PROCEEDINGS OF SCIENCE

An extension to the Lüscher's finite volume method above inelastic threashold (formalism)

Noriyoshi ISHII*
Department of Physics, The University of Tokyo
E-mail: ishii@ribf.riken.jp

for HAL-QCD Collaboration

An extension of the Lüscher's finite volume method above inelastic thresholds is proposed. It is fulfilled by extendind the procedure recently proposed by HAL-QCD Collaboration for a single channel system. Focusing on the asymptotic behaviors of the Nambu-Bethe-Salpeter (NBS) wave functions (equal-time) near spatial infinity, a coupled channel extension of effective Schrödinger equation is constructed by introducing an energy-independent interaction kernel. Because the NBS wave functions contain the information of T-matrix at long distance, S-matrix can be obtained by solving the coupled channel effective Schrödinger equation in the infinite volume.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010
June 14-19, 2010
Villasimius, Italy

*Speaker.

1. Introduction

The standard method to calculate the scattering phase shift on the lattice is the Lüscher's finite volume method[1]. It utilizes the shift of energy spectra in the finite periodic box to calculate the scattering phase. The formalism is restricted to the elastic scattering region. One attempted to extend it above inelastic thresholds [2]. Although it can lead to a constraint imposed on the matrix elements of S-matrix, S-matrix elements cannot be obtained separately.

We propose a different method to extend it above the inelastic threshold. This is fulfilled by extending the procedure recently proposed by HAL-QCD Collaboration [3, 4]. We focus on the asymptotic behaviors of the equal-time Nambu-Bethe-Salpeter (NBS) wave functions near the spatial infinity, where the information of T-matrix is contained. We construct a coupled channel effective Schrödinger equation by introducing an energy-independent interaction kernel, so that it can generate all the NBS wave functions simultaneously. Once such an interaction kernel is constructed in the finite volume, S-matrix is obtained by solving the effective Schrodinger equation in the infinite volume. (The resulting wave functions are nothing but the NBS wave functions, which contain the information of T-matrix in their long distance part.) In this paper, we use $N\Lambda$-$N\Sigma$ coupled system to formalize the procedure.

Contents are organized as follows. In Sect. 2, we consider the asymptotic long distance behavior of the NBS wave functions for $N\Lambda$-$N\Sigma$ coupled system. We will see that they contain T-matrix in their long distance part. In Sect. 3, we extend an effective Schrödinger equation for the coupled channel system by introducing an energy-independent interaction kernel. The extension is performed so that it can generate all the NBS wave functions simultaneously as solutions. In Sect. 4, we derive the factorization formula, which plays a key role in constructing an effective Schrödinger equation in a coupled channel system in Sect. 3.

2. Asymptotic behaviors of equal-time Nambu-Bethe-Salpeter (NBS) wave functions for a coupled channel system

We consider $N\Lambda$-$N\Sigma$ coupled system with $m_N \simeq 940$ MeV, $m_\Lambda \simeq 1115$ MeV, and $m_\Sigma \simeq 1190$ MeV. For notational simplicity, we treat them as bosons. We are interested in the energy region

$$m_N + m_\Lambda \le E \le m_N + m_\Lambda + m_\pi, \tag{2.1}$$

which is a combined region of (i) elastic $N\Lambda$ region: $m_N + m_\Lambda \le E \le m_N + m_\Sigma$ and (ii) $N\Lambda$-$N\Sigma$ coupling region above the $N\Sigma$ threshold: $m_N + m_\Sigma \le E \le m_N + m_\Lambda + m_\pi$.

