Italy

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1. Introduction

The discovery at Belle and Babar of new, unexpected charmonium states has led to a renaissance in charmonium spectroscopy. After a hiatus of many years the observation of states such as the X(3872) [1], Y(4260) [2] and Y(3940) [3] presents a new challenge for the theoretical physics community. In principal, lattice QCD can determine the complete spectrum of states (including exotics and hybrids). Such precision spectroscopy requires control of systematic uncertainties as well as reliable identification of excited states (which fall faster than ground states), statistical precision of the intrinisically noisy gluonic excitations in hybrids, isoscalar states and glueballs and a method to handle threshold effects including mixings and disconnected contributions.

In this paper we describe how using a new algorithm for quark propagation on dynamical anisotropic lattices greatly improves statistical precision as well as facilitating the inclusion of disconnected and threshold effects at the few-percent-level of precision. Preliminary results for the charmonium spectrum are presented and future plans are discussed.

1.1 Technical details

In this study we use an anisotropic lattice to enhance the temporal resolution. The anisotropy is tuned nonperturbatively such that $\xi = a_s/a_t = 3.5$ as described in Ref. [4]. The gauge action is tree-level Symanzik-improved while quarks are simulated with a tree-level tadpole-improved Sheikholeslami-Wohlert action, including spatial stout links [5] for quark propagation. Table 1 lists some properties of the lattices used in this study while a full description of the Hadron Spectrum Collaboration lattices can be found in Ref. [6]. A detailed study of the light isovector meson spectrum and the light baryon spectrum have already been carried out on these ensembles [7, 8]. A comprehensive determination of the charmonium excited state spectrum in the quenched theory is described in Ref. [9]. This work extends that earlier study.

N_f	Volume	m_π	$a^{-1}_{(m_\Omega)}$
3	$12^3 \times 96$	702MeV	4.76 GeV
2+1	$16^3 \times 128$	392MeV	5.67 GeV

Table 1: Simulation parameters for the lattices used in this study.

We exploit the anisotropic lattices to treat the charm quarks relativistically, with the same Sheikholeslami-Wohlert action used for the light quarks. In both the temporal and spatial directions am_{charm} is less than unity. For the $16^3 \times 128$ lattices the combination $a_t m_{\text{charm}}$ is $\sim 1/4$ while in the spatial direction $a_s m_{\text{charm}}$ is less than unity. Since measurements are made in the temporal direction errors of $\mathcal{O}(a_s m_{\text{charm}})$ are not expected to play any significant systematic role. Nevertheless, a determination of the dispersion relation for the η_c is underway.

2. Theoretical details

2.1 Spin asssignment in lattice QCD

On a cubic lattice states are characterised by their transformation properties under the octahedral group, O_h and fall into one of ten irreducible representations of this group (including parity).

Continuum spin assignment is made by forming the representations of O_h subduced from $SO(3)$. Care must then be taken so that the correct degeneracies across lattice irreps are reproduced as the continuum is approached. This can be particularly difficult in charmonium where small hyperfine splittings must be disentangled from amongst these degeneracies. However, using the amplitude of operators as in Ref. [9] has proven very effective and will be incorporated in future work.

2.2 Operator construction

The preliminary results presented at this conference were determined from a subset of operators that can be formed from single-site as well as one-link and two-link displacements, described by $\mathcal{O}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)\psi_\beta(x)$, $\mathcal{O}^i_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)\psi_\beta(x+\hat{\imath})$ and $\mathcal{O}^{ij}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)U_j(x+\hat{\imath})\psi_\beta(x+\hat{\imath}+\hat{\jmath})$, from which the S, P and some D waves have been determined. A more complete basis construction is underway and will be used in a variational analysis of the $16^3 \times 128$ dataset.

2.3 Distillation

The charmonium spectrum is studied using a new algorithm for quark propagation called distillation. This method was described at this conference [10, 11, 12, 13, 14] and in detail in Ref. [15]. In brief, distillation amounts to a redefinition of smearing. It stems from the observation that optimal smearing results in fields with support in a very low dimensional vector space compared to the original ones. A projection operator is defined onto a low-dimensional space of fields,

$$\Box = \sum_{k=1}^{M} v^{(k)} \otimes v^{(k)*}. \tag{2.1}$$

Typically $v^{(k)}$ is chosen to be the k^{th} lowest eigenvector of Δ^2, the lattice three-dimensional gauge-covariant laplace operator. Two point correlation functions are written in the usual way as

