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Renormalized Polyakov loop in the Fixed Scale Approach

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I compute Polyakov loop, the deconfinement order parameter, for $SU(2)$ lattice gauge theory using the fixed scale approach for several different scales and show how to obtain a renormalized physical order parameter. The generalization to other gauge theories, including quenched or full QCD, is straightforward.

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1. Introduction

Phase transitions in various spin models have served as wonderful examples for the study of quark-hadron transition in quantum chromodynamics (QCD) and the related $SU(N_c)$ gauge theories, where N_c is the number of colours. While average magnetization serves as the order parameter in the former case, the Polyakov loop, L, defined as the product of the timelike gauge links at a given site, is the order parameter for the deconfinement transition [1]. On an Euclidean $N_\sigma^3 \times N_\tau$ lattice $L(\vec{x})$ is defined at a site $\vec{x}$ as

$$L(\vec{x}) = \frac{1}{N_c}\ \mathrm{Tr}\ \Pi_{x_0=1}^{N_\tau}\ U^4(\vec{x}, x_0)\ , \tag{1.1}$$

where $U^\mu(x)$ are the gauge variables associated with the directed links in the μth direction, $\mu = 1,4$. As in the spin models again, it is convenient to define its average over the spatial volume, $\bar{L} = \sum_{\vec{x}} L(\vec{x})/N_\sigma^3$. $\langle|\bar{L}|\rangle$ was used to establish a second order deconfinement transition in numerical simulations of the $SU(2)$ pure gauge theory. Since then it has been used for similar studies of the deconfinement phase transitions for a variety of N_c [2], for establishing the universality [3] of the continuum limit, as well as for theories with dynamical quarks [4]. Further, the predicted universality[5] of critical indices has also been numerically verified [6]. Indeed, one hopes to be able to construct effective actions [7] for L in a Wilsonian RG approach. These will be similar to the spin models in the same universality class but with possibly additional interaction terms. A large number of models of quark-hadron transitions use the Polyakov loop as the order parameter for the deconfinement transition as well.

An order parameter should be physical, i.e., independent of the the lattice size. This is indeed so for spin models for sufficiently large lattices. For $SU(N_c)$ gauge theories, this requirement means in addition independence from the lattice spacing a in the continuum limit. Furthermore, it must be so in *both* the phases it seeks to distinguish. As is the case for any bare Wilson loop, the Polyakov loop, needs to be renormalized for this to be true. Since the bare Polyakov loop is further known to decrease progressively with N_τ, suggesting it to be zero in the continuum limit in the high temperature phase, renormalized L is even more desirable to have.

2. Results

The physical interpretation of the order parameter as a measure of the free energy of a single quark, $\langle\bar{L}(T)\rangle = \exp(-F_Q(T)/T)$ provides a straightforward clue for renormalization. Since many years various attempts to remove the divergent contribution in the single quark free in the continuum limit have been made. These include computations employing lattice perturbation theory [8], use of the heavy quark-antiquark free energy [9], fits to $\langle\bar{L}\rangle$ on N_τ-grids [10] and an iterative direct renormalization procedure [11] for $\langle\bar{L}\rangle$ among others.

Here I advocate [12] another, perhaps better, method to define renormalized $\langle\bar{L}\rangle$. Let me elaborate why this maybe so. The definition of Ref. [9] needs heavy quark potential at short distances. Lattice artifacts are at their worse when one is at such short distances, with maximal violation of the rotational invariance. Finite volume of the lattice also enters in defining the large distance between the heavy quarks, or Polyakov loops. Similarly the iterative procedure used in Ref. [11] to obtain the renormalization constants needs large lattices in both spatial and temporal