We consider two incoming states at the same energy E as $|N(\vec{p})\Lambda(-\vec{p}), in\rangle$ and $|N(\vec{q})\Sigma(-\vec{q}), in\rangle$. $\vec{p}$ and $\vec{q}$ denote the asymptotic momenta for $N\Lambda$ and $N\Sigma$ systems, respectively. They are related to E as

$$E = \sqrt{m_N^2 + \vec{p}^2} + \sqrt{m_\Lambda^2 + \vec{p}^2} = \sqrt{m_N^2 + \vec{q}^2} + \sqrt{m_\Sigma^2 + \vec{q}^2}. \tag{2.2}$$

We define the NBS wave functions for an incoming state $|N(\vec{p})\Lambda(-\vec{p}), in\rangle$ as

$$\psi_{N\Lambda;N\Lambda}(x_1, x_2; E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0|T[N(x_1)\Lambda(x_2)]|N(\vec{p})\Lambda(-\vec{p}), in\rangle \tag{2.3}$$

$$\psi_{N\Sigma;N\Lambda}(x_1, x_2; E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0|T[N(x_1)\Sigma(x_2)]|N(\vec{p})\Lambda(-\vec{p}), in\rangle, \tag{2.4}$$

and the NBS wave functions for an incoming state $|N(\vec{q})\Sigma(-\vec{q}),in\rangle$ as

$$\psi_{N\Lambda;N\Sigma}(x_1,x_2;E) \equiv Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0|T[N(x_1)\Lambda(x_2)]|N(\vec{q})\Sigma(-\vec{q}),in\rangle \tag{2.5}$$
$$\psi_{N\Sigma;N\Sigma}(x_1,x_2;E) \equiv Z_N^{-1/2} Z_\Sigma^{-1/2} \langle 0|T[N(x_1)\Sigma(x_2)]|N(\vec{q})\Sigma(-\vec{q}),in\rangle, \tag{2.6}$$

where $N(x)$, $\Lambda(x)$ and $\Sigma(x)$ denote local composite interpolating fields for nucleon, Λ and Σ baryons, respectively. Z_N, Z_Λ and Z_Σ denote the normalization factors involved in the limit $N(x) \to Z_N^{1/2} N_{out}(x)$, $\Lambda(x) \to Z_\Lambda^{1/2}\Lambda_{out}(x)$, and $\Sigma(x) \to Z_\Sigma^{1/2}\Sigma_{out}(x)$, respectively, as $x_0 \to +\infty$. By using the reduction formula, these NBS wave functions are related to S-matrix as

$$\langle N(p_1')\Lambda(p_2'),out|N(\vec{p})\Lambda(-\vec{p}),in\rangle \tag{2.7}$$
$$= \text{disc.} + \int d^4x_1 d^4x_2 e^{ip_1'x_1} \left(\Box_1 + m_N^2\right) e^{ip_2'x_2} \left(\Box_2 + m_\Lambda^2\right) \psi_{N\Lambda;N\Lambda}(x_1,x_2;E)$$
$$\langle N(q_1')\Sigma(q_2'),out|N(\vec{p})\Lambda(-\vec{p}),in\rangle \tag{2.8}$$
$$= \int d^4x_1 d^4x_2 e^{iq_1'x_1} \left(\Box_1 + m_N^2\right) e^{iq_2'x_2} \left(\Box_2 + m_\Sigma^2\right) \psi_{N\Sigma;N\Lambda}(x_1,x_2;E),$$

and

$$\langle N(p_1')\Lambda(p_2'),out|N(\vec{q})\Sigma(-\vec{q}),in\rangle \tag{2.9}$$
$$= \int d^4x_1 d^4x_2 e^{ip_1'x_1} \left(\Box_1 + m_N^2\right) e^{ip_2'x_2} \left(\Box_2 + m_\Lambda^2\right) \psi_{N\Lambda;N\Sigma}(x_1,x_2;E)$$
$$\langle N(q_1')\Sigma(q_2'),out|N(\vec{q})\Sigma(-\vec{q}),in\rangle \tag{2.10}$$
$$= \text{disc.} + \int d^4x_1 d^4x_2 e^{iq_1'x_1} \left(\Box_1 + m_N^2\right) e^{iq_2'x_2} \left(\Box_2 + m_\Sigma^2\right) \psi_{N\Sigma;N\Sigma}(x_1,x_2;E),$$

where "disc." stands for disconnected terms, and $\Box_i \equiv \partial^2/\partial t_i^2 - \vec{\nabla}_i^2$ denotes d'Alembert operator. These relations lead us to the following asymptotic behaviors of equal-time restrictions of NBS wave functions near spatial infinity ($|\vec{x}-\vec{y}| \to \infty$) as

