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Light meson form factors in $N_f = 2+1$ QCD with dynamical overlap quarks

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We report on our calculation of pion and kaon form factors in three-flavor QCD using the overlap quark action. Gauge ensembles are generated on a $16^3 \times 48$ lattice at a lattice spacing of 0.11 fm with pion masses down to 310 MeV. Connected and disconnected meson correaltors are calculated using the all-to-all quark propagator. We present our preliminary analysis on the chiral behavior of the electromagnetic and scalar form factors as well as a comparison of the shape of the $K \to \pi$ form factors with experiment.

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1. Introduction

One of the major goals of lattice QCD is a precise determination of hadron form factors. Electromagnetic (EM) and scalar form factors of pions and kaons are fundamental observables in hadron physics. A detailed comparison between their chiral behavior on the lattice and that in chiral perturbation theory (ChPT) may provide a good testing ground for these theoretical tools as well as a determination of unknown parameters in ChPT, namely the low-energy constants (LECs). Reliable calculation of the form factors of semileptonic weak decays, such as the $K \to \pi l \nu$ decays, is important in the search for new physics through a precise determination of CKM matrix elements.

In this article, we report on our calculation of these light meson form factors. Chiral symmetry is exactly preserved in our simulations, that enables us to directly compare our results with ChPT. We use the all-to-all quark propagator in order to accurately calculate connected and disconnected correlators of pions and kaons.

2. Numerical simulations

Our gauge ensembles of $N_f = 2+1$ QCD are generated using the Iwasaki gauge and the overlap quark actions with a topology fixing term [1], which remarkably reduces the computational cost. While we only explore the trivial topological sector at the moment, the effect of fixing topology on the pion form factors turned out to be under control, typically a few %, in our previous study in two-flavor QCD [2]. The lattice size is $N_s^3 \times N_t = 16^3 \times 48$ and the lattice spacing is 0.112(1) fm with the Ω baryon mass used as input. We simulate four values of degenerate up and down quark masses $m_{ud} = 0.015$, 0.025, 0.035 and 0.050, which cover a range of the pion mass $310 \lesssim M_\pi[\mathrm{GeV}] \lesssim 560$. Measurements of the form factors are carried out with the periodic boundary condition at a single value of the strange quark mass $m_s = 0.080$, which is very close to the physical value 0.081 fixed from M_K. We have accumulated 2,500 HMC trajectories at each combination of m_{ud} and m_s.

We calculate meson correlators using the all-to-all quark propagator [3]. Let us consider an expansion of the propagator using the eigenmodes (λ_k, u_k) $(k = 1, \ldots, 12N_s^3N_t)$ of the overlap-Dirac operator, namely $D^{-1}(x,y) = \sum_k \lambda_k^{-1} u_k(x) u_k^\dagger(y)$. It is expected that low-lying modes dominantly contribute to low-energy observables, such as the form factors. We exactly evaluate this important contribution by using 160 eigenmodes for each configuration. Remaining contribution from the higher modes is estimated stochastically by using the noise method. We refer readers to Ref. [2] for further details on our measurement method using the all-to-all propagator.

We calculate three- and two-point functions

$$C^{P\mathscr{O}P'}_{\phi\phi'}(\Delta t, \Delta t', \mathbf{p}, \mathbf{p}') = \frac{1}{N_t N_s^3} \sum_{\mathbf{x},t} \sum_{\mathbf{x}',\mathbf{x}''} \langle P'_{\phi'}(\mathbf{x}'', t + \Delta t + \Delta t') \mathscr{O}(\mathbf{x}', t + \Delta t) P^\dagger_\phi(\mathbf{x}, t) \rangle \times e^{-i\mathbf{p}'(\mathbf{x}''-\mathbf{x}') - i\mathbf{p}(\mathbf{x}'-\mathbf{x})}, \quad (2.1)$$