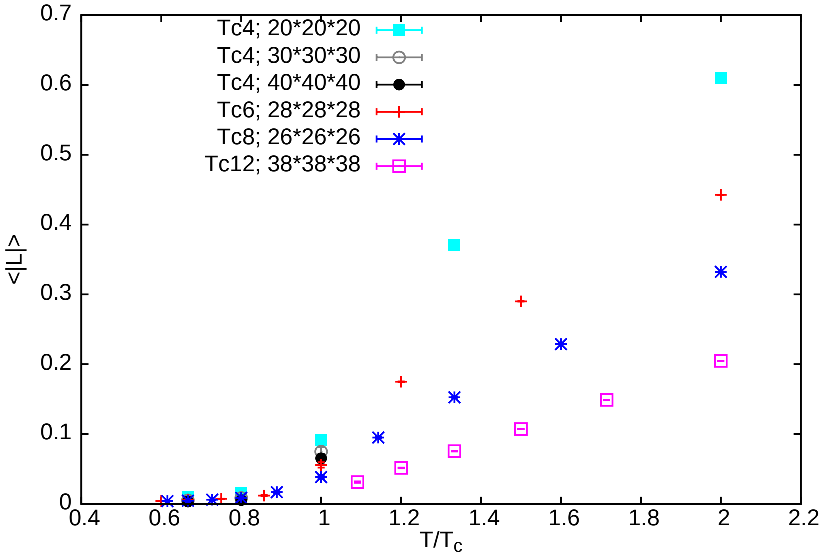

Figure 1: The average Polyakov loop as a function of T/T_c for four different scales. The lattice sizes are as indicated in the key.

directions. Physically perhaps an undesirable aspect of the definition of Ref. [11] is that it works only on the plasma side, i.e., for $T \geq T_c$, where T_c is the position of the peak in the Polyakov loop susceptibility. The definition [9] has so far been employed only in the $T \geq T_c$ for pure gauge theories for which L is an order parameter. It would clearly be nice if the renormalization procedure is applicable to the usually employed $\langle|\bar{L}|\rangle$, which is used as an order parameter on finite volumes.

I obtain a renormalized Polyakov loop which is valid for both the phases below and above T_c [12]. It can be defined in any spatial volume, and it becomes the true order parameter in the infinite spatial volume limit. Of course, it is also physical, i.e., N_τ-independent on finite volumes as well. Indeed, it seems to work rather well for a range of temporal lattice sizes, including $N_\tau \geq 4$. I use the fixed scale approach [13] to do so. It was introduced to minimize the computational costs for the zero temperature simulations needed to subtract the vacuum contribution in thermodynamic quantities such as the pressure and to isolate pure thermal effects in computation of T_c [14]. Furthermore, its advantage is that all the simulations stay on the line of constant physics in a straightforward way. What I argue is that it is indeed this advantage which also permits an easy renormalization of the Polyakov loop. Although these considerations are general, and apply to any $SU(N)$ gauge theory as well as any quark representation, I shall consider below the simplest case of the $SU(2)$ lattice gauge theory to illustrate how and why it works.

Recall that the temperature T is varied in this approach by varying N_τ, holding the lattice spacing a, or equivalently the gauge coupling $\beta = 2N_c/g^2$ fixed. The single quark free energy $F_b(N_\tau, a)$ is then obtained from the $\bar{L}$ by the canonical relation,

$$\ln\langle|\bar{L}|\rangle = -aN_\tau F_b(N_\tau, a) \ . \tag{2.1}$$

The subscript b reminds us that one obtains the bare free energy this way. If the chosen coupling is β_c, corresponding to the position of the peak of the $|L|$-susceptibility in the usual fixed N_τ approach,

