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The chiral and angular momentum content of the ρ-mesons in lattice QCD

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The variational method allows one to study the mixing of interpolators with different chiral transformation properties in the nonperturbatively determined physical state. It is then possible to define and calculate in a gauge-invariant manner the chiral as well as the partial wave content of the quark-antiquark component of a meson in the infrared, where mass is generated. Using a unitary transformation from the chiral basis to the ${}^{2S+1}L_J$ basis one may extract the partial wave content of a meson. We present results for the ρ- and ρ'-mesons using a simulation with $N_f = 2$ dynamical quarks, all for lattice spacings close to 0.15 fm. Our results indicate a strong chiral symmetry breaking in the ρ state and its simple ${}^3S_1$-wave composition in the infrared. For the ρ'-meson we find a small chiral symmetry breaking in the infrared as well as a leading contribution of the ${}^3D_1$ partial wave, which is contradictory to the quark model.

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1. Introduction

A central question in QCD is the mass generation mechanism and its interconnection with confinement and chiral symmetry breaking. The angular momentum generation of hadrons is another related question. Chiral symmetry is dynamically broken in the QCD vacuum which is evidenced by the absence of parity doublets in the low-lying hadron spectrum and by the existence of the pion as a pseudo Goldstone boson. It follows from the trace anomaly that the hadron mass almost completely consists of the energy of the quantized gluonic field. However, this statement tells us nothing about mechanism of mass generation. It is widely believed that the spontaneous breaking of chiral symmetry, i.e., the quark condensate of the vacuum, is prominently responsible for the mass generation of hadrons like the nucleon or ρ-meson. This was observed in various microscopical models and in the QCD sum rule approach [1, 2].

Chiral symmetry breaking in the vacuum explains a phenomenological success of the quark model, at least for the ground states. Namely, almost massless light quarks acquire their effective (dynamical, constituent) mass at low momenta via their coupling with the quark condensate. This large mass renders the problem effectively non-relativistic and the ground state ρ is a $^3S_1$ state in the quark model language [3, 4]. Traditionally the excitation spectrum is also described within the quark model and the first excited state of the ρ-meson, $\rho(1450)$, is believed to be the first radial excitation, i.e., a $^3S_1$ state [3, 4].

At the same time there are indications that chiral symmetry is effectively restored in the high-lying spectrum [5, 6]. This would imply that the mass generation mechanism in highly excited hadrons is different and the quark condensate of the vacuum is of little importance. It would also imply that the constituent quark model language is inadequate for excited hadrons. To resolve the issue one needs direct information about the hadron structure, which can be supplied in ab initio lattice simulations. Here we present a way to reconstruct in dynamical simulations a chiral as well as an angular momentum decomposition of the leading quark-antiquark component of mesons at physical, infrared scale.

The variational method [7] provides a tool to study the hadron wave function in lattice QCD calculations. One uses a set of interpolators $\{O_1, O_2, \ldots, O_N\}$, which have the quantum numbers of the state of interest, and computes the cross-correlation matrix,

$$C_{ij}(t) = \left\langle\, O_i(t)\, O_j^\dagger(0)\, \right\rangle . \tag{1.1}$$

One solves a generalized eigenvalue problem; assuming that the set of interpolators $\{O_i\}$ is complete enough, the wave function is related to the eigenvectors obtained. We are interested in the reconstruction of the leading quark-antiquark component of the low lying mesons. Therefore we need interpolators that allow us to define such a component in a unique way.

In [5] a classification of all non-exotic quark-antiquark states (interpolators) in the light meson sector according to the transformation properties with respect to the $SU(2)_L \times SU(2)_R$ and $U(1)_A$ was presented. If no explicit excitation of the gluonic field with non-vacuum quantum numbers is present, this basis is a complete one for a quark-antiquark system and we can define and investigate chiral symmetry breaking. Namely, one can reconstruct a decomposition for a given meson in terms of different representations of the chiral group by diagonalizing the cross-correlation matrix from (1.1). The eigenvectors describe the quark-antiquark content in terms of different chiral representations. If we observe components with different transformation properties in terms of $SU(2)_L \times SU(2)_R$ and $U(1)_A$, then we conclude that chiral symmetry is broken in that state.

In order to establish a connection to the quark model, it is interesting to reconstruct the meson composition in terms of the ${}^{2S+1}L_J$ basis, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ are the standard angular momenta in the two-body system. Such a decomposition of the leading quark-antiquark component in terms of the ${}^{2S+1}L_J$ basis in the infrared, i.e., where the hadron mass is generated, would tell us to which degree the quark model language is adequate for a given state.

