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Highly excited and exotic meson spectroscopy from dynamical lattice QCD

Christopher E. Thomas* **(for the Hadron Spectrum Collaboration)**
Jefferson Laboratory
E-mail: thomasc@jlab.org

We discuss recent progress in extracting highly excited and exotic meson spectra from lattice QCD. A new method for identifying the continuum spin of states is reviewed and its application to the light meson sector is presented. A combination of techniques and dynamical anisotropic lattices have enabled us to extract, and confidently identify the spin of, an extensive spectrum of excited states. Highlights include many states with exotic quantum numbers and, for the first time in a lattice QCD calculation, spin-four states. We conclude with some comments on future prospects

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1. Introduction

A complete understanding of the strong interaction requires a calculation of the spectrum of QCD and comparison against high quality experimental data. Lattice QCD provides one first-principles method for computing the spectrum and the calculation of the masses of the lowest-lying states has long been an important benchmark of such calculations. However, extracting clean signals for excited states and exotics has proven difficult. Recently, the Hadron Spectrum Collaboration has made significant progress in studying excited states, states of high spin and exotics using lattice QCD calculations. Techniques for extracting spectra [1] and radiative transition amplitudes [2, 3] were first testing in a quenched calculation in the charmonium sector, a system which is less computationally demanding and where there is a profusion of experimental data. These methods are now starting to be applied to the light meson sector [4, 5] with an eventual aim of calculating excited and exotic meson photocouplings, relevant for, amongst other things, the GlueX experiment, part of Jefferson Lab's 12 GeV upgrade, which will perform a comprehensive investigation of the spectrum of light mesons in photoproduction

Over the last few years there has been a resurgence of experimental interest in meson spectroscopy, particularly in the charmonium sector where a wealth of data has emerged from the B-factories, CLEO-c, BES and other experiments, in turn fuelling much theoretical work. Of particular interest are states with exotic quantum numbers, such as $J^{PC} = 0^{+-}$, 1^{-+} and 2^{+-}, and a particular aim of the GlueX experiment is to produce such exotic mesons. Such a state signals physics beyond that of just a quark-antiquark pair, one possibility being a hybrid meson where the gluonic degrees of freedom play a non-trivial role and another being a multiquark or molecular mesons where there is at least one extra quark-antiquark pair.

In Section 2 we outline the techniques used to extract excited spectra and in Section 3 we highlight some results, referring to Refs. [4, 5] for full details and references. We conclude in Section 4 with some comments on future prospects.

2. Extracting excited meson spectra and spin identification

As described in Ref. [5], we calculate the two-point correlation functions, $C(t) = <0|\mathscr{O}_i(t)\mathscr{O}_j(0)|0>$, for a large basis of operators and then use the variational method. The variational method boils down to solving a generalised eigenvector problem where the eigenvalues give E_n and the eigenvectors are related to the matrix elements, $Z_i^{(n)} \equiv < n|O_i|0 >$ (the overlaps of operators onto states). A major advantage of this method is that, because of the orthogonality of the eigenvectors, it can separate states that are close together in mass, whereas extracting nearly degenerate states in a multi-exponential fit to a single correlator is a rather unstable procedure with noisy data. The variational method also gives the linear combination of operators, $\Omega^{(n)}$, which has the 'best' overlap onto a particular state n. The Z values can give information about the nature of states[1, 6, 5].

Our interpolating operators, $\mathscr{O}$, are constructed from a fermion bilinear with gamma matrices and lattice-discretised gauge-covariant derivatives, overall projected onto definite momentum $\vec{p}$, $\mathscr{O} = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\bar{\psi}(x)\Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x)$, and are chosen to have the quantum numbers of the states under investigation[1]. We construct the operators in such a way that, in the continuum, each operator overlaps only onto states with a certain J^{PC}.

[1]We only consider $\vec{p} = \vec{0}$ here.

J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

Table 1: Continuum spins subduced into lattice irreps $\Lambda(\text{dim})$ [5]

In lattice QCD the theory is discretised on a finite four-dimensional Euclidean hypercubic grid: the three-dimensional rotational symmetry of the continuum is reduced to that of the octahedral group, equivalent to the symmetry group of a cube. Therefore, states are no longer labelled by the continuum spin, J, but rather by lattice irreducible representations (*irreps*), Λ. There are only five single-cover lattice irreps for each parity and charge conjugation (A_1, T_1, T_2, E and A_2) and the various components of a spin-J meson are distributed across the various irreps, known as subduction, as shown in Table 1. In general, the components of each J are spread across multiple irreps and each irrep contains many J. In the continuum limit, where full rotational symmetry is restored, the components subduced into two different irreps (e.g. the components of a spin-two state subduced into the T_2 and E irreps) will be degenerate, but at finite lattice spacing they will be split.

