PROCEEDINGS OF SCIENCE

Baryon axial charges and momentum fractions with $N_f = 2+1$ dynamical fermions

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QCDSF/UKQCD Collaboration

In this talk we report on recent results of the QCDSF/UKQCD Collaboration on investigations of baryon structure using configurations generated with $N_f = 2+1$ dynamical flavours of $\mathcal{O}(a)$-improved Wilson fermions. With the strange quark mass as an additional dynamical degree of freedom in our simulations we avoid the need for a partially quenched approximation when investigating the properties of particles containing a strange quark, e.g. the hyperons. In particular, we will focus on the nucleon and hyperon axial coupling constants and quark momentum fractions.

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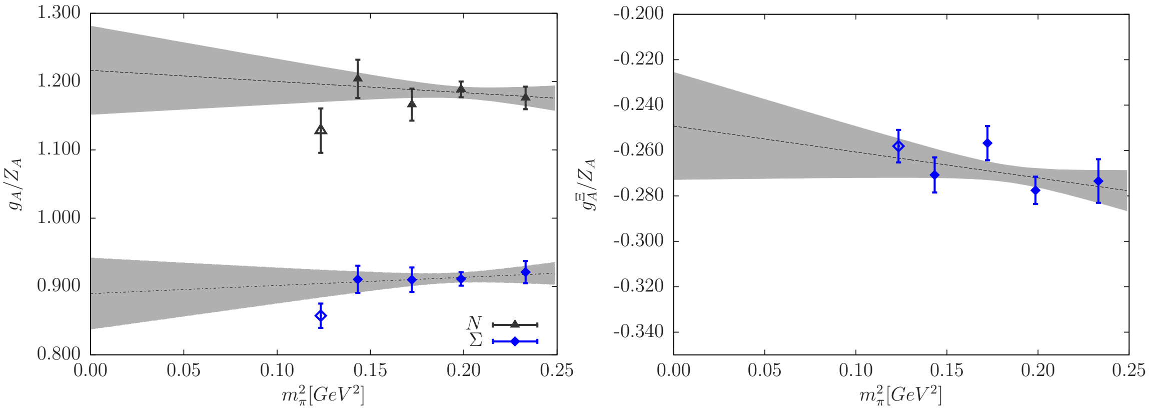


(a) Axial charge (unrenormalised) for N and Σ.

(b) Axial charge (unrenormalised) for Ξ

Figure 1: Unrenormalised baryon axial charges. The data points depicted with open symbols were omitted for the two-parameter linear fits.

1. Introduction

The axial coupling constant of the nucleon is important as it governs neutron β-decay and also provides a quantitative measure of spontaneous chiral symmetry breaking. It is also related to the first moment of the helicity dependent quark distribution functions, $g_A = \Delta u - \Delta d$. It has been studied theoretically as well as experimentally for many years and its value, $g_A = 1.2695(29)$, is known to very high accuracy. Hence it is an important quantity to study on the lattice, and since it is relatively clean to calculate (zero momentum, isovector), it serves as useful yardstick for lattice simulations of nucleon structure.

While there has been much work on the (experimentally well-known) nucleon axial coupling, there have only been a handful of lattice investigations of the axial coupling constants of the other octet baryons [1 – 3], which are relatively poorly known experimentally. These constants are important since at leading order of SU(3) heavy baryon chiral perturbation theory (χPT), these coupling constants are linear combinations of the universal coupling constants D and F, which enter the chiral expansion of every baryonic quantity.

Much of our knowledge about QCD and the structure of the nucleon has been derived from deep inelastic scattering experiments where cross sections are determined by its structure functions. Through the operator product expansion, the first moments of these structure functions are directly related to the momentum fractions carried by the quarks and gluons in the, e.g., nucleon, $\langle x \rangle_{q,g}$. While the quark momentum fractions of the nucleon and pion have received much attention for many years (see, e.g., [4] for a recent review), there have to date been no investigations of the $SU(3)_{\text{flavour}}$ symmetry breaking effects of the quark momentum fractions of the hyperons. The obvious question that arises in this context is: *"How is the momentum of the hyperon distributed amongst its light and strange quark constituents?"*

We present preliminary results from the QCDSF/UKQCD Collaboration for the octet hyperon axial charges, g_{BB}, and quark momentum fractions, $\langle x \rangle_q$, determined from $N_f = 2+1$ lattice QCD.

