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Pseudoscalar decay constants from $N_f = 2+1+1$ twisted mass lattice QCD

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We present first results for the pseudoscalar decay constants f_K, f_D and f_{D_s} from lattice QCD with $N_f = 2+1+1$ flavours of dynamical quarks. The lattice simulations have been performed by the European Twisted Mass collaboration (ETMC) using maximally twisted mass quarks. For the pseudoscalar decay constants we follow a mixed action approach by using so called Osterwalder-Seiler fermions in the valence sector for strange and charm quarks. The data for two values of the lattice spacing and several values of the up/down quark mass is analysed using chiral perturbation theory.

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ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	L/a
B35.32	1.95	0.0035	0.135	0.170	32
B55.32	1.95	0.0055	0.135	0.170	32
B75.32	1.95	0.0075	0.135	0.170	32
B85.24	1.95	0.0085	0.135	0.170	24
D20.48	2.10	0.0020	0.120	0.1385	48
D30.48	2.10	0.0030	0.120	0.1385	48

Table 1: The ensembles used in this investigation. The notation of ref. [6] is used for labeling the ensembles.

1. Introduction

The decay constants of the Pion, Kaon, the D- and the D_s-meson are phenomenologically interesting quantities, not least because the ratio f_K/f_π together with the well known $|V_{ud}|$ can be used to determine $|V_{us}|$. While experimentally f_π and f_K are well known and f_D and f_{D_s} less so, lattice QCD is in principle able to provide calculations of all of these from first principles. And recent advances in the field allow now also for statistically precise determinations with $N_f = 2$ and $N_f = 2+1$ dynamical quark flavours [1, 2, 3, 4, 5], with systematic uncertainties more or less under control.

In this proceeding contribution we are going to present yet another determination of the aforementioned decay constants, however with a dynamical charm quark in place, i.e. for QCD with $N_f = 2+1+1$ quark flavours.

2. Set-up

We use gauge configurations as produced by the European Twisted Mass Collaboration (ETMC) with $N_f = 2+1+1$ flavours of Wilson twisted mass quarks and Iwasaki gauge action. The set-up is described in ref. [6] and the ensembles used in this investigation are summarised in table 1. The twisted mass Dirac operator in the light – i.e. up/down – sector reads [7]

$$D_\ell = D_W + m_0 + i\mu_\ell\gamma_5\tau_3 \tag{2.1}$$

and in the strange/charm sector [8]

$$D_h = D_W + m_0 + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3\,, \tag{2.2}$$

where D_W is the Wilson Dirac operator. The value of m_0 was tuned to its critical value as discussed in refs. [9, 6] in order to realise automatic $\mathcal{O}(a)$ improvement at maximal twist [10]. Note that the bare twisted masses $\mu_{\sigma,\delta}$ are related to the bare strange and charm quark masses via the relation

$$m_{c,s} = \mu_\sigma \;\pm\; (Z_\mathrm{P}/Z_\mathrm{S})\;\mu_\delta \tag{2.3}$$

with pseudoscalar and scalar renormalisation constants Z_P and Z_S.

In the strange and charm quark sector, we use a mixed action approach with Osterwalder-Seiler (OS) valence quarks by formally introducing a twisted doublet for valence strange and charm quark

	$\beta = 1.95$	$\beta = 2.10$
$a\mu_s$	0.0130	0.0110
	0.0145	0.0120
	0.0160	0.0130
	0.0180	0.0150
	0.0210	0.0180
		0.0200

Table 2: Bare values of the valence quark masses in the strange region for $\beta = 1.95$ and $\beta = 2.10$.

flavours [11, 12], a set-up without flavour mixing artifacts in the valence sector. The two actions in the sea and the valence sector can be matched by tuning the bare values of valence strange and charm such that unitary Kaon and D-meson masses are reproduced. Unfortunately, it turned out that the unitary Kaon and D-meson masses were not exactly tuned to their physical values and currently we are still lacking ensembles which would allow to interpolate to the corresponding physical values. Hence, for the time being we vary the valence quark masses to interpolate the valence Kaon and D-meson masses to their physical values, as discussed below. This approach has been successfully applied to the ETMC $N_f = 2$ flavour gauge configurations in ref. [4].