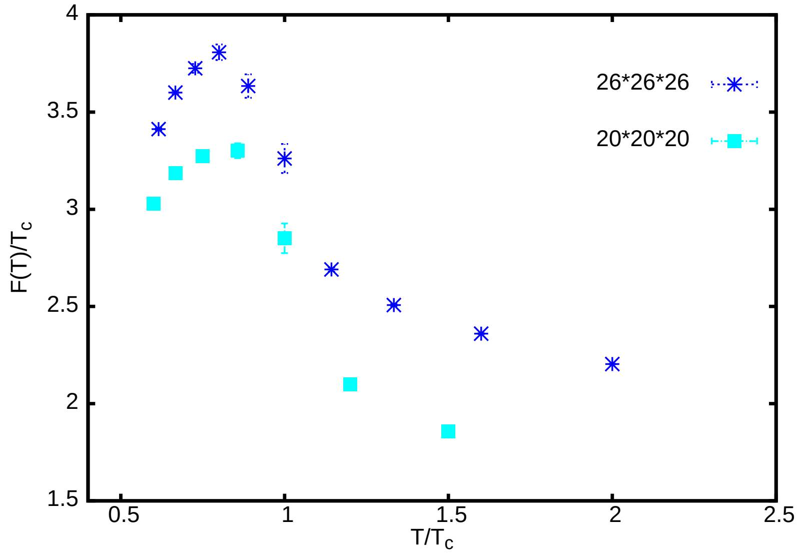

Figure 2: The heavy quark free energy F a function of T/T_c for two different scales. The lattice sizes are as indicated in the key.

and it lies in the scaling region, then the physical deconfinement temperature $T_c = 1/N_{\tau,c}a_c$, and $T/T_c = N_{\tau,c}/N_\tau$ in the fixed scale approach, with the free energy given by $F_b(T/T_c, a_c)$. Writing it as a sum of a divergent and a regular contribution, one has $a_cF_b(T/T_c, a_c) = a_cF(T/T_c, a_c) - a_cA(a_c)$, where A is the divergent free energy in physical units. Clearly, the divergent contribution will be *same* at all temperatures in the fixed scale approach since it depends only on a_c.

Since β_c, or a_c, is known precisely for the Wilson action of the $SU(2)$ theory for many different N_τ, I chose four different scales labelled T_{c4}, T_{c6}, T_{c8}, T_{c12} corresponding to the known transition couplings on $N_\tau = 4$, 6 [15] and 8, 12 [16] respectively: $\beta_{c1} = 2.2991$, $\beta_{c2} = 2.4265$, $\beta_{c3} = 2.5104$, and $\beta_{c4} = 2.6355$. Note that T/T_c is given simply by n/N_τ with $n = 4$, 6, 8 and 12 respectively. Employing then $N_\tau = 3$ to 12, I varied the temperature in the range $2 \geq T/T_c \geq 0.6$. Note that fixed scale a_c leads to a constant spatial volume in physical units in each case in contrast to the usual fixed N_τ approach where the spatial volume varies with T. I used a variety of spatial lattice sizes.

Figure 1 shows the results for the thermal expectation value of $\bar{L}$ as a function of the temperature in the units of T_c. In most cases, I used both a random and an ordered start. The errors are corrected for autocorrelations. The agreement in the data for the two starts suggest the statistics of 200K iterations to be sufficient. As expected, the four different scales, T_{c4} , T_{c6} , T_{c8} , and T_{c12} lead to four different curves for the order parameter. One also sees the known feature of $\langle \bar{L} \rangle \to 0$ as $a_c \to 0$ even in the deconfined region. Figure 2 displays the behaviour of the bare free energy for just two scales, obtained by using the eq.(2.1). The stars are for the scale T_{c8} while the squares are for T_{c6} corresponding to the higher lattice spacing of the two. The figure reinforces the expectation of the effect of the divergent free energy, since the free energy increases with the decrease in the lattice cut-off a_c.

Any two different scales, a_{c1} and a_{c2} have their respective divergent contributions, $a_{c1}A(a_{c1})$ and $a_{c2}A(a_{c2})$. Multiplying eq.(2.1) by N_j, for $j = 1$ and 2 corresponding to the critical N_τ for the