$$C^{PP'}_{\phi\phi'}(\Delta t, \mathbf{p}) = \frac{1}{N_t N_s^3} \sum_{\mathbf{x},t} \sum_{\mathbf{x}'} \langle P'_{\phi'}(\mathbf{x}', t + \Delta t) P^\dagger_\phi(\mathbf{x}, t) \rangle e^{-i\mathbf{p}(\mathbf{x}'-\mathbf{x})}, \quad (2.2)$$

where $P^\dagger_\phi$ ($P'^\dagger_{\phi'}$) represents an interpolating operator for the initial (final) meson with a smearing function ϕ (ϕ'), and $\mathscr{O}$ is either the EM current (J_μ), weak current (V_μ), or scalar operator (S). Using the all-to-all propagator, we can accurately calculate these correlators by taking the average over the location of the meson source, namely $(\mathbf{x}, t)$ in Eqs. (2.1) and (2.2). In addition, the correlators

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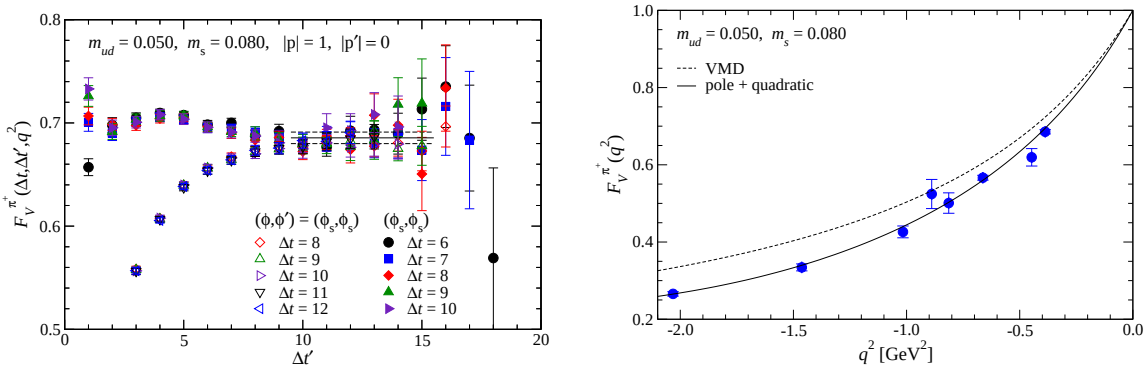

Figure 1: Left panel: effective value $F_V^{\pi^+}(\Delta t,\Delta t';q^2)$ with local (open symbols) and smeared operators for pion source and sink (filled symbols). Note that we can take arbitrary combinations of $(\Delta t,\Delta t')$ by using the all-to-all propagator. Right panel : $F_V^{\pi^+}(q^2)$ as a function of q^2. We also plot the q^2 dependence expected from the VMD hypothesis by the dashed line.

with different choices of $\phi^{(\prime)}$ and $\mathbf{p}^{(\prime)}$ can be calculated with small additional costs. In this study, we take two choices of $\phi^{(\prime)}$, namely local $\phi_l(r)=\delta_{r,0}$ and exponential functions $\phi_s(r)=e^{-0.4r}$, and 27 choices of $\mathbf{p}^{(\prime)}$ with $|\mathbf{p}^{(\prime)}|\le\sqrt{3}$, which cover a region of the momentum transfer $-2.0\lesssim q^2[\mathrm{GeV}^2]\lesssim 0$. Note that meson momenta $p^{(\prime)}$ are shown in units of $2\pi/(N_s a)$ in this article.