$$\psi_{N\Lambda;N\Lambda}(\vec{x}-\vec{y};E) \equiv \lim_{x_0\to+0} \psi_{N\Lambda;N\Lambda}(\vec{x},x_0,\vec{y},y_0=0;E) \tag{2.11}$$
$$\simeq e^{i\vec{p}\cdot\vec{r}} + \frac{2}{E_N(p)+E_\Lambda(p)} \mathscr{T}_{N\Lambda;N\Lambda}(s) \frac{e^{ipr}}{pr} + \cdots$$
$$\psi_{N\Sigma;N\Lambda}(\vec{x}-\vec{y};E) \equiv \lim_{x_0\to+0} \psi_{N\Sigma;N\Lambda}(\vec{x},x_0,\vec{y},y_0=0;E) \tag{2.12}$$
$$\simeq \frac{2}{E_N(q)+E_\Sigma(q)} \mathscr{T}_{N\Sigma;N\Lambda}(s) \frac{e^{iqr}}{qr} + \cdots,$$

and

$$\psi_{N\Lambda;N\Sigma}(\vec{x}-\vec{y};E) \equiv \lim_{x_0\to+0} \psi_{N\Lambda;N\Sigma}(\vec{x},x_0,\vec{y},y_0=0;E) \tag{2.13}$$
$$\simeq \frac{2}{E_N(p)+E_\Lambda(p)} \mathscr{T}_{N\Lambda;N\Sigma}(s) \frac{e^{ipr}}{pr} + \cdots$$
$$\psi_{N\Sigma;N\Sigma}(\vec{x}-\vec{y};E) \equiv \lim_{x_0\to+0} \psi_{N\Sigma;N\Sigma}(\vec{x},x_0,\vec{y},y_0=0;E) \tag{2.14}$$
$$\simeq e^{i\vec{q}\cdot\vec{r}} + \frac{2}{E_N(q)+E_\Sigma(q)} \mathscr{T}_{N\Sigma;N\Sigma}(s) \frac{e^{iqr}}{qr} + \cdots,$$

PoS(Lattice 2010)145

where $\mathscr{T}$ denotes the on-shell T-matrix, i.e., $\langle f, out|i, in\rangle = \mathbb{I} + i(2\pi)^4\delta^4(P_f - P_i)\mathscr{T}_{f,i}$, $E_N(p) \equiv \sqrt{m_N^2 + \vec{p}^2}$, $E_\Lambda(p) \equiv \sqrt{m_\Lambda^2 + \vec{p}^2}$, $E_\Sigma(p) \equiv \sqrt{m_\Sigma^2 + \vec{p}^2}$, and $\vec{r} \equiv \vec{x} - \vec{y}$.

Derivations of these asymptotic behaviors of NBS wave functions are similar to single-channel case [4, 5, 6]. Here, we give a brief example of $\psi_{N\Lambda;N\Lambda}(\vec{r};\vec{p})$ case.

$$\begin{aligned}\psi_{N\Lambda;N\Lambda}(\vec{r};E) &= Z_N^{-1/2} Z_\Lambda^{-1/2} \langle 0|N(\vec{r})\Lambda(\vec{0})|N(\vec{p})\Lambda(-\vec{p}), in\rangle \\ &= Z_N^{-1/2} Z_\Lambda^{-1/2} \int \frac{d^3k}{(2\pi)^3\, 2E_N(k)} \langle 0|N(\vec{r})|N(\vec{k})\rangle\langle N(\vec{k})|\Lambda(0)|N(\vec{p})\Lambda(-\vec{p}), in\rangle + I(\vec{r}),\end{aligned} \tag{2.15}$$

where $I(\vec{r})$ is referred to as the inelastic contribution, which corresponds to contributions not associated with single nucleon intermediate state. Since $I(\vec{r})$ does not propagate long distance, we will neglect this term below. Because of Eq. (2.7), we have an equality