The ${}^{2S+1}L_J$ angular momentum basis and the chiral basis are both complete for a two-particle system. They are connected to each other via a unitary transformation [8]. Each state of the chiral basis can be uniquely represented in terms of the ${}^{2S+1}L_J$ states. Then, diagonalizing the cross-correlation matrix, built from interpolators with definite chiral transformation properties, one can obtain the partial wave decomposition of the leading Fock component. This method can in principle be applied to any meson, here we use as an example the vector meson ρ and its first excitation $\rho(1450)$ [9, 10].

2. Chiral classification and the transformation to the angular momentum basis

The classification of the quark-antiquark states and interpolators with respect to representations of $SU(2)_L \times SU(2)_R$ was done in [5]. We are interested in the quark-antiquark component of the ground state ρ-meson and its first excitation. There are two possible chiral representations that are compatible with the quantum numbers of the ρ-meson, which have drastically different chiral transformation properties. Assuming that chiral symmetry is not broken, then one has two independent states. The first state is $|(0,1)\oplus(1,0);1\,1^{--}\rangle$; it can be created from the vacuum by the standard vector current

$$O_V = \overline{q}\gamma^i\,\vec{\tau}\,q\,. \tag{2.1}$$

Its chiral partner is the a_1 meson. The other state is $|(1/2,1/2)_b;1\,1^{--}\rangle$, which can be created by the pseudotensor operator,

$$O_T = \overline{q}\sigma^{0i}\,\vec{\tau}\,q\,, \tag{2.2}$$

and its chiral partner is the h_1 meson. Here, $\vec{\tau}$ denotes the vector of isospin Pauli matrices.

Chiral symmetry breaking in the state implies that the state should be a mixture of both representations. If it is a superposition of both representations with approximately equal weights, then the chiral symmetry is maximally violated in this state. If, on the contrary, one of the representations strongly dominates over the other representation, one could speak about effective chiral restoration in this state.

These chiral representations can be transferred into the ${}^{2S+1}L_J$ basis, using the unitary transformation [8]

$$\begin{pmatrix} |(0,1)\oplus(1,0);1\,1^{--}\rangle \\ |(1/2,1/2)_b;1\,1^{--}\rangle \end{pmatrix} = U \cdot \begin{pmatrix} |1;{}^3S_1\rangle \\ |1;{}^3D_1\rangle \end{pmatrix}\,, \tag{2.3}$$

where U is given by

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}\,. \tag{2.4}$$

Thus, using the interpolators O_V and O_T from (2.1) and (2.2) for the diagonalization of the cross-correlation matrix, we are able to reconstruct the partial wave content of the leading $\bar{q}q$-Fock component of the ρ-meson.

3. Reconstruction of the wave function using the variational method

We briefly want to discuss the basic features of the variational method [7] and how to analyse the decomposition of the ρ-mesons. The time propagation properties of the normalized physical states $|n\rangle$ are given by

$$\langle n(t)|m(0)\rangle = \delta_{nm}\,\mathrm{e}^{-E_n t}\ . \tag{3.1}$$

The lattice interpolators O_i are typically not normalized and are projected to zero spatial momentum. The cross-correlation matrix from Eq. (1.1) can be written as

$$C_{ij}(t) = \left\langle\, O_i(t)\, O_j^\dagger(0)\,\right\rangle\ = \sum_n a_i^{(n)}\, a_j^{(n)*}\,\mathrm{e}^{-E_n t}\ , \tag{3.2}$$

where the coefficients $a_i^{(n)}$ give us the overlap of the physical state $|n\rangle$ with the lattice interpolator O_i,

$$a_i^{(n)} = \langle 0|O_i|n\rangle\ . \tag{3.3}$$

The two chiral representations $(0,1)\oplus(1,0)$ and $(1/2,1/2)_b$ form a complete and orthogonal basis (with respect to the chiral group) for ρ-mesons. Consequently, using the variational method we are able to study the mixing of the two representations in both ρ and ρ' states.

Following the lines of [9] one can show that the ratio of couplings can be obtained as

$$\frac{a_i^{(n)}(t)}{a_k^{(n)}(t)} = \frac{C_{ij}(t)\, u_j^{(n)}(t)}{C_{kj}(t)\, u_j^{(n)}(t)}\ . \tag{3.4}$$

The ratio of the vector to pseudotensor couplings, $a_V^{(n)}/a_T^{(n)}$, tells us about the chiral symmetry-breaking in the states $n=\rho,\rho'$.