The determination of the (unitary) K-meson mass is described in ref. [13]. Its decay constant can be determined from

$$f_K = (m_\ell + m_s)\frac{\langle 0|\tilde{P}_K|K\rangle}{m_K^2}\,, \tag{2.4}$$

where $\tilde{P}_K$ represents the projection to the physical Kaon interpolating operator as discussed in ref [13]. m_s is the strange quark masses defined in eq. (2.3). The pseudoscalar decay constant f_{PS} in the valence sector is determined from

$$f_{\mathrm{PS}} = \left(\mu_{val}^{(1)} + \mu_{val}^{(2)}\right)\frac{|\langle 0|P|PS\rangle|}{m_{\mathrm{PS}}\sinh m_{\mathrm{PS}}}\,, \tag{2.5}$$

where $P = \bar{q}_1\gamma_5 q_2$ with quark fields q_1, q_2 suitably chosen for the desired quark content. The meson mass m_{PS} and the matrix element $|\langle 0|P|PS\rangle|$ entering eq. (2.5) have been extracted from a single state fit of the corresponding two-point pseudoscalar correlation function. The replacement of m_{PS} with $\sinh m_{\mathrm{PS}}$ in the lattice definition (2.5) of the decay constant helps in reducing discretisation errors for heavy meson masses [4].

Any value for f_{PS} and m_{PS} depends on the mass-value of the light dynamical quark (sea strange and charm quark mass values are fixed for a given β-value) and two valence quark masses, which we denote by $f_{\mathrm{PS}}(\mu_\ell^{sea}, \mu_1^{val}, \mu_2^{val})$. We have investigated several values for the valence quarks: $\mu^{val} = \mu_\ell^{sea} = \mu_\ell$ and five to six values in the strange and charm quark region $\mu^{val} = \mu_{s,c}$. The ensembles used are summarised in table 1 and the bare values of the valence strange and charm quark masses in tables 2 and 3.

3. Results

One may expect large differences in between the unitary set-up and the mixed action set-up at finite values of the lattice spacing. In order to investigate this point we have determined f_K in the

	$\beta = 1.95$	$\beta = 2.10$
$a\mu_c$	0.200	0.1650
	0.215	0.1800
	0.240	0.2000
	0.260	0.2250
		0.2500

Table 3: Bare values of the valence quark masses in the charm region for $\beta = 1.95$ and $\beta = 2.10$.

unitary and the mixed action set-up after matching the Kaon mass. The result is shown in figure 1 where we plot f_K as a function of the squared pion mass. Both quantities are in units of f_0, the pion decay constant in the chiral limit as determined in ref. [6]. Within errors unitary and mixed set-up determination of f_K agree. Moreover, results for the two available values of the lattice spacing agree within errors. This indicates small lattice and small unitarity breaking artifacts at least in f_K. Note that the ratio $Z_\mathrm{P}/Z_\mathrm{S}$ used for the determination of the unitary f_K has been determined by matching the bare value of the OS strange quark mass to μ_σ and μ_δ via eq. (2.3).