κ_l	κ_s	$m_\pi[GeV]$	$m_K[GeV]$	$N \times N_T$	$m_\pi L$	N_{meas}
0.120830	0.121040	0.481	0.420	24x48	4.63	2500
0.120900	0.120900	0.443	0.443	24x48	4.28	4000
0.120950	0.120800	0.414	0.459	24x48	3.99	2500
0.121000	0.120700	0.377	0.473	24x48	3.63	2500
0.121040	0.120620	0.350	0.485	24x48	3.37	2500

Table 1: Simulation parameters for $N_f = 2+1$ dynamical fermions with two mass-degenerate light quarks and one strange quark. The simulation parameter β was chosen to $\beta = 5.50$ which corresponds to a lattice spacing of $a = 0.078(3)$fm.

2. Simulation Details

Our gauge field configurations have been generated with $N_f = 2+1$ flavours of dynamical fermions, using the Symanzik improved gluon action and nonperturbatively $\mathscr{O}(a)$ improved Wilson fermions [5]. We choose our quark masses by first finding the $SU(3)_{\text{flavour}}$-symmetric point where flavour singlet quantities take on their physical values and vary the individual quark masses while keeping the singlet quark mass $\overline{m}_q = (m_u + m_d + m_s)/3 = (2m_l + m_s)/3$ constant [6]. Simulations are performed on lattice volumes of $24^3 \times 48$ with lattice spacing, $a = 0.078(3)$fm. A summary of the parameter space spanned by our dynamical configurations can be found in Table 1. More details regarding the tuning of our simulation parameters are given in Ref. [6].

3. Axial charges g_A

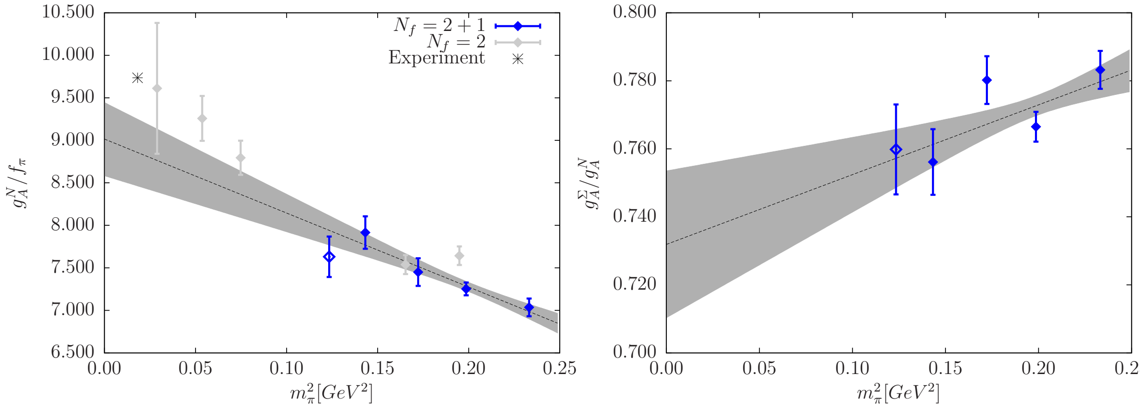
The axial charge of baryon states, in particular the nucleon, has been the subject of both theoretical and experimental studies for many years. The axial charges are defined as the axial vector form factor at zero four-momentum transfer, $g_A = G_A(0)$, which is obtained from the matrix element for the baryon, B

$$\langle B(p',s')|A_\mu^{u-d}|B(p,s)\rangle = \bar{u}_B(p',s')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5\frac{q_\mu}{2m_N}G_P(q^2)\right]u_B(p,s)\ , \tag{3.1}$$

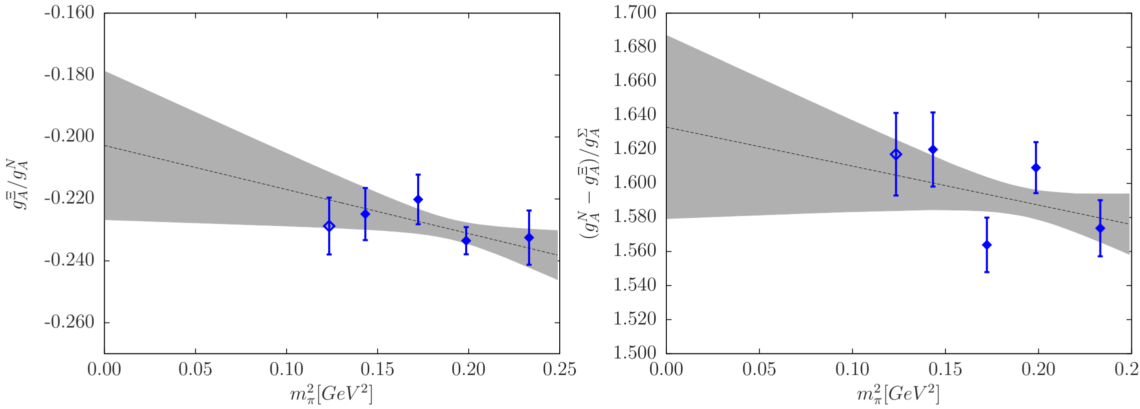
where $q = p' - p$ denotes the 4-momentum transfer and $u_B(p,s)$ is the spinor for the baryon, B, with momentum p and spin vector s and G_P is the induced pseudoscalar form factor. The isovector axial current is defined as $A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$ where u and d denote the up and down quark fields, respectively. We work in the limit of exact isospin invariance, i.e. u and d quarks are assumed to be degenerate in mass. The states are normalised according to $\langle p',s'|p,s\rangle = (2\pi)^3 2p^0\delta(\mathbf{p}-\mathbf{p}')\delta_{ss'}$, we take $s^2 = -m_B^2$ and m_B is the baryon mass. Thus the axial charge is given by the forward matrix element $\langle B(p,s)|A_\mu^{u-d}|B(p,s)\rangle = 2g_A^B s_\mu$. In parton model language, the forward matrix elements of the axial current are related to the fraction of the spin of the baryon carried by the quarks. Denoting by $\langle 1\rangle_{\Delta q}^B$ the contribution of the quark, q, to the spin of the baryon, B, one has