$$C(t_1,t_0) = \mathrm{Tr}\Box(t_1)\Gamma_1\Box(t_1)M_c^{-1}(t_1,t_0)\Box(t_0)\Gamma_0\Box(t_0)M_{\bar{c}}^{-1}(t_0,t_1), \tag{2.2}$$

where $\Gamma_{0,1}$ are the creation operators to make mesons with the appropriate quantum numbers. Inserting the definition of the distillation operator this becomes a trace over a product of rank-M matrices,

$$C_{ab}(t_1,t_0) = \mathrm{Tr}\phi_a(t_1)\tau(t_1,t_0)\phi_b(t_0)\tau(t_0,t_1), \tag{2.3}$$

with $\phi_a^{(i,j)} = v^{(i)*}\Gamma_a v^{(j)}$ and $\tau^{(i,j)} = v^{(i)*}(t_1)M^{-1}(t_1,t_0)v^{(j)}(t_0)$. This factorisation means the choice of source and sink operators (which only appear in the $\phi_a^{(i,j)}$) is independent of the quark propagation which is encoded in the $\tau^{(i,j)}$, called perambulators. While the calculation of perambulators is costly, they can be stored and used *a posteriori* to correlate any source and sink operators. In addition, distillation does not preclude using stochastic methods and results using this combination are reported at this conference [11, 12]. Figure 1 shows the effective masses of the η_c and the h_c at timeslice five as a function of the number of eigenvectors included, on the $12^3 \times 96$ lattices. It shows that for increasing number of eigenvectors the effective mass falls, indictating an earlier onset of the plateau. The same study is being repeated on the $16^3 \times 128$ lattices to determine the optimal number of eigenvectors to use.

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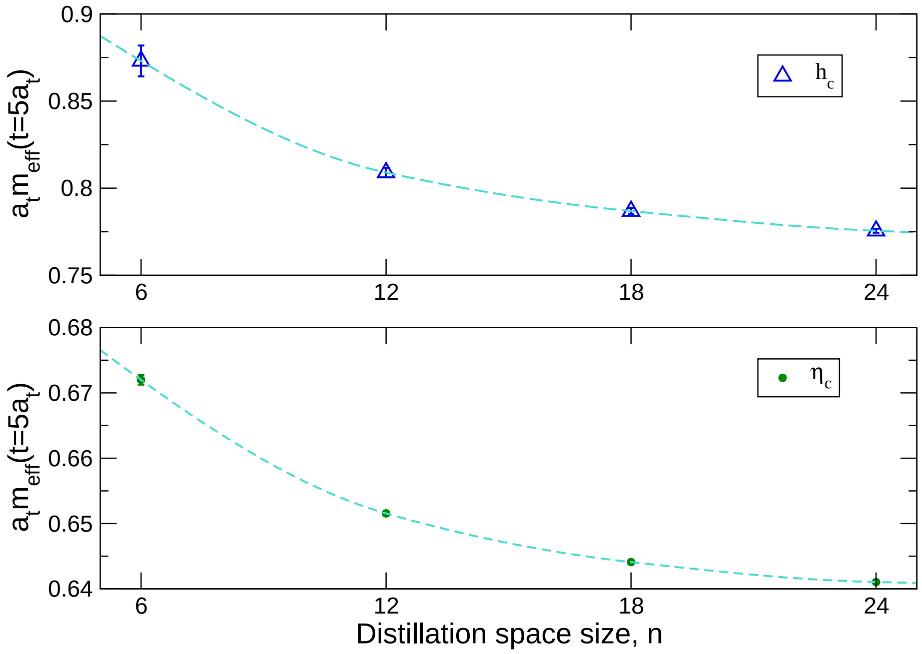

Figure 1: The effective mass at timeslice five as a function of the number of eigenvectors included in the distillation space for an S wave (η_c) and P wave (h_c). This study was carried out on the $12^3 \times 96$ lattice.

3. Results

Preliminary results for connected charmonium correlators for S, P and D wave states as well as the 1^{-+} hybrid are presented on the $16^3 \times 128$ lattices for 32 configurations with inversion on all timeslices in the perambulators. An investigation of distillation for disconnected contributions and mixings for states above threshold is also presented on $12^3 \times 96$ lattices with 100 configurations.