In order to determine the physical value of f_K we fit SU(2) χPT formulae to our pion and Kaon decay constants data [14, 3] simultaneously according to

$$\begin{aligned} f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_\ell) &= f_0\cdot(1-2\,\xi_{ll}\ln\xi_{ll}+b\,\xi_{ll})\,, && (3.1)\\ f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_s) &= (f_0^{(K)}+f_m^{(K)}\,\xi_{ss})\cdot \\ &\quad\cdot\left[1-\frac{3}{4}\xi_{ll}\ln\xi_{ll}+(b_0^{(K)}+b_m^{(K)}\,\xi_{ss})\,\xi_{ll}\right] && (3.2)\end{aligned}$$

where

$$\xi_{XY} = \frac{m_{PS}^2(\mu_\ell,\mu_X,\mu_Y)}{(4\pi f_0)^2} \tag{3.3}$$

are expressed in our analysis as a function of meson masses[1]. We correct our data for finite size effects using NLO χPT [15, 16]

$$\begin{aligned} f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_\ell;L) &= f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_\ell)\cdot[1-2\,\xi_{ll}\,\tilde{g}_1(L,\xi_{ll})]\,,\\ f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_s;L) &= f_\mathrm{PS}(\mu_\ell,\mu_\ell,\mu_s)\cdot\left[1-\frac{3}{4}\,\xi_{ll}\,\tilde{g}_1(L,\xi_{ll})\right]\,.\end{aligned} \tag{3.4}$$

We fit to our $\beta = 1.95$ and $\beta = 2.10$ data simultaneously. We do not include lattice artifacts of $\mathcal{O}(a^2)$ in our fit, because the amount of data is not sufficient to do so: including these effects lets the fits become instable. Moreover, the data is fittable without these terms with $\chi^2/\mathrm{dof} = 50/30$.

The physical input to our fits is $f_\pi = 130.7$ MeV, $m_\pi = 135$ MeV and $m_K = 497.7$ MeV. In figure 2 we show pion and Kaon decay constants as a function of the squared pion mass m_π^2. The corresponding Kaon decay constant is determined at a value of $m_\mathrm{PS}^2(\mu_\ell,\mu_s,\mu_s) = 2m_K^2 - m_\pi^2$. For the figure we have interpolated our data linearly to these values. As a result we obtain $f_K/f_\pi = 1.224(13)$, $f_K = 160(2)$ MeV and $\bar{\ell}_4 = 4.78(2)$ with statistical errors only as determined from a bootstrap analysis.

[1] We use the normalization in which $f_\pi = 130.7\,\mathrm{MeV}$.

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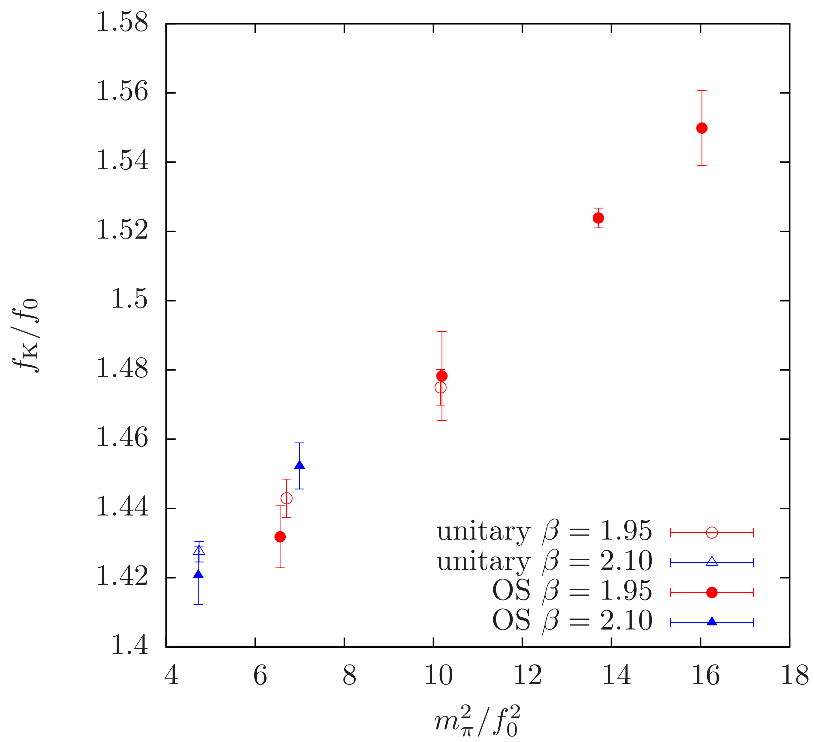

Figure 1: f_K/f_0 as a function of the pion mass squared. Results for the unitary and the mixed action f_K are shown for two values of the lattice spacing.

