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Critical behaviour of the compact $3d\ U(1)$ gauge theory at finite temperature

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Critical properties of the compact three-dimensional $U(1)$ lattice gauge theory are explored at finite temperatures. The critical point of the deconfinement phase transition, critical indices and the string tension are studied numerically on lattices with temporal extension $N_t = 8$ and spatial extension ranging from $L = 32$ to $L = 256$. The critical indices, which govern the behaviour across the deconfinement phase transition, are generally expected to coincide with the critical indices of the two-dimensional XY model. It is found that the determination of the infinite volume critical point differs from the pseudo-critical coupling at $L = 32$, found earlier in the literature and implicitly assumed as the onset value of the deconfined phase. The critical index ν computed from the scaling of the pseudocritical couplings agrees well with the value $\nu = 1/2$ of the XY model. The computation of the index η brings to a value larger than expected. The possible reasons for such behaviour are discussed.

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1. Introduction

This article deals with the compact three-dimensional ($3d$) $U(1)$ lattice gauge theory (LGT), whose partition function can be written as

$$Z(\beta_t,\beta_s) = \int_0^{2\pi} \prod_{x\in\Lambda} \prod_{n=0}^{2} \frac{d\omega_n(x)}{2\pi} \ \exp S[\omega] \ , \tag{1.1}$$

where Λ is an $L^2 \times N_t$ lattice, S is the Wilson action, which reads

$$S[\omega] = \beta_s \sum_{p_s} \cos\omega(p_s) + \beta_t \sum_{p_t} \cos\omega(p_t) \tag{1.2}$$

and sums run over all space-like (p_s) and time-like (p_t) plaquettes. The plaquette angles $\omega(p)$ are defined in the standard way. The anisotropic couplings β_t and β_s are defined in Ref. [1]. To study the theory at finite temperature, periodic boundary conditions in the temporal direction are imposed on the gauge fields.

At zero temperature the theory is confining at all values of the bare coupling constant [2], while at finite temperature the theory undergoes a deconfinement phase transition. It is well known that the partition function of the $3d$ $U(1)$ LGT in the Villain formulation coincides with that of the $2d$ XY model in the leading order of the high-temperature expansion [3]. When combined with the universality conjecture by Svetitsky-Yaffe [4], this result leads to conclude that the deconfinement phase transition belongs to the universality class of the $2d$ XY model, which is known to have Berezinskii-Kosterlitz-Thouless (BKT) phase transition of infinite order [5, 6]. In particular, one might expect the critical behaviour of the Polyakov loop correlation function $\Gamma(R)$ to be governed by the following expressions

$$\Gamma(R) \asymp \frac{1}{R^{\eta(T)}} \ , \tag{1.3}$$

for $\beta \geq \beta_c$ and

$$\Gamma(R) \asymp \exp\left[-R/\xi(t)\right] \ , \tag{1.4}$$

for $\beta < \beta_c$, $t = \beta_c/\beta - 1$. Here, $R \gg 1$ is the distance between test charges, T is the temperature and $\xi \sim e^{bt^{-\nu}}$ is the correlation length. Such behaviour of ξ defines the so-called *essential scaling*. The critical indices $\eta(T)$ and ν are known from the renormalization-group (RG) analysis of the XY model: $\eta(T_c) = 1/4$ and $\nu = 1/2$, where T_c is the BKT critical point.

The direct numerical check of these predictions was performed on lattices $L^2 \times N_t$ with $L = 16, 32$ and $N_t = 4, 6, 8$ in Ref. [7]. Though the authors of Ref. [7] confirm the expected BKT nature of the phase transition, the reported critical index is almost three times that predicted for the XY model, $\eta(T_c) \approx 0.78$. More recent numerical simulations of Ref. [8] have been mostly concentrated on the study of the properties of the high-temperature phase. In these papers it was found that, for the isotropic lattice $\beta_s = \beta_t = \beta$ with $L = 32$ and $N_t = 8$, the pseudo-critical point is $\beta_{pc} = 2.30(2)$ for Ref. [7] and $\beta_c \approx 2.346(2)$ for Ref. [8]. Values of β above these values were taken implicitly as belonging to the deconfined phase.

In Ref. [1] we have studied the model on extremely anisotropic lattice with $\beta_s = 0$. By a simple analytical analysis we showed that in the limits of both small and large β_t such anisotropic model reduces to the $2d$ XY model with some effective couplings. Then we performed numerical simulations of the effective spin model for the Polyakov loop which can be exactly computed in the limit $\beta_s = 0$. We used lattices with $N_t = 1,4,8$ and with the spatial extent $L \in [64,256]$ and found that the index η is well compatible with the XY value. We may thus assume that, at least in the limit $\beta_s = 0$, the 3d $U(1)$ LGT does belong to the universality class of the XY model.

Here we consider the isotropic model on the lattice with $N_t = 8$. Our strategy is the following: we postulate that the scaling laws of the XY model are valid and use them to determine the critical indices of the gauge model. In doing so we have encountered certain surprises: (i) the infinite volume critical coupling turned out to be essentially higher than the values for the pseudo-critical couplings reported in Refs. [7, 8]. As a consequence, the values of β used in Ref. [8] to study the deconfinement phase lie well inside the confinement phase when the thermodynamic limit is considered; (ii) the index ν extracted from the scaling of the pseudo-critical couplings with L does agree well with the expected XY value $\nu = 1/2$, but the index η was found to be strikingly different from the XY value, namely $\eta \approx 0.50$. While the value $\eta \approx 0.78$ obtained in Ref. [7] could, in principle be attributed to rather small lattices used, $L = 32$, and to an incorrect location of the critical point, our result is almost insensitive to varying the spatial extent if L is large enough.