$$\begin{aligned}&\langle N(k_1)|\Lambda(0)|N(p_1)\Lambda(p_2), in\rangle \\ &= \langle 0|\Lambda(0) a_{N,in}(k_1)|N(p_1)\Lambda(p_2), in\rangle \\ &\quad + iZ_N^{-1/2} \int d^4x_1 e^{ik_1x_1} \left(\Box_1 + m_N^2\right) \langle 0|T[N(x_1)\Lambda(0)]|N(p_1)\Lambda(p_2), in\rangle \\ &= Z_\Lambda^{1/2}\, 2E_N(p_1)\,(2\pi)^3 \delta^3(k_1 - p_1) + Z_\Lambda^{1/2} \frac{\mathscr{T}(N(k_1)\Lambda(p_1+p_2-k_1); N(p_1)\Lambda(p_2))}{-(k_1 - p_1 - p_2)^2 + m_\Lambda^2 - i\varepsilon},\end{aligned} \tag{2.16}$$

which is used to replace the second factor in the integrand of Eq. (2.15) as

$$\begin{aligned}\psi_{N\Lambda;N\Lambda}(\vec{r};E) &= e^{i\vec{p}\cdot\vec{r}} + \int \frac{d^3k}{(2\pi)^3\, 2E_N(k)} \\ &\quad \times \frac{1}{E_\Lambda(k) - E_N(k) + E_N(p) + E_\Lambda(p)} \cdot \frac{\mathscr{T}(N(\vec{k})\Lambda(-\vec{k}); N(\vec{p})\Lambda(-\vec{p}))}{E_\Lambda(k) + E_N(k) - E_\Lambda(p) - E_N(p) - i\varepsilon} e^{i\vec{k}\cdot\vec{r}}.\end{aligned} \tag{2.17}$$

Eq. (2.11) is arrived at by performing the integration. The integration can be carried out by noticing that the propagating degrees of freedom come from the pole contribution, i.e., $E_\Lambda(k) + E_N(k) \simeq E_\Lambda(p) + E_N(p)$, where the off-shell $\mathscr{T}$ can be replaced by the on-shell $\mathscr{T}$. The derivations of Eq. (2.12), Eq. (2.13) and Eq. (2.14) are similar.

3. An extension of effective Schrödinger equation for a coupled channel system

To construct an effective Schrödinger equation, we define the following K functions by multiplying Helmholtz operators on the NBS wave functions as

$$\begin{aligned}K_{N\Lambda;N\Lambda}(\vec{x};E) &\equiv \left(\triangle + \vec{p}^2\right) \psi_{N\Lambda;N\Lambda}(\vec{x};E) \\ K_{N\Sigma;N\Lambda}(\vec{x};E) &\equiv \left(\triangle + \vec{q}^2\right) \psi_{N\Sigma;N\Lambda}(\vec{x};E),\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}K_{N\Lambda;N\Sigma}(\vec{x};E) &\equiv \left(\triangle + \vec{p}^2\right) \psi_{N\Lambda;N\Sigma}(\vec{x};E) \\ K_{N\Sigma;N\Sigma}(\vec{x};E) &\equiv \left(\triangle + \vec{q}^2\right) \psi_{N\Sigma;N\Sigma}(\vec{x};E).\end{aligned} \tag{3.2}$$

These Helmholtz operators filter out the propagating degrees of freedom in the asymptotic forms of NBS wave functions. As a result, these K functions become localized objects, i.e., K is nonzero only within the range of interaction. We factorize these K functions as

$$\begin{aligned} &\begin{bmatrix} K_{N\Lambda;N\Lambda}(\vec{x};E) & K_{N\Lambda;N\Sigma}(\vec{x};E) \\ K_{N\Sigma;N\Lambda}(\vec{x};E) & K_{N\Sigma;N\Sigma}(\vec{x};E) \end{bmatrix} \\ &= \int d^3y \begin{bmatrix} U_{N\Lambda;N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda;N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma;N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma;N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{y};E) & \psi_{N\Lambda;N\Sigma}(\vec{y};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{y};E) & \psi_{N\Sigma;N\Sigma}(\vec{y};E) \end{bmatrix}, \end{aligned} \tag{3.3}$$

with an E-independent kernel U. (The proof of the factorization is given in Sect. 4.) Since K is localized, U has to be localized as well.