4. Defining the resolution scale

If we probe the hadron structure with the local interpolators, then we study the hadron decomposition at the scale fixed by the lattice spacing a. For a reasonably small a this scale is close to the ultraviolet scale. However, we are interested in the hadron content at the infrared scales, where mass is generated. For this purpose we cannot use a large a, because matching with the continuum QCD will be lost. Given a fixed, reasonably small lattice spacing a a large infrared scale R (i.e., small resolution scale $1/R$) can be achieved by gauge-invariant smearing of the point-like interpolators. We smear the quark field (sources) in spatial directions over the size R in physical units, such that $R/a \gg 1$. Then, even in the continuum limit $a \to 0$ we probe the hadron content at the infrared scale fixed by R. Such a definition of the resolution is similar to the experimental one, where an external probe is sensitive only to quark fields (it is blind to gluonic fields) at a resolution that is determined by the momentum transfer in spatial directions.

Set	β_{LW}	$a\,m_0$	#conf	a [fm]	m_π [MeV]	m_ρ [MeV]
A	4.70	-0.050	200	0.1507(17)	526(7)	911(11)
B	4.65	-0.060	300	0.1500(12)	469(5)	870(10)
C	4.58	-0.077	300	0.1440(12)	323(5)	795(15)

Table 1: Details of the lattice simulation: the leading value β_{LW} of the gauge coupling, the bare mass parameter $a\,m_0$ of the CI action, the number of analyzed configurations, the lattice spacing a, the pseudoscalar mass m_π, and the vector meson mass m_ρ (cf. [13] for more details).

The smearing procedure itself is done using Jacobi smearing [11]. A smearing operator M acts on a point-like source S_0,

$$S = M\,S_0\ , \qquad M = \sum_{n=0}^{N} (\kappa H)^n\ . \tag{4.1}$$

The hopping term H is given by

$$H = \sum_{k=1}^{3} \left[U_k(x,t)\,\delta_{x+\hat{k},y} + U_k^\dagger(x-\hat{k},t)\,\delta_{x-\hat{k},y} \right]\ . \tag{4.2}$$

It creates approximately a Gaussian profile of the width R for each quark field of the smeared interpolator.

5. Lattice simulation details and results

Like in our previous analysis of excited hadrons [12, 13], we use the Lüscher-Weisz gauge action [14] and the Chirally Improved (CI) Dirac operator, which has better chiral properties than the Wilson Dirac operator [15]. For this study three sets of dynamical gauge configurations, all for lattice size of $16^3 \times 32$, including two mass-degenerate light sea quarks are used (for details see Tab. 1).

We include in our cross-correlation matrix the four interpolators

$$O_1 = \bar{u}_n\,\gamma^i\,d_n\ , \qquad O_2 = \bar{u}_w\,\gamma^i\,d_w\ , \tag{5.1}$$

$$O_3 = \bar{u}_n\,\gamma^t\,\gamma^i\,d_n\ , \qquad O_4 = \bar{u}_w\,\gamma^t\,\gamma^i\,d_w\ . \tag{5.2}$$

Here γ^i denotes one of the spatial Dirac matrices and γ^t the γ-matrix in (Euclidean) time direction. The subscripts n and w (for narrow and wide) denote the two smearing widths, $R \approx 0.34$ fm and 0.67 fm, respectively. With these interpolators we are able to extract both the ground state mass and the mass of the first excited state of the ρ-meson, see the l.h.s. of Fig. 1.

On the r.h.s. of Fig. 1 we show the R-dependence of the ratio from Eq. (3.4) for the case a_V/a_T both for the ground state ρ-meson and its first excited state. This ratio of the vector to the pseudotensor coupling to the states shows us their decomposition in terms of the $(0,1)\oplus(1,0)$ and $(1/2,1/2)_b$ representations. For the ground state at the largest value of $R \approx 0.67$ fm this ratio is approximately 1.2, i.e., we see a strong mixture of the two representations in the wave function of the ground state ρ-meson. Inverting the unitarity transformation from Eq. (2.3) results