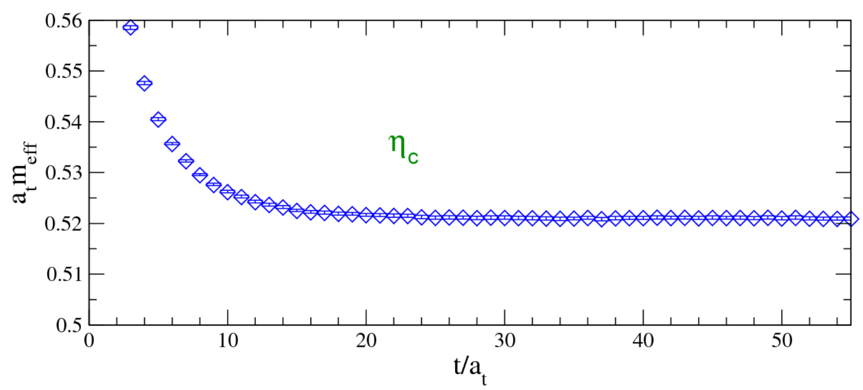

Figure 2: The effective mass of the η_c, on a $16^3 \times 128$ lattice for 32 configurations and inversions on all timeslices. The precision on the fitted mass is less than 1MeV.

Figure 2 shows the effective mass of the η_c, where the statistical errors are below the percent level. The fitted mass is determined to a precision of less than 1MeV. In Figure 3 the effective mass of the 1^{-+} hybrid meson is shown. In this case the error on the fitted mass is below 2%, corresponding to

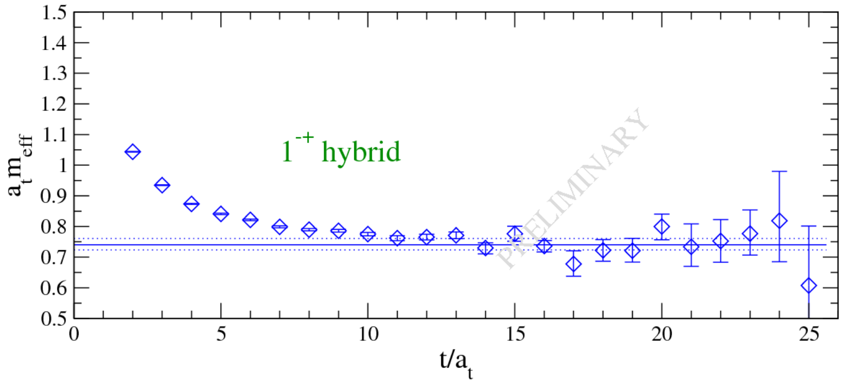

Figure 3: The effective mass of a charmonium hybrid - the 1^{-+} state. The error on the fitted mass is $< 2\%$ corresponding to $\sim$ 60MeV.

approximately 60MeV. In this analysis single correlators are fitted to single exponentials and errors are determined from 100 bootstrap samples while the fit ranges were chosen by eye.

The preliminary spectrum of states is shown in Figure 4. The states are labelled assuming a quark-model spin identification. A comprehensive analysis is underway using the techniques developed in Ref. [9] to identify the states. The results are presented in lattice units in this exploratory analysis. Note that the precision on both S-waves (the η_c and J/Ψ) is at the percent level and the

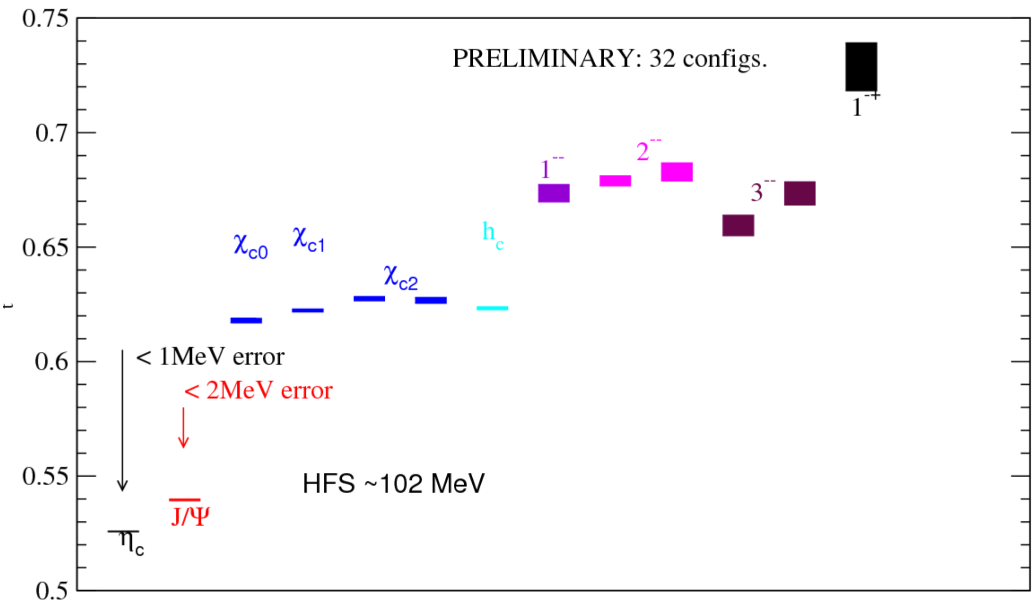

Figure 4: A preliminary spectrum of S, P and D waves and the 1^{-+} hybrid.