3. Electromagnetic form factors

We calculate an effective value of the pion EM form factor from

$$F_V^{\pi^+}(\Delta t,\Delta t';q^2) = \frac{2M_\pi}{E_\pi(|\mathbf{p}|)+E_\pi(|\mathbf{p}'|)}\frac{R^{\pi J\pi}_{\phi\phi'}(\Delta t,\Delta t';q^2)}{R^{\pi J\pi}_{\phi\phi'}(\Delta t,\Delta t';0)}, \tag{3.1}$$

$$R^{\pi J\pi}_{\phi\phi'}(\Delta t,\Delta t';q^2) = \frac{C^{\pi J\pi}_{\phi\phi'}(\Delta t,\Delta t';\mathbf{p},\mathbf{p}')}{C^{\pi\pi}_{\phi\phi_l}(\Delta t;\mathbf{p})C^{\pi\pi}_{\phi_l\phi'}(\Delta t';\mathbf{p}')}, \tag{3.2}$$

where we use the dispersion relation to estimate $E_\pi(|\mathbf{p}|)=\sqrt{M_\pi^2+\mathbf{p}^2}$ at $|\mathbf{p}|\neq 0$. As shown in the left panel of Fig. 1 we can reliably identify the plateau of $F_V^{\pi^+}(\Delta t,\Delta t';q^2)$ by using different combination of the smearing functions (ϕ,ϕ'). The EM form factor $F_V^{\pi^+}(q^2)$ is determined by a constant fit to $F_V^{\pi^+}(\Delta t,\Delta t';q^2)$. The statistical accuracy is typically a few % because of the average over the source location mentioned above. Although we ignore the finite volume correction to $F_V^{\pi^+}(q^2)$ in this preliminary report, it turned out to be comparable with the statistical error of a few % level in our previous study in two-flavor QCD at similar values of m_{ud} and $N_s a$ [2].

As seen in the right panel of Fig. 1, the q^2 dependence of $F_V^{\pi^+}(q^2)$ is close to the ρ meson pole $1/(1-q^2/M_\rho^2)$ expected from the vector meson dominance (VMD) hypothesis. We then assume that the small deviation due to the higher poles or cuts can be approximated by a polynomial of q^2, and use a parametrization

$$F_V^{\pi^+}(q^2) = \frac{1}{1-q^2/M_\rho^2}+c_1 q^2+c_2(q^2)^2 = 1+\frac{1}{6}\langle r^2\rangle_V^{\pi^+} q^2+\cdots. \tag{3.3}$$

to determine the pion charge radius $\langle r^2\rangle_V^{\pi^+}$. Since the deviation of $F_V^{\pi^+}(q^2)$ from VMD is small, we obtain reasonable $\chi^2/\mathrm{d.o.f.}\sim 1$, and $\langle r^2\rangle_V^{\pi^+}$ does not change significantly by the inclusion of higher order corrections of $O(q^6)$.

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At NLO of $SU(3)$ ChPT, $\langle r^2\rangle_V^{\pi^+}$ is given by [4]

$$\langle r^2\rangle_V^{\pi^+} = \frac{1}{2NF_0^2}(-3+24NL_9^r) - 2\nu_\pi - \nu_K, \quad \nu_P = \frac{1}{2NF_0^2}\ln\left[\frac{M_P^2}{\mu^2}\right] \quad (P=\pi,K), \tag{3.4}$$

where $N=(4\pi)^2$ and the renormalization scale μ is set to $4\pi F_0$. We fix F_0 to 52 MeV determined from our study of the meson decay constants [5]. This is significantly smaller than the phenomenological value ~ 88 MeV [6] and enhances the chiral logarithms $\nu_{\pi,K}$. As shown in Fig. 2, the NLO formula fails to reproduce the quark mass dependence of our data (dashed line). The extrapolation becomes consistent with experiment by including a NNLO analytic term $\propto M_\pi^2$ (solid line). We note that significant NNLO contributions have been observed also in our two-flavor studies in a similar region of m_{ud} [2].

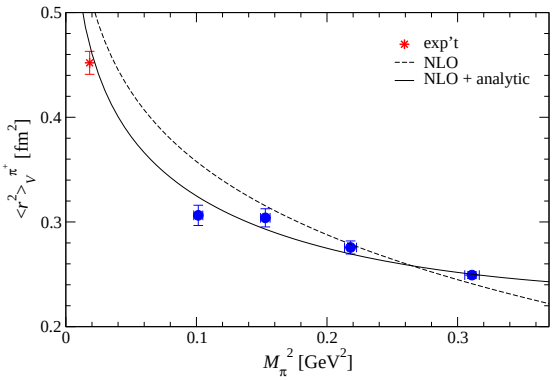

Figure 2: Chiral extrapolation of $\langle r^2\rangle_V^{\pi^+}$ using the one-loop formula (3.4) (dashed line) and that with a higher order analytic term (solid line). The experimental value [7] is plotted by a star.