The fit also allows us to determine the values of the lattice spacings at $\beta = 1.95$ and $\beta = 2.10$ and the value of f_0. All these quantities agree very well with the results obtained in ref. [6].

Following the procedure described in ref. [4] we have also analysed the data for f_D and f_{D_s} using $\mathrm{SU}(2)$ heavy meson chiral perturbation theory [17]. We consider here the expansions of

$$f_{D_s}\sqrt{m_{D_s}} \quad \text{and} \quad \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} \tag{3.5}$$

including terms proportional $a^2 m_{D_s}^2$ and $1/m_{D_s}$. For details we refer to ref. [4]. Our first results for f_D and f_{D_s} are very encouraging, however, quoting results requires a better control of the systematics involved in this investigation.

4. Summary and Outlook

We have presented the first determination of the pseudoscalar decay constants f_K, f_D and f_{D_s} from lattice QCD with dynamical up, down, strange and charm quark flavours. We have used a mixed action approach with Osterwalder-Seiler valence quarks on a maximally twisted mass sea. The analysis indicates small lattice and unitarity breaking artifacts. The preliminary results are $f_K/f_\pi = 1.224(13)$ and $f_K = 160(2)$ MeV obtained with a $\mathrm{SU}(2)$ chiral perturbation theory fit. The errors are statistical only. Using the results of ref. [18] this translates to $|V_{us}| = 0.220(2)$.

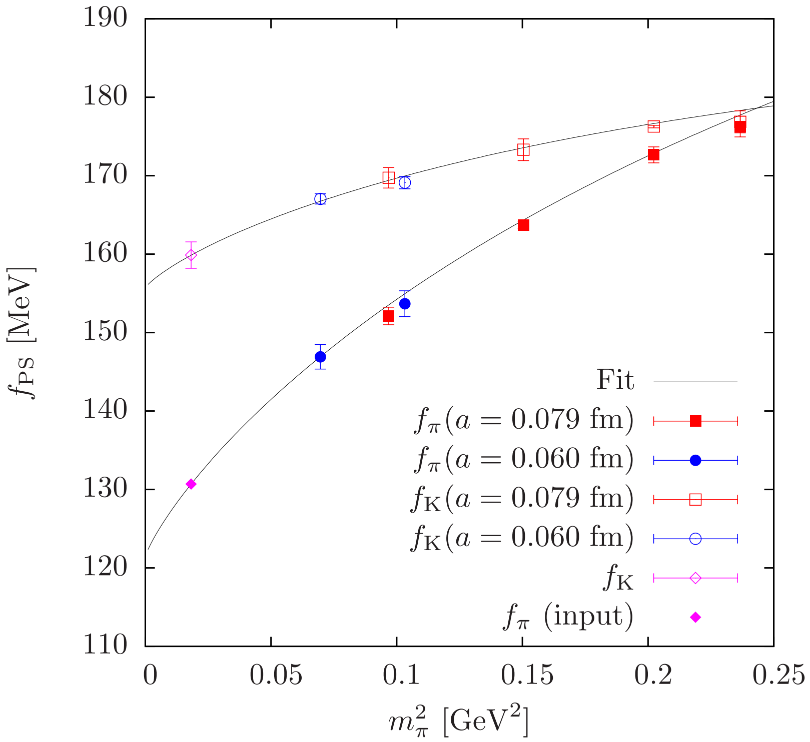

Figure 2: Pion and Kaon decay constants as a function of m_π^2. We show data for two values of the lattice spacing $a = 0.079$ fm and $a = 0.060$ fm, corresponding to $\beta = 1.95$ and $\beta = 2.10$.