$$\langle B(p,s)|\bar{q}\gamma_\mu\gamma_5 q|B(p,s)\rangle = 2\langle 1\rangle_{\Delta q}^B s_\mu \tag{3.2}$$

Thus for the nucleon we write $g_A^N = \langle 1\rangle_{\Delta u}^N - \langle 1\rangle_{\Delta d}^N$.



(a) Nucleon axial charge over pion decay constant (b) g_A^Σ / g_A^N



(c) g_A^Ξ / g_A^N (d) $(g_A^N - g_A^\Xi)/g_A^\Sigma$

Figure 2: Ratios of unrenormalised baryon axial charges where the renormalisation constant cancels.

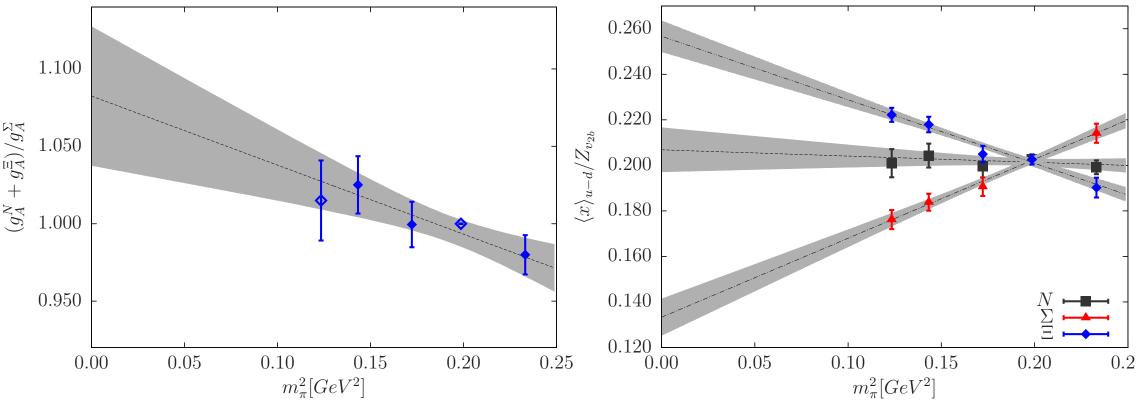
Figure 1a (1b) shows the unrenormalised axial charge for the nucleon and the Σ (Ξ). It is well known that the nucleon axial charge is sensitive to finite size effects (FSE) [7], and so we suspect that the results for N and Σ at the lightest pion masses are suffering from FSE.

With the current data we can not do better than a first approximation with a linear two-parameter fit to find the unrenormalised axial charge at the physical point.

The next step would be to renormalise our results, however as yet Z_A is unknown for these ensembles so we instead consider ratios where the renormalisation constant cancels. The first ratio we consider is shown in Fig. 2a where we plot the ratio of the axial charge of the nucleon g_A^N over the pion decay constant $f_{\pi^\pm}$. Since the renormalisation constants cancel in the ratio, we are able to compare our results to the experimental value [8] and to our $N_f = 2$ results [9]. Except for the lightest pion mass, which is possibly due to FSE, the measurements show a trend towards the experimental value and agree very well with the $N_f = 2$ results.