The kaon EM form factors are calculated from the ratios (3.1) and (3.2) but with kaon source and sink. Results are plotted as a function of q^2 in Fig. 3. The neutral kaon form factor $F_V^{K^0}(q^2)$ originates from a difference between the contributions from the down ($\bar{d}\gamma_\mu d$) and strange quark currents ($\bar{s}\gamma_\mu s$), and is much smaller than the charged one $F_V^{K^+}(q^2)$. We obtain significant signal for $F_V^{K^0}(q^2)$ with our statistical accuracy improved by using the all-to-all propagator. Similar to $F_V^{\pi^+}(q^2)$, the q^2 dependence of both $F_V^{K^+}(q^2)$ and $F_V^{K^0}(q^2)$ is close to that of VMD

$$F_V^{K^+}(q^2) = \frac{2}{3}\frac{1}{1-q^2/M_\rho^2} + \frac{1}{3}\frac{1}{1-q^2/M_\phi^2}, \quad F_V^{K^0}(q^2) = -\frac{1}{3}\frac{1}{1-q^2/M_\rho^2} + \frac{1}{3}\frac{1}{1-q^2/M_\phi^2}. \tag{3.5}$$

We determine charge radii, $\langle r^2\rangle_V^{K^+}$ and $\langle r^2\rangle_V^{K^0}$, using a fitting form with these vector meson poles plus a polynomial correction up to $O(q^4)$.

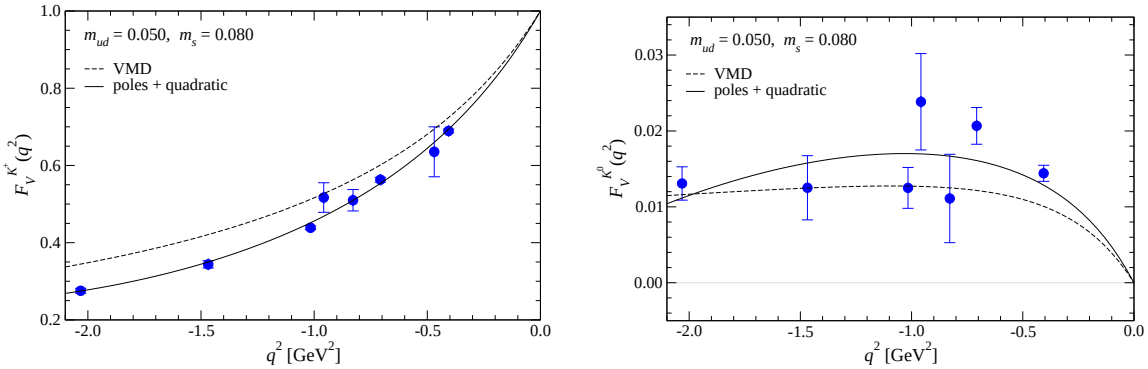

Figure 3: Electromagnetic form factors of charged (left panel) and neutral kaons (right panel) as a function of q^2. Dashed lines show the pole dependence of Eq.(3.5).