2. Numerical results

We simulated the system on lattices of the type $L^2 \times N_t$, with $N_t = 8$ fixed and L increasing towards the thermodynamic limit (for details, see Ref. [9]). In the adopted Monte Carlo algorithm a sweep consisted in a mixture of one Metropolis update and five microcanonical steps. Measurements were taken every 10 sweeps in order to reduce the autocorrelation and the typical statistics per run was about 100k. The error analysis was performed by the jackknife method over bins at different blocking levels.

The observable used as a probe of the two phases of the finite temperature 3d $U(1)$ LGT is the Polyakov loop, defined as

$$P(\vec{x}) = \prod_t U_0(\vec{x},t) \ , \tag{2.1}$$

where $U_0(\vec{x},t)$ is the temporal link attached at the spatial point $\vec{x}$. The effective theory for the Polyakov loop is two-dimensional and possesses global $U(1)$ symmetry. Since the global symmetry cannot be broken spontaneously in two dimensions owing to the Mermin-Wagner-Coleman theorem, the expectation value of the Polyakov loop vanishes in the thermodynamic limit. On a finite lattice $\langle \sum_{\vec{x}} P(\vec{x}) \rangle = 0$ due to $U(1)$ symmetry (if the boundary conditions used preserve the symmetry). This is confirmed by the numerical analysis on the periodic lattice: in the confined (small β) phase the values taken by the Polyakov loop in a typical Monte Carlo ensemble scatter around the origin of the complex plane forming a uniform cloud, whereas in the deconfined (high β) phase they distribute on a ring, the thermal average being equal to zero in both cases. What really feels the transition is then the absolute value of P, which has been chosen to be the order parameter in this work.

At finite volume the transition manifests through a peak in the magnetic susceptibility of the Polyakov loop, defined as

$$\chi_L = L^2(\langle |P|^2 \rangle - \langle |P| \rangle^2) \quad , \quad P = \frac{1}{L^2} \sum_x P(\vec{x}) \quad . \tag{2.2}$$

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The value of the coupling at which this happens is the pseudo-critical coupling, β_{pc}. By increasing the spatial volume, the position of the peak moves towards the (nonuniversal) infinite volume critical coupling, β_c. The value of β_{pc} for a given L is determined by interpolating the values of the susceptibility χ_L around the peak by a Lorentzian function. In Table 1 we summarize the resulting values of β_{pc} and the peak values of the susceptibility χ_L for the several volumes considered in this work (we included also the determination for $L = 32$, taken from the first paper in Ref. [8]).

Table 1: β_{pc} and peak value of the Polyakov loop susceptibility χ_L on the lattices $L^2 \times 8$.

L	β_{pc}	$\chi_{L,\max}$
32	2.346(2), Ref. [8]	
48	2.4238(67)	12.93(41)
64	2.4719(39)	20.09(66)
96	2.5648(96)	38.8(1.6)
128	2.6526(59)	60.1(3.5)
150	2.68(1)	92.6(8.0)
200	2.7336(69)	144(12)
256	2.7780(40)	220(20)

There are, in principle, two hypotheses to be tested in order to locate the infinite volume critical coupling β_c: first order and BKT transition. The hypothesis of first order transition is not incompatible with data for the peak susceptibility for $L \geq 128$. However, the corresponding scaling law for the pseudo-critical couplings,

$$\beta_{pc} = \beta_c + \frac{A}{L^2} \quad , \tag{2.3}$$

seems to be ruled out by our data (χ^2/d.o.f equal to 5.6 for $L \geq 96$, 3.7 for $L \geq 128$, 2.1 for $L \geq 96$).

Assuming the essential scaling of the BKT transition, *i.e.* $\xi \sim e^{bt^{-\nu}}$, the scaling law for β_{pc} becomes

$$\beta_{pc} = \beta_c + \frac{A}{(\ln L + B)^{\frac{1}{\nu}}} \quad . \tag{2.4}$$

A 4-parameter fit of the data for $\beta_{pc}(L)$ given in Table 1 with the law given in Eq. (2.4) leads to unstable values of the parameters. Instead, when the parameter ν is fixed at the XY value, $\nu = 1/2$, the fit is stable for lattices with size not smaller than L, leading to an estimated value of the infinite volume critical coupling, $\beta_c = 3.06(11)$ (see Ref. [9]).

By finite size scaling (FSS) analysis at β_c, we can extract other critical indices. An interesting one is the magnetic critical index, η, which enters the scaling law

$$\chi_L(\beta_c) \sim L^{2-\eta} \quad . \tag{2.5}$$

Actually in this law one should consider logarithmic corrections (see Refs. [10, 11] and references therein) and, indeed, recent works on the XY universality class generally include them. However, taking these corrections into account for extracting critical indices calls for very large lattices even in the XY model; for the theory under consideration to be computationally tractable, we have no choice but to neglect logarithmic corrections.

Setting the coupling β at the value of our best estimation for β_c, i.e. $\beta = 3.06$, we determined the susceptibilities $\chi_L(\beta_c)$ for several volumes. Then, following FSS, we fitted the results with the law $\chi_L(\beta_c) = AL^{2-\eta}$ and got

$$A = 0.0171(10), \quad \eta = 0.496(15) \