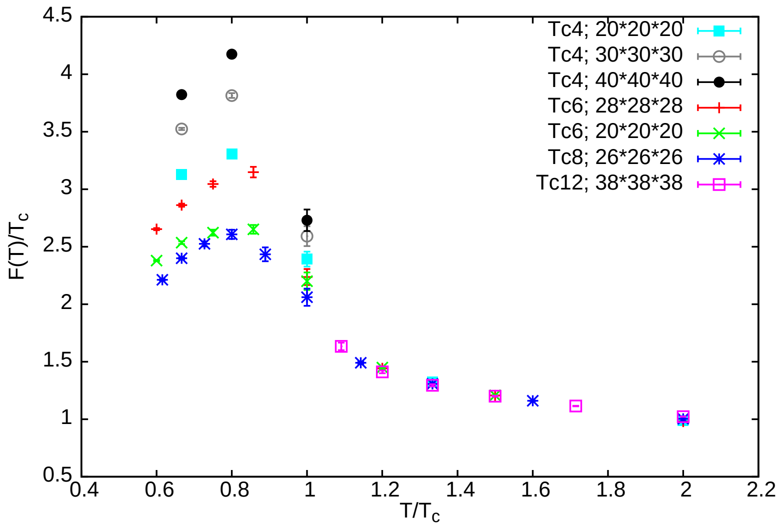

Figure 3: Heavy quark free energy F as a function of T/T_c with a constant shift, as explained in the text.

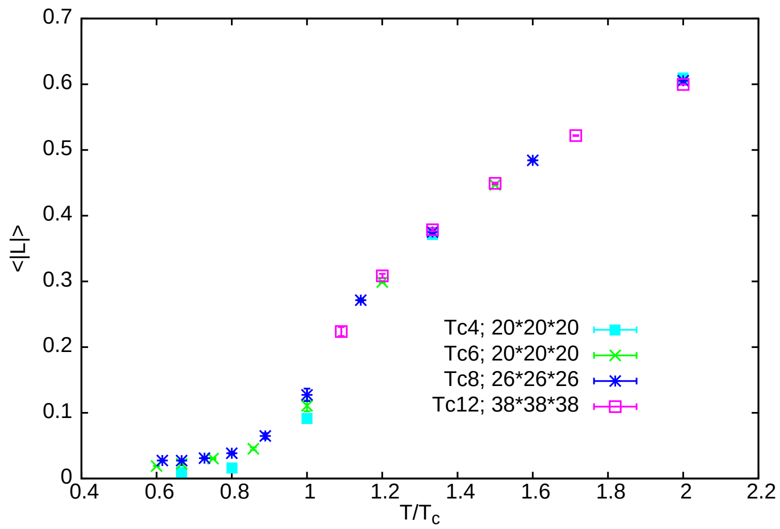

Figure 4: Renormalized Polyakov loop versus T/T_c using the shifted free energy of the upper figure, as explained in the text.

scale choices above, i.e, 6 and 8, one obtains

$$\frac{T}{T_c}\ln\langle|\bar{L}|\rangle = -\frac{F_b(T/T_c, a_{cj})}{T_c}\ , \tag{2.2}$$

where $F_b(T/T_c, a_{cj})/T_c = F(T/T_c, a_{cj})/T_c - A(a_{cj})/T_c$. Thus the free energies at the same temperatures but two different scales are related by a mere constant, $[A(a_{c1}) - A(a_{c2})]/T_c$. For the four scales considered here, this implies 3 such constants. Figure 3 shows the results for the free energy with three constant shifts in the free energy determined by demanding coincidence at the highest