A simple method for assigning continuum spins would be to extrapolate to the continuum limit and then identify degeneracies across irreps according to the patterns given in Table 1. However, apart from the high computational cost of performing the calculations on successively finer lattices, there are many approximate degeneracies in the spectrum and it is easy to confuse these with those arising from discretisation effects. For example, the $\chi_{c0,1,2}$ triplet of states in charmonium is split only by the small spin-orbit interaction. These states would appear as a state in each of the irreps A_1^{++}, T_1^{++}, T_2^{++} and E^{++}, exactly the same pattern as a spin-four state split by discretisation effects.

An important recent advance has been the development of a method for spin-identification without extrapolating to the continuum limit, instead considering the overlap of states onto carefully constructed operators. Ref. [4] contains a short exposition of this method and Ref. [5] a more detailed discussion and more extensive results. First, the relative magnitudes of the extracted Z values can be considered and it is found that each state has large overlap only onto operators of a single spin; Fig. 1 shows an example of this in the Λ^{--} irreps. We emphasise that in the continuum limit each of the operators only overlaps onto states with a particular J^{PC}. Second, states are matched across different irreps and the Z values of common operators are compared. At finite lattice spacings small deviations from equality are expected and Refs. [4, 5] show that these Z values agree well across irreps. In the following section we show some first results from the application of this method.

3. Highlights of results

For these calculations we used dynamical anisotropic lattices [7, 8] with spatial lattice spacing,

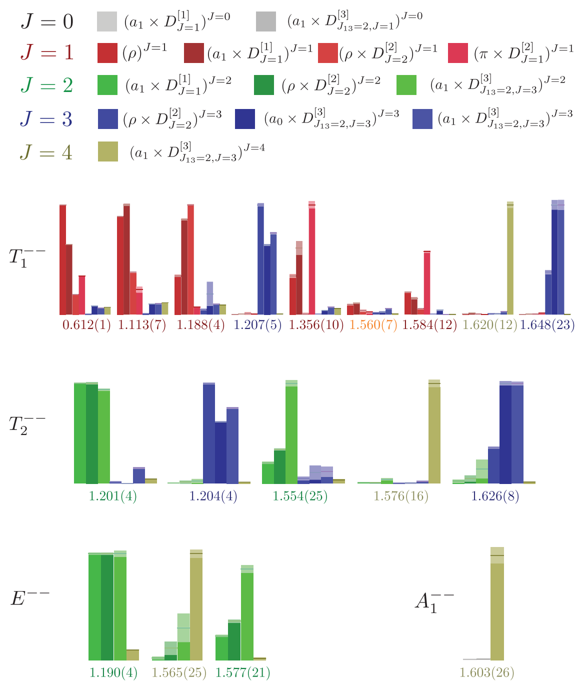

Figure 1: From Ref. [5]. Overlaps, Z, of a selection of operators onto states labelled by m/m_Ω in each lattice irrep, Λ^{--}. Z's are normalised so that the largest value across all states is equal to 1. Lighter area at the head of each bar represents the one sigma statistical uncertainly.

$a_s \sim 0.12$ fm and a finer temporal lattice spacing, $a_t = a_s/3.5$. Excited state correlation functions decay faster than the ground state and at large times are swamped by the signals for lower states. A fine temporal lattice spacing therefore aids their resolution and anisotropic lattices allow this without the increased cost (for a fixed spatial volume) of a finer spatial lattice spacing. The distillation method [9] was used for correlator construction; an overview of this method was given in a plenary talk at this conference, Ref. [10] and references therein.

The bare strange quark mass was held fixed and the two degenerate light quark masses varied, corresponding to $m_\pi \approx 700$, 520, 440 and 400 MeV. We used two volumes, 16^3 and 20^3 in lattice units, corresponding to spatial extents of ≈ 2.0 fm and 2.4 fm respectively; in both cases the temporal extent is 128 in temporal lattice units. In the first results only connected Wick contractions were computed, giving access to isovector states and kaons. More details on the datasets used are given in Ref. [5].

As an example of the results, in Fig. 2 we show the spin-identified isovector (octet) spectrum with $m_\pi \approx 700$MeV on both volumes. Note that this m_π corresponds to degenerate light and strange quarks, i.e. $SU(3)$ symmetry, and so all mesons in an octet are degenerate ($m_\pi = m_K = m_{\eta_8}$). Boxes represent the extracted mass scaled by the Ω baryon mass and one sigma statistical uncertainties. It can be seen that this method has enabled the extraction and spin-identification of many states across all PC combinations. There are many states with exotic quantum numbers, extracted with

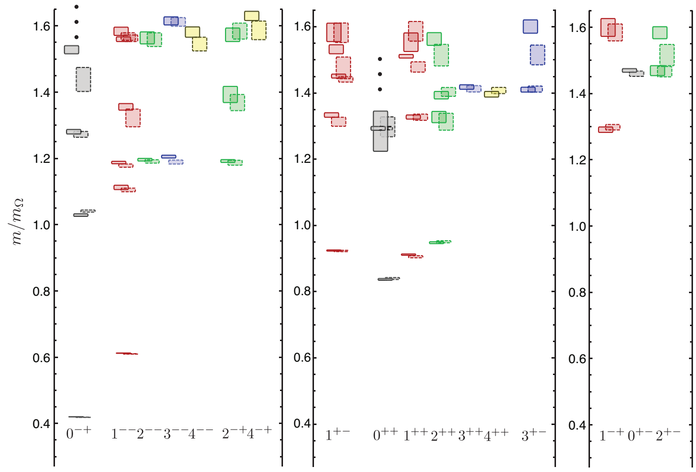

Figure 2: From Ref. [5]. Spin-identified isovector (octet) spectrum with $m_\pi \approx 700$ MeV on 16^3 (solid) and 20^3 (dashed) volumes. Ellipses indicate that there are heavier states with a given J^{PC} but that they are not well determined in this calculation.

a precision comparable to that of nonexotic states. For the first time in a lattice QCD calculation, spin-four mesons have been identified. There is really no statistically significant variation between the two volumes.