R	a_0	a_1	χ^2/dof	quality	value
g_A^Σ/g_A^N	0.732(22)	0.21(11)	6.043090	0.002374	0.736(22)
g_A^Ξ/g_A^N	-0.203(24)	-0.14(13)	1.299522	0.272662	-0.205(24)
$(g_A^N - g_A^\Xi)/g_A^\Sigma$	1.633(54)	-0.23(28)	6.498347	0.001506	1.629(54)
$(g_A^N + g_A^\Xi)/g_A^\Sigma$	1.082(45)	-0.45(23)	0.311016	0.577057	1.074(45)

Table 2: Ratios of baryon axial charges in the chiral limit. Extrapolations to the physical point were obtained via a two-parameter linear fit model $R = a_0 + a_1 m_\pi^2$.



(a) $(g_A^N + g_A^\Xi)/g_A^\Sigma$

(b) Unrenormalised quark momentum fractions

Figure 3: Left side: Ratio of baryon axial charge. Right side: Quark momentum fractions for the N, Σ, and Ξ.

4. Ratios of Axial Charges

In the case of exact flavour-SU(3) symmetry, the axial charges of the N, Σ, and Ξ ground states are connected by the following simple relations in terms of the SU(3) constants, F and D [10, 11]

$$g_A^N = F + D \qquad g_A^\Sigma = 2F \qquad g_A^\Xi = D - F.$$

As before, since we have not yet determined Z_A, we will only consider ratios of the baryon axial charges in which the renormalisation constant cancels:

$$\frac{g_A^\Sigma}{g_A^N} = \frac{2F}{F+D} \qquad \frac{g_A^\Xi}{g_A^N} = \frac{F-D}{F+D} \qquad \frac{g_A^N - g_A^\Xi}{g_A^\Sigma} = \frac{D}{F} \qquad \frac{g_A^N + g_A^\Xi}{g_A^\Sigma} = 1$$

These ratios are shown in Figs. 2b - 3a as a function of m_π^2. Once again, for these results we use a linear extrapolation to obtain preliminary predictions at the physical quark masses which we display in Table 2. These preliminary results are in excellent agreement with earlier lattice [1, 3] and quark model [12] determinations.

From a fit to the experimental data taking model independent leading SU(3) breaking contributions to the axial current matrix elements into account Savage and Walden [13] found the following values: $F = 0.47(7)$ and $D = 0.79(10)$. Combining the central values the following ratios are obtained: $g_A^\Sigma/g_A^N = 0.75$, $g_A^\Xi/g_A^N = -0.25$, and $(g_A^N - g_A^\Xi)/g_A^\Sigma = 1.68$. Thus, our results are in good accordance with their results.

5. Momentum Fractions

The first moment of a baryon's, B, unpolarised quark distribution function, $q(x)$ gives the total fraction of the baryon's momentum carried by the quark, q, $\langle x\rangle_q^B$

$$\langle B(p)|\overline{q}\gamma^{\{\mu} i\overset{\leftrightarrow}{D}{}^{\nu\}} q|B(p)\rangle = 2\langle x\rangle_q^B p^{\{\mu} p^{\nu\}} \ , \tag{5.1}$$

where $\overset{\leftrightarrow}{D} = (\vec{D} - \overset{\leftarrow}{D})/2$ is the forward/backward covariant derivative.

We determine the individual connected quark contributions, $\langle x\rangle_q^B$, and take the difference of the doubly and singly represented quark contributions ($(D-S)$, which is $(u-d)$ in the nucleon) so that the disconnected contributions cancel. As in the previous section for the axial charges, the renormalisation constant for the momentum fractions $Z_{v_{2b}}$ has not yet been determined, and so we are not yet able to make any quantitative predictions for the quark momentum fractions of the hyperons.

Figure 3b shows $(D-S)$ quark momentum fractions for the nucleon $(u-d)$, $\Sigma(u-s)$, and $\Xi(s-u)$. Here we see clear evidence for $SU(3)_{\text{flavour}}$-symmetry breaking effects as the fractions carried by the light and strange quark fan out from the symmetric point as we decrease the pion mass. This result indicates that the larger contribution to the baryon momentum is carried by the (heavier) strange quark, and that this contribution increases (and in turn, the light quark contribution decreases) as the strange (light) quark mass is increased (decreased) towards its physical value.

6. Conclusions

We have presented preliminary results from the QCDSF/UKQCD collaboration for the octet hyperon axial coupling constants and quark momentum fractions from a simulation in $N_f = 2+1$ lattice QCD.

Our results for the hyperon axial charges agree well with earlier lattice results and show a hint of $SU(3)_{\text{flavour}}$-symmetry breaking effects. The quark momentum fractions of the octet hyperons, on the other hand, show strong $SU(3)_{\text{flavour}}$-symmetry breaking effects, with the heavier strange quark contributing a larger fraction to the total baryon momentum than the light quarks.

An obvious feature that is currently lacking from these results is a determination of the renormalisation constants for the local operators considered here. These calculations are now underway and will allow us to make more quantitative predictions in the near future.

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