A comparison of our results for f_K and f_K/f_π to results available in the literature shows that to the current level of accuracy there is no difference visible, neither to $N_f = 2$ flavour results [4] nor to $N_f = 2+1$ flavour results [3, 2, 1].

A similar analysis has been performed for f_D and f_{D_s}. In the case of these charmed quantities it will be in particular interesting to understand whether lattice artifacts proportional to $a^2 m_c^2$ are small enough in our lattice set-up to allow for a precise determination of the corresponding quantities. However, the $N_f = 2$ results presented in ref. [4] give rise to optimism that also f_D and f_{D_s} can be reliably determined in our set-up.

Clearly the results presented here need a better understanding of the systematic uncertainties. This includes in particular the dependence of the decay constants on the sea strange and charm quark mass values. ETMC is currently producing ensembles to investigate this point. These new ensembles should eventually allow to interpolate to the physical values of m_K and m_D. Moreover, more ensembles with different light quark mass values are required for $\beta = 2.10$ to better control lattice artifacts, for which also a third value of the lattice spacing is desirable.

On the analysis site we are currently implementing different fit formulae (see for instance refs. [5, 19]) in order to better understand the extrapolation in the various quark masses.

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References

[1] **MILC** Collaboration, C. Aubin *et al.*, Phys. Rev. **D70**, 114501 (2004), `arXiv:hep-lat/0407028`.

[2] **HPQCD** Collaboration, E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu, Phys. Rev. Lett. **100**, 062002 (2008), `arXiv:0706.1726 [hep-lat]`.

[3] **RBC-UKQCD** Collaboration, C. Allton *et al.*, Phys. Rev. **D78**, 114509 (2008), `arXiv:0804.0473 [hep-lat]`.

[4] B. Blossier *et al.*, JHEP **07**, 043 (2009), `arXiv:0904.0954 [hep-lat]`.

[5] S. Dürr *et al.*, Phys. Rev. **D81**, 054507 (2010), `arXiv:1001.4692 [hep-lat]`.

[6] **ETM** Collaboration, R. Baron *et al.*, JHEP **06**, 111 (2010), `arXiv:1004.5284 [hep-lat]`.

[7] **ALPHA** Collaboration, R. Frezzotti, P. A. Grassi, S. Sint and P. Weisz, JHEP **08**, 058 (2001), `hep-lat/0101001`.

[8] R. Frezzotti and G. C. Rossi, Nucl. Phys. Proc. Suppl. **128**, 193 (2004), `hep-lat/0311008`.

[9] T. Chiarappa *et al.*, Eur. Phys. J. **C50**, 373 (2007), `arXiv:hep-lat/0606011`.

[10] R. Frezzotti and G. C. Rossi, JHEP **08**, 007 (2004), `hep-lat/0306014`.

[11] R. Frezzotti and G. C. Rossi, JHEP **10**, 070 (2004), `arXiv:hep-lat/0407002`.

[12] **ETM** Collaboration, B. Blossier *et al.*, JHEP **04**, 020 (2008), `arXiv:0709.4574 [hep-lat]`.

[13] **ETM** Collaboration, R. Baron *et al.*, `arXiv:1005.2042 [hep-lat]`.

[14] J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984).

[15] J. Gasser and H. Leutwyler, Phys. Lett. **B188**, 477 (1987).

[16] D. Becirevic and G. Villadoro, Phys. Rev. **D69**, 054010 (2004), `arXiv:hep-lat/0311028`.

[17] S. R. Sharpe and Y. Zhang, Phys. Rev. **D53**, 5125 (1996), `arXiv:hep-lat/9510037`.

[18] W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004), `arXiv:hep-ph/0402299`.

[19] V. Bernard, S. Descotes-Genon and G. Toucas, `arXiv:1009.5066 [hep-ph]`.