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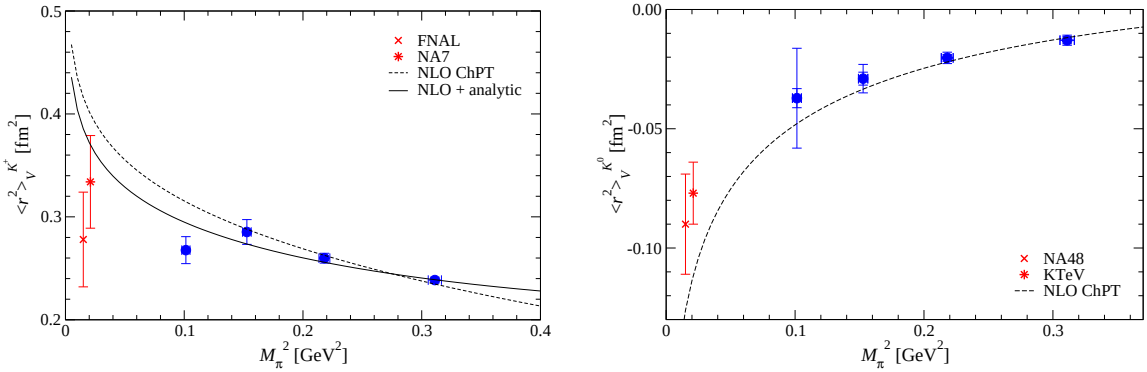

Figure 4: Chiral extrapolation of charge radii of charged (left panel) and neutral kaons (right panel). Crosses and stars are experimental values [7].

Figure 4 shows the chiral extrapolation of $\langle r^2 \rangle_V^{K^+}$ and $\langle r^2 \rangle_V^{K^0}$ based on NLO ChPT

$$\langle r^2 \rangle_V^{K^+} = \frac{1}{2NF_0^2}(-3+24NL_9^r) - \nu_\pi - 2\nu_K, \quad \langle r^2 \rangle_V^{K^0} = \nu_\pi - \nu_K. \tag{3.6}$$

For K^+, we again observe that the NLO fit leads to a large value of χ^2/d.o.f. ~ 3.2. The extrapolation becomes closer to experiment with acceptable χ^2/d.o.f. (~ 0.6) by including a NNLO analytic term.

Since K^0 does not directly couple to photons, the NLO expression of $\langle r^2 \rangle_V^{K^0}$ does not have analytic terms and hence $O(p^4)$ LECs. The dashed line in the right panel of Fig. 4 is a parameter-free prediction with F_0 determined from the decay constants. Our data are consistent with this NLO prediction. For a more rigorous comparison, calculations with twisted boundary conditions are currently underway to reduce the large systematic uncertainty due to the lack of data near $q^2 = 0$.

4. Pion scalar form factor

In this report, we consider the scalar form factor normalized at a reference value of $q^2_{\rm ref}$, namely $F_S^\pi(q^2)/F_S^\pi(q^2_{\rm ref})$, since it has sufficient information to determine the scalar radius $\langle r^2 \rangle_S^\pi$ and does not need a non-perturbative renormalization of S. We set $|q^2_{\rm ref}|$ to our smallest non-zero $|q^2|$, where

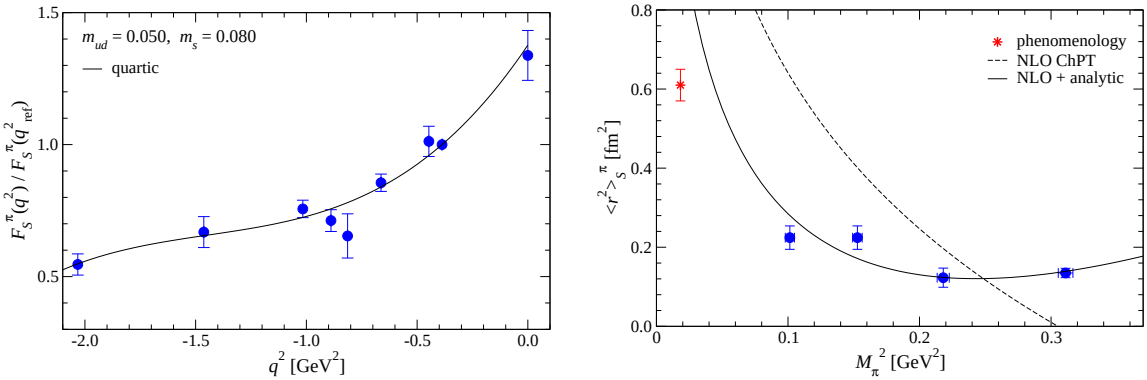

Figure 5: Left panel: $F_S^\pi(q^2)/F_S^\pi(q^2_{\rm ref})$ as a function of q^2. Right panel: chiral extrapolation of $\langle r^2 \rangle_S^\pi$. The star shows a phenomenological estimate from $\pi\pi$ scattering [8].