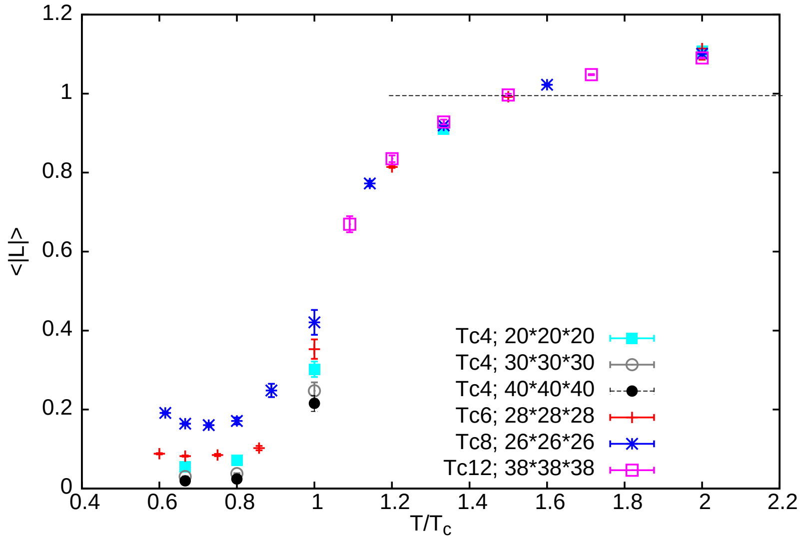

Figure 5: Renormalized Polyakov loop versus T/T_c after subtracting the would-be divergent contribution.

$T = 2T_c$. A universal curve for the free energy seems to result as a result for a wide range of $T > T_c$. The results for the low temperature phase are seen to be volume dependent, as expected. In the infinite volume limit, the free energy should increase to infinity in the confined phase whereas it should essentially remain constant in the deconfined phase. Such an expectation is indeed borne out by the results in the Figure 3. For the same physical volume, the free energy appears to be a-independent in the $T < T_c$ phase as well, as seen by comparing the crosses and the stars.

Finally, it should now be clear how one can obtain a universal curve for the order parameter from the universal free energy curve. The $\langle|\bar{L}|\rangle$ corresponding to scale β_{c2} should simply be multiplied by the factor $\exp(N_\tau[A(a_{c1})/T_c - A(a_{c2})/T_c])$ and then the date will lie on a universal curve. This is exhibited in Figure 4 for *all* the four scales. It is worth noting that the same universal order parameter results in *both* below and above T_c by fixing only three constants for the four scales exhibited. The entire low and high temperature region of the order parameter is uniquely fixed, and appears to be universal.

From the Figure 4, it appears as though the approach of $\langle|\bar{L}|\rangle$ to unity is slow and from below. It is, however, known since long [17] that perturbation theory predicts $L \to 1$ from *above* at very large T : $L = 1 + C_3 g^3 + \mathscr{O}(g^4)$, where $c_3(N_c) > 0$ is a constant. The solution to this apparent paradox can be traced to the usual fact that a renormalized quantity depends on the scale chosen to define the scheme for renormalization. In my case, the inclusion of a constant free energy $A(a_c)/T_c$ for the chosen scale a_c defines the choice. The details of the shape of the physical order parameter are therefore scale-dependent in the plasma phase but it is universal none the less once a choice is made. Moreover, any further change of scale leads to a computable change in the shape. Indeed, in order to mimic the perturbative renormalization scheme I estimated the point-divergent contribution. At the highest temperature $2T_c$, I fitted the results for the four scales to $-\ln\langle|\bar{L}_j|\rangle = F(2T_c)/2T_c + B \cdot N_{\tau j}/2$. Having thus determined the coefficient B of the would-be divergent contribution at the scale T_{c4}, I eliminated the B-dependent contribution at that scale. The renormalized $\bar{L}$ at the other three scales were related to it by the same shifts as before. Figure 5 displays the resultant $\langle|\bar{L}|\rangle$. It crosses unity at about 1.5 T_c. Since all the multiplying factors tend to unity at large T, the approach of this $\bar{L}$ to unity is from above at large T. Note also that large spatial volumes, aspect ratio of ~ 10, are needed for this $L \simeq 0$ in the low T phase.

3. Summary

In conclusion, I showed that the fixed scale approach leads to a natural definition of a physical, N_τ-independent, order parameter which is defined in both the confined and the deconfined phases. The definition itself does not depend on any lattice artifacts or the lattice size in the deconfined phase , and works very well for even coarse lattices ($a \leq 1/4T_c$). Moreover, it displays the expected behaviour in the confined phase as the physical volume is increased, suggesting that the so determined physical free energy of a single quark in the confined phase, F, goes to infinity in the infinite volume limit. Eliminating the point-divergent contribution leads to a high temperature behaviour consistent with perturbation theory. It is straightforward to generalize this idea to $SU(N_c)$ gauge theories and QCD as well as to sources in higher representations.

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