If such a factorization is possible, we have

$$\begin{aligned} &\begin{bmatrix} \left(\triangle+\vec{p}^2\right)\psi_{N\Lambda;N\Lambda}(\vec{x};E) & \left(\triangle+\vec{p}^2\right)\psi_{N\Lambda;N\Sigma}(\vec{x};E) \\ \left(\triangle+\vec{q}^2\right)\psi_{N\Sigma;N\Lambda}(\vec{x};E) & \left(\triangle+\vec{q}^2\right)\psi_{N\Sigma;N\Sigma}(\vec{x};E) \end{bmatrix} \\ &= \int d^3y \begin{bmatrix} U_{N\Lambda;N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda;N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma;N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma;N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{y};E) & \psi_{N\Lambda;N\Sigma}(\vec{y};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{y};E) & \psi_{N\Sigma;N\Sigma}(\vec{y};E) \end{bmatrix}. \end{aligned} \tag{3.4}$$

This implies that any linear combinations of NBS wave functions with arbitrary complex numbers α and β as

$$\begin{cases} \psi_{N\Lambda}(\vec{x};E) \equiv \alpha\,\psi_{N\Lambda;N\Lambda}(\vec{x};E)+\beta\,\psi_{N\Lambda;N\Sigma}(\vec{x};E) \\ \psi_{N\Sigma}(\vec{x};E) \equiv \alpha\,\psi_{N\Sigma;N\Lambda}(\vec{x};E)+\beta\,\psi_{N\Sigma;N\Sigma}(\vec{x};E) \end{cases} \tag{3.5}$$

satisfy

$$\begin{bmatrix} \left(\triangle+\vec{p}^2\right)\psi_{N\Lambda}(\vec{x};E) \\ \left(\triangle+\vec{q}^2\right)\psi_{N\Sigma}(\vec{x};E) \end{bmatrix} = \int d^3y \begin{bmatrix} U_{N\Lambda;N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda;N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma;N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma;N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda}(\vec{y};E) \\ \psi_{N\Sigma}(\vec{y};E) \end{bmatrix}, \tag{3.6}$$

which serves as an coupled channel extension of the effective Schrödinger equation.

Several comments are in order.

(i) Eq. (3.6) does not depend on a particular choice of boundary condition, since α and β can be arbitrarily chosen. This is a desirable property, which makes it possible to determine the interaction kernel U in the finite volume. (The volume has to be sufficiently large compared to the range of the interaction.) Once U is determined in the finite volume, S-matrix can be obtained by solving Eq. (3.6) in the infinite volume. (Eq. (3.6) generates NBS wave functions as solutions, which contain the information of T-matrix in their long distance part.)

(ii) The interaction kernel U is most generally a non-local object. As is described in Ref. [4], a practical way to construct such a non-local interaction kernel U is to rely on the derivative expansion, which makes it possible to construct U order by order in the derivative expansion by using a finite number of NBS wave functions of varying energy E and angular momentum L. The most challenging step in fulfilling this procedure is the construction of NBS wave functions at excited energies with variational method. This turns out to be feasible. (See Ref. [7], where this procedure is applied to $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$ coupled system.)

(iii) In the non-relativistic limit, $\vec{p}^2$ and $\vec{q}^2$ are approximately related as $\vec{q}^2/(2\mu_{N\Sigma}) \simeq \vec{p}^2/(2\mu_{N\Lambda}) + m_\Lambda - m_\Sigma$, where $\mu_{N\Sigma}$ and $m_{N\Lambda}$ denote the reduced masses for $N\Sigma$ and $N\Lambda$ systems, respectively. In this limit, Eq. (3.6) reduces to an eigenvalue problem used in Ref. [2]:

$$\begin{bmatrix} -\frac{\triangle}{2\mu_{N\Lambda}} + \widetilde{U}_{N\Lambda;N\Lambda} & \widetilde{U}_{N\Lambda;N\Sigma} \\ \widetilde{U}_{N\Sigma;N\Lambda} & m_\Sigma - m_\Lambda - \frac{\triangle}{2\mu_{N\Sigma}} + \widetilde{U}_{N\Sigma;N\Sigma} \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda}(\vec{x};E) \\ \psi_{N\Sigma}(\vec{x};E) \end{bmatrix} = E_{\rm NR} \begin{bmatrix} \psi_{N\Lambda}(\vec{x};E) \\ \psi_{N\Sigma}(\vec{x};E) \end{bmatrix}, \quad (3.7)$$