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we do not need to subtract the contribution from the vacuum expectation value of S. In the left panel of Fig. 5, we plot $F_S^\pi(q^2)/F_S^\pi(q^2_{\rm ref})$ determined from ratios similar to Eqs. (3.1) and (3.2) (see Ref. [2] for details). Due to the lack of the knowledge of scalar resonances at the simulated quark masses, we parametrize the q^2 dependence of $F_S^\pi(q^2)$ by a generic polynomial form

$$F_S^\pi(q^2) = F_S^\pi(0)\left(1+\frac{1}{6}\langle r^2\rangle_S^\pi q^2 + d_2(q^2)^2 + d_3(q^2)^3 + d_4(q^2)^4\right). \tag{4.1}$$

We then fit results for $\langle r^2\rangle_S^\pi$ to the NLO chiral expansion

$$\langle r^2\rangle_S^\pi = \frac{1}{NF_0^2}\{-8+24N(2L_4^r+L_5^r)\} - 12\nu_\pi - 3\nu_K. \tag{4.2}$$

The pion-loop logarithm is 6 times larger than that in $\langle r^2\rangle_V^{\pi^+}$ and is further enhanced by our small value of F_0. As shown in the right panel of Fig.5, the NLO expression has a strong m_{ud} dependence and can not describe our data leading to χ^2/d.o.f. $\sim O(100)$. This is largely reduced to ~ 7 by including a NNLO analytic term $\propto M_\pi^2$ suggesting that the consistency with $SU(3)$ ChPT should be studied by including full NNLO corrections as in our previous study in two-flavor QCD [2].

5. Kaon weak decay form factors

We calculate the vector and scalar form factors of the $K\to\pi$ decays, namely $f_+(q^2)$ and $f_0(q^2)=f_+(q^2)+f_-(q^2)q^2/(M_K^2-M_\pi^2)$, by using ratios of kaon and pion correlators proposed in previous studies [9]. For instance, $\xi(q^2)=f_-(q^2)/f_+(q^2)$, which is needed to convert $f_+(q^2)$ to $f_0(q^2)$ (and vice versa), is determined from a double ratio

$$\frac{C^{KV_k\pi}_{\phi\phi'}(\Delta t,\Delta t',\mathbf{p},\mathbf{p}')C^{KV_4K}_{\phi\phi'}(\Delta t,\Delta t',\mathbf{p},\mathbf{p}')}{C^{KV_4\pi}_{\phi\phi'}(\Delta t,\Delta t',\mathbf{p},\mathbf{p}')C^{KV_kK}_{\phi\phi'}(\Delta t,\Delta t',\mathbf{p},\mathbf{p}')}. \tag{5.1}$$

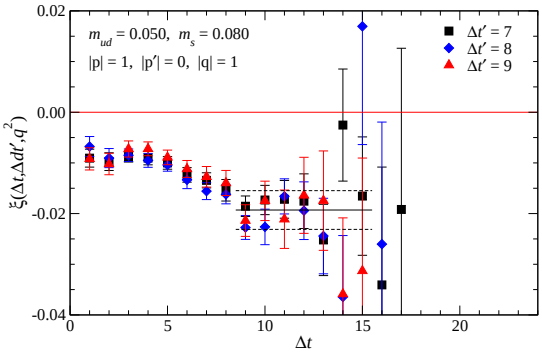

Figure 6: Effective value of ξ extracted from ratio Eq. (5.1).