As discussed in Ref. [5], the spectrum of states shows many features of the $n^{2S+1}L_J$ pattern predicted by quark potential models; we refer to that reference for a more detailed discussion of this pattern. The right pane of Fig. 2 shows a set of states with exotic J^{PC}, not possible with just a quark-antiquark pair. Such states could be hybrid mesons, where the gluonic field is excited, or multiquark or molecular mesons where there are extra quark-antiquark pairs. These states have a large overlap onto operators containing the commutator of two covariant derivatives, i.e. the field strength tensor, suggesting that the gluonic field plays a non-trivial role in those states. This, along with there being no clear evidence for multi-meson states in the extracted spectrum, as argued in Ref. [5], suggests a hybrid nature for these states. There are also a number of states with nonexotic J^{PC} which do not appear to fit in to the quark model pattern. These states again all have significant overlap onto operators containing the commutator of two covariant derivatives and are of comparable mass to the exotics, suggesting that they have a hybrid nature. Such nonexotic hybrids are predicted in models of hybrids and can mix with conventional mesons having the same J^{PC}.

In Ref. [5] we show how the spectrum varies as we move away from the $SU(3)$ flavour point, lowering the mass of the two degenerate light quarks and keeping the remaining strange quark heavy. Therein we also discuss kaons and “strangeonium”. Here we will only highlight some results for the exotic states, shown in Fig. 3 with a comparison to some previous results for the 1^{-+}. It can be seen how using anisotropic lattices, a large basis of carefully constructed operators and high statistics have enabled us to extract the 1^{-+} more precisely, with precision comparable to that of the nonexotic states, and also reliably extract other states with exotic quantum numbers.

4. Conclusions

In summary, we have discussed recent progress in extracting excited meson spectra from lattice

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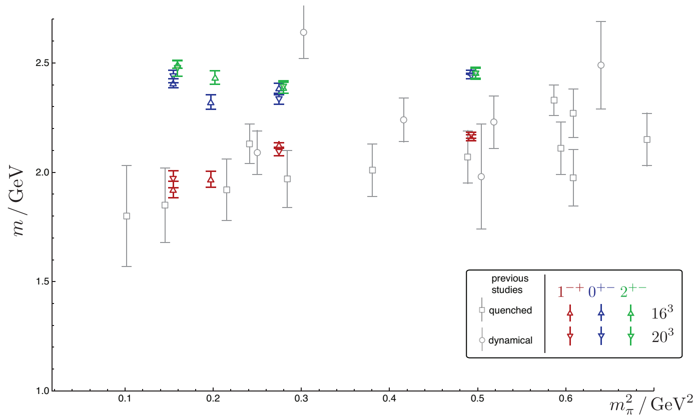

Figure 3: From Ref. [5]. Summary of extracted isovector exotic states with $J^{PC} = 1^{-+}, 0^{+-}, 2^{+-}$ as a function of m_π^2. For comparison we show some results for the 1^{-+} from Refs. [11 – 16].

QCD calculations and highlighted some results, referring to Refs. [4, 5] for more results and details. On all mass sets and volumes we are able to reliably extract and identify a large number of excited states across all *PC* combinations, including many states with exotics quantum numbers (0^{+-}, 1^{-+} and 2^{+-}), hinting at explicit gluonic degrees of freedom. For the first time in such a calculation, we have extracted spin-four states (4^{++}, 4^{-+} and 4^{--}). We have argued that the pattern of extracted states shows features predicted by quark-potential models, along with other states, both exotic and nonexotic, that do not seem to lie in that classification.

In Ref. [5] we argue that there is no clear evidence for multi-particle states in our extracted spectrum and that to study such states we need to construct operators with a larger number of quark fields, e.g. operators which are the product of two fermion bilinears. Such constructions should give us access to multi-particle energy levels and a denser spectrum of states which can be interpreted in terms of resonances using techniques like Lüscher's and its extensions.

Further avenues of investigation include the computation of disconnected two-point correlation functions giving access to isoscalars and application of these methods to the baryon sector [17]. It is also possible to perform a more detailed model dependent interpretation of the extracted spectrum, for example determining the mixing between conventional vector mesons and vector hybrids. An eventual aim is to calculate the light meson photocouplings which will be relevant for, amongst other things, the GlueX experiment at Jefferson Lab's 12 GeV upgrade.

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