where $\widetilde{U}_{N\Lambda;N\Lambda} \equiv U_{N\Lambda;N\Lambda}/(2\mu_{N\Lambda})$, $\widetilde{U}_{N\Lambda;N\Sigma} \equiv U_{N\Lambda;N\Sigma}/(2\mu_{N\Lambda})$, $\widetilde{U}_{N\Sigma;N\Lambda} \equiv U_{N\Sigma;N\Lambda}/(2\mu_{N\Sigma})$, $\widetilde{U}_{N\Sigma;N\Sigma} \equiv U_{N\Sigma;N\Sigma}/(2\mu_{N\Sigma})$, and $E_{\rm NR} \equiv \vec{p}^2/(2\mu_{N\Lambda})$.

(iv) In constructing Eq. (3.6), we assumed the orthogonality of NBS wave functions. If the violation of orthogonality becomes severe, NBS wave functions should be orthogonalized before all the above procedures begin. The orthogonalization should be performed without affecting the long distance behaviors, where information of T-matrix is contained.

4. Proof of the factorization of K function

We assume that the following NBS wave functions are linearly independent

$$\left\{ \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{x};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{x};E) \end{bmatrix}, \begin{bmatrix} \psi_{N\Lambda;N\Sigma}(\vec{x};E) \\ \psi_{N\Sigma;N\Sigma}(\vec{x};E) \end{bmatrix} \right\}_{m_N+m_\Lambda \le E \le m_N+m_\Lambda+m_\pi}. \quad (4.1)$$

There exists a dual basis

$$\left\{ [\widetilde{\psi}_{N\Lambda;N\Lambda}(\vec{x};E), \widetilde{\psi}_{N\Lambda;N\Sigma}(\vec{x};E)], \quad [\widetilde{\psi}_{N\Sigma;N\Lambda}(\vec{x};E), \widetilde{\psi}_{N\Sigma;N\Sigma}(\vec{x};E)] \right\}_{m_N+m_\Lambda \le E \le m_N+m_\Lambda+m_\pi}, \quad (4.2)$$

which serves as a "left inverse" as an integration operator in the following sense:

$$\int d^3y \begin{bmatrix} \widetilde{\psi}_{N\Lambda;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Lambda;N\Sigma}(\vec{y};E') \\ \widetilde{\psi}_{N\Sigma;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Sigma;N\Sigma}(\vec{y};E') \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{y};E) & \psi_{N\Lambda;N\Sigma}(\vec{y};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{y};E) & \psi_{N\Sigma;N\Sigma}(\vec{y};E) \end{bmatrix} = (2\pi)\delta(E-E'). \quad (4.3)$$

Now, the factorization can be verifi ed in the following way:

$$\begin{aligned} &\begin{bmatrix} K_{N\Lambda;N\Lambda}(\vec{x};E) & K_{N\Lambda;N\Sigma}(\vec{x};E) \\ K_{N\Sigma;N\Lambda}(\vec{x};E) & K_{N\Sigma;N\Sigma}(\vec{x};E) \end{bmatrix} \qquad (4.4) \\ &= \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda;N\Lambda}(\vec{x};E') & K_{N\Lambda;N\Sigma}(\vec{x};E') \\ K_{N\Sigma;N\Lambda}(\vec{x};E') & K_{N\Sigma;N\Sigma}(\vec{x};E') \end{bmatrix} \\ &\qquad \times \int d^3y \begin{bmatrix} \widetilde{\psi}_{N\Lambda;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Lambda;N\Sigma}(\vec{y};E') \\ \widetilde{\psi}_{N\Sigma;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Sigma;N\Sigma}(\vec{y};E') \end{bmatrix} \cdot \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{y};E) & \psi_{N\Lambda;N\Sigma}(\vec{y};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{y};E) & \psi_{N\Sigma;N\Sigma}(\vec{y};E) \end{bmatrix} \\ &= \int d^3y \left\{ \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda;N\Lambda}(\vec{x};E') & K_{N\Lambda;N\Sigma}(\vec{x};E') \\ K_{N\Sigma;N\Lambda}(\vec{x};E') & K_{N\Sigma;N\Sigma}(\vec{x};E') \end{bmatrix} \cdot \begin{bmatrix} \widetilde{\psi}_{N\Lambda;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Lambda;N\Sigma}(\vec{y};E') \\ \widetilde{\psi}_{N\Sigma;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Sigma;N\Sigma}(\vec{y};E') \end{bmatrix} \right\} \\ &\qquad \times \begin{bmatrix} \psi_{N\Lambda;N\Lambda}(\vec{y};E) & \psi_{N\Lambda;N\Sigma}(\vec{y};E) \\ \psi_{N\Sigma;N\Lambda}(\vec{y};E) & \psi_{N\Sigma;N\Sigma}(\vec{y};E) \end{bmatrix}. \end{aligned}$$

We arrive at the factorization formula Eq. (3.3) by defining an interaction kernel U as

$$\begin{bmatrix} U_{N\Lambda;N\Lambda}(\vec{x},\vec{y}) & U_{N\Lambda;N\Sigma}(\vec{x},\vec{y}) \\ U_{N\Sigma;N\Lambda}(\vec{x},\vec{y}) & U_{N\Sigma;N\Sigma}(\vec{x},\vec{y}) \end{bmatrix} \equiv \int \frac{dE'}{2\pi} \begin{bmatrix} K_{N\Lambda;N\Lambda}(\vec{x};E') & K_{N\Lambda;N\Sigma}(\vec{x};E') \\ K_{N\Sigma;N\Lambda}(\vec{x};E') & K_{N\Sigma;N\Sigma}(\vec{x};E') \end{bmatrix} \cdot \begin{bmatrix} \widetilde{\psi}_{N\Lambda;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Lambda;N\Sigma}(\vec{y};E') \\ \widetilde{\psi}_{N\Sigma;N\Lambda}(\vec{y};E') & \widetilde{\psi}_{N\Sigma;N\Sigma}(\vec{y};E') \end{bmatrix}. \tag{4.5}$$

Note that, due to the integration of E', the interaction kernel U does not depend on the energy E.

5. Summary

We have proposed an extension of the Lüscher's finite volume method above the inelastic threshold. We have considered $N\Lambda$-$N\Sigma$ coupled system as an example. We have seen that equal-time Nambu-Bethe-Salpeter (NBS) wave functions contain the information of T-matrix in their long distance part. We have constructed an effective Schrödinger equation for a coupled channel system by introducing an energy-independent interaction kernel, so that it can generate all the NBS wave functions in the energy region $m_N + m_\Lambda \le E \le m_N + m_\Lambda + m_\pi$. Since this interaction kernel is localized, it can be constructed by using lattice QCD calculation in a finite volume. Once it is constructed in the finite volume, S-matrix can be obtained by solving the effective Schrödinger equation in the infinite volume.

It is interesting to apply this method to the hyperon systems, where only a limited amount of experimental information is available. The first attempt to apply this method to $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$ coupled system is reported in Ref. [7], where the interaction kernel is constructed at the leading order of the derivative expansion by using NBS wave functions obtained by the variational method.

Acknowledgments

This work was supported in part by Grant-in-Aid of the Japanese Ministry of Education, Science, Sport and Culture (No. 22540268) and in part by a Grand-in-Aid for Specially Promoted Research (13002001).

References

[1] M. Lüscher, Nucl. Phys. **B354**, 531 (1991).

[2] C. Liu, X. Feng, S. He, Int. J. Mod. Phys. **A21**, 847 (2006).

[3] N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. **99**, 022001 (2007).

[4] S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys. **123**, 89 (2010).

[5] C.-D.J. Lin, G. Martinelli, C.T. Sachrajda, M. Testa, Nucl. Phys. **B619**, 467 (2001).

[6] S. Aoki et al. (CP-PACS Collaboration), Phys. Rev. **D71**, 094504 (2005).

[7] K. Sasaki for HAL-QCD Coll., PoS **LAT2010**, 157 (2010).

PoS(Lattice 2010)145