splitting between these states is approximately 102MeV. The hyperfine splitting is known to depend on the lattice spacing as well as the light quark masses so it is not surprising that it is low, nevertheless it is encouraging that with the simulation parameters used here the result is reasonably close to the experimental value.

To make a complete determination of the charmonium spectrum requires, in addition to precision data for connected correlators, information about the contribution of disconnected correlators while for states above threshold the effects of mixings with for example multi-hadron states is needed. This has been very challenging for lattice calculations to date, requiring all elements of the heavy quark propagators determined precisely. Using distillation simplifies these calculations enormously and very encouraging results have already been determined in the light hadron sector [7].

3.1 Disconnected correlators and threshold effects

In charmonium the contribution of disconnected diagrams to the η_c is expected to be small due to OZI suppression. A measurement has proven difficult however as all-to-all propagators are needed for a direct measurement and in addition the signal in the charm sector falls rapidly into the noise. Figure 5 (left pane) shows the disconnected correlator determined on the $12^3 \times 96$ lattices. Using distillation the correlation function is simplified to two separate traces over the distillation spaces, written,

$$D_{ab}(t',t) = \mathrm{Tr}\left[\phi_a(t)\tau(t,t)\right]\mathrm{Tr}\left[\phi_b(t')\tau^{\dagger}(t',t')\right], \qquad (3.1)$$

using the notation of Section 2.3. The statistical errors on the data points are at the ten percent level and the signal persists through six timeslices.

In charmonium only the $1S$, $2S$ and ground states of the P waves lie below the $D\bar{D}$ threshold for open charm decay. To make an unambiguous identification of states above threshold requires analysis of operators that couple to multi-particle states. Once again, distillation makes this calculation

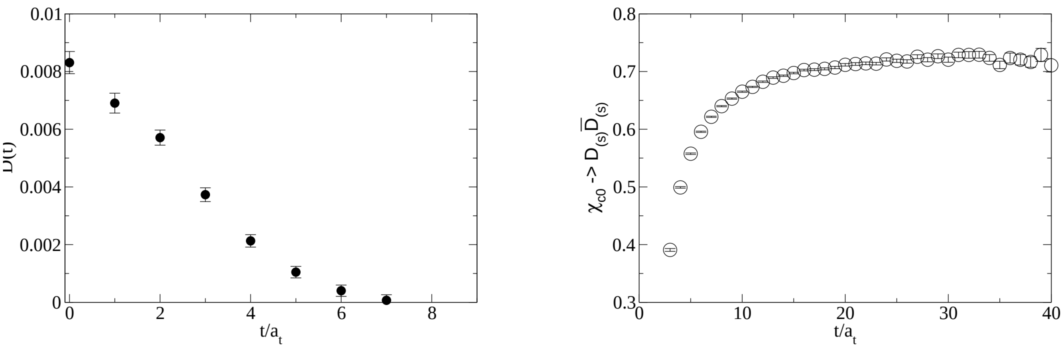

Figure 5: The left plot shows the disconnected correlator for the η_c on the $12^3 \times 96$ lattice. The right plot shows the correlator for the decay $\chi_{c0} \to D_{(s)}\bar{D}_{(s)}$ relevant for threshold effects, again on a $12^3 \times 96$ lattice.

relatively straightforward. The right pane in Figure 5 shows the correlator for the $\chi_{c0} \to D(s)\bar{D}_{(s)}$. We find a convincing signal with small statistical errors.

4. Conclusions

A first look at charmonium spectroscopy on the $N_f = 2+1$ dynamical anisotropic lattices of the Hadron Spectrum Collaboration is presented. Distillation is used to improve statistical precision and to simplify the determination of disconnected correlators and mixings required to study threshold effects. The preliminary spectrum of ground-state S, P and D waves and a hybrid was found to be precisely determined and in good agreement with experiment. A comprehensive study of the spectrum is underway.

Acknowledments

It is a pleasure to thank my colleagues in the Hadron Spectrum Collaboration. SR is supported by the Research Executive Agency (REA) of the European Union under Grant Agreement number PITN-GA-2009-238353 (ITN STRONGnet).

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