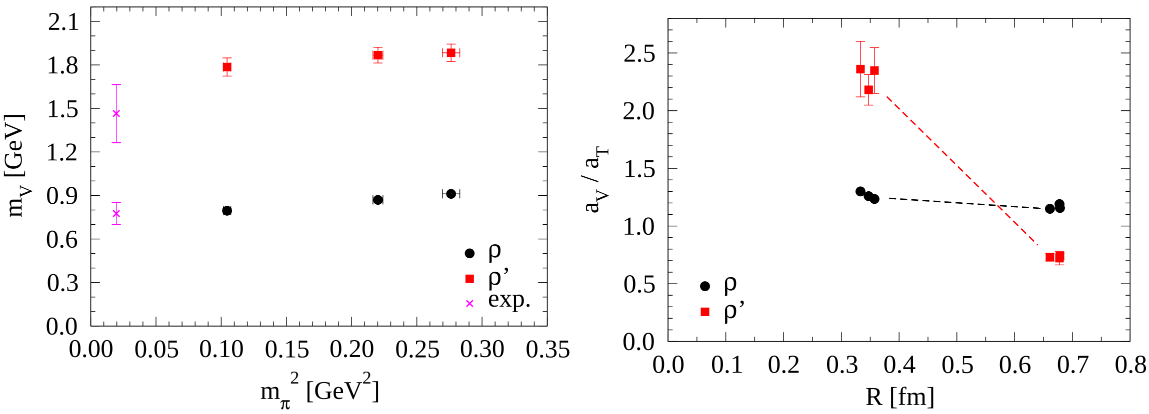

Figure 1: L.h.s.: The vector meson mass m_V is plotted against m_π^2 for all three sets. Black circles represent the ground state, ρ, and red squares represent the first excitation, ρ'. The experimental values are depicted as magenta crosses with decay width indicated. R.h.s.: The ratio a_V/a_T is plotted against the smearing width R for all three data sets. Black circles represent the ground state and red squares the first excitation. Broken lines are drawn only to guide the eye (color online).

in the fact that the vector meson is predominantly a $^3S_1$ state with a tiny admixture of a $^3D_1$ wave, $0.997|^3S_1\rangle - 0.073|^3D_1\rangle$. This result indicates that the $\rho(770)$ at the scale fixed by the meson size is approximately a $^3S_1$ state – in agreement with the quark model language.

However, the situation changes for the first excited state, $\rho' = \rho(1450)$. In this case a strong dependence of the ratio on the infrared scale is observed. Extrapolating the results to the scale of the ρ' size, $R \sim 0.8 - 1$ fm, one expects a significant contribution from the $(1/2,1/2)_b$ representation and a contribution of the other representation is suppressed. This indicates a smooth onset of effective chiral restoration.

The interpretation is as follows. From the conformal symmetry of QCD one expects that in the deep ultraviolet the pseudotensor interpolator decouples from the ρ-mesons. This can also be seen from the non-vanishing anomalous dimension of the pseudotensor operator, implying its decoupling in the ultraviolet limit. Thus, the ratio a_V/a_T must increase for small R. At large R the ratio determines a degree of chiral symmetry breaking in the infrared region, where mass is generated.

In the $\rho(770)$ meson chiral symmetry is strongly broken since this state is a strong mixture of $(0,1)\oplus(1,0)$ and $(1/2,1/2)_b$ with approximately equal weights. Consequently, its "would-be chiral partners" have a much larger mass: $a_1(1260)$ and $h_1(1170)$. To these low lying states we cannot assign any chiral representation. For the $\rho(1450)$ the contribution from $(1/2,1/2)_b$ is much bigger than the contribution of the other representation. One then predicts that in the same energy region there must exist an h_1 (and not an a_1) meson as an approximate chiral partner of $\rho(1450)$. And in fact there is a state $h_1(1380)$ and no a_1 state in the same energy region. The second excited ρ-meson, the $\rho(1700)$, should then be dominated by the representation $(0,1)\oplus(1,0)$. This assumption is favored by the existence of the $a_1(1640)$ state. There is no room for this $a_1(1640)$ meson within the the quark model [3, 4].

Although we do not have the precise value of the ratio a_V/a_T for $\rho(1450)$ at large $R \sim 0.8 -$

1fm, it is indicative that this value is small. Then we are able to give a qualitative estimate for the angular momentum content of the $\rho(1450)$ in the infrared. Assuming a vanishing ratio the state would have the following partial wave content,

$$\sqrt{\frac{1}{3}}\,|{}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|{}^3D_1\rangle\ . \tag{5.3}$$

This shows a leading contribution of the ${}^3D_1$ wave. Possible small deviations of the ratio from zero do not change this qualitative conclusion. This result is inconsistent with ρ' to be a radial excitation of the ground state ρ-meson, i.e., an ${}^3S_1$ state, as predicted by the quark model [3, 4].

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