This involves the three-point functions with spatial components V_k and non-zero meson momenta $\mathbf{p}^{(\prime)}$, that are quite noisy if naively calculated. By using the all-to-all propagator, we obtain a clear signal for $\xi(q^2)$ as shown in Fig. 6. Both of $f_+(q^2)$ and $f_0(q^2)$ are determined with the statistical accuracy of typically a few % even at nonzero $\mathbf{p}^{(\prime)}$.

We plot $f_0(q^2)$ as a function of q^2 in the left panel of Fig.7. The q^2 dependence of both $f_{+,0}(q^2)$ is well described by either the single pole or quadratic form

$$f_X(q^2) = \frac{f_X}{1-q^2/M_X^2}, \qquad f_X(q^2) = 1+c_Xq^2+d_X(q^2)^2 \quad (X=+,0), \tag{5.2}$$

The normalized slopes $\lambda'_X = M^2_{\pi,\rm phys}c_X$ are measured in recent experiments [7]. Our results $\lambda'_+ = 2.01(25)\times10^{-2}$ and $\lambda'_0 = 1.54(20)\times10^{-2}$ extrapolated to $M_{\pi,\rm phys}$ are in good agreement with the experiments as shown in the right panel of Fig. 7. We also observe that the normalized curvature $\lambda''_+ = 2M^4_{\pi,\rm phys}d_+ = 0.08(10)\times10^{-2}$ is also consistent with the experimental value $0.20(5)\times10^{-2}$.

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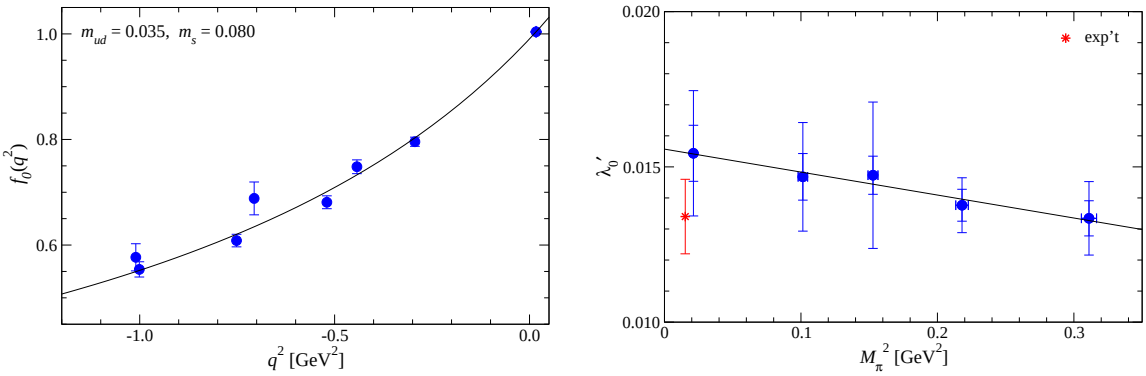

Figure 7: Left panel: $f_0(q^2)$ as a function of q^2. The solid line shows the quadratic parametrization in Eq. (5.2). Right panel: linear chiral extrapolation of λ_0' compared with experiment [7].

6. Conclusions

We report on our calculation of the light meson form factors in three-flavor QCD with overlap quarks. For the EM and scalar form factors, we observe that the mild m_{ud} dependence of our data can not be described by NLO ChPT. We are planning to extend our analysis to NNLO. To this end, it is helpful to calculate various observables in order to constrain many $O(p^4)$ and $O(p^6)$ LECs involved in NNLO chiral expansions. For instance, we calculate the kaon EM form factors with small additional costs using the all-to-all propagator.

We also confirm that the shape of the $K \to \pi$ decay form factor is in good agreement with experiment. Our calculations are being extended to a larger volume with twisted boundary conditions in order to carry out a chiral extrapolation of $f_+(0)$ with controlled systematic uncertainties, which is essential for a reliable estimate of $|V_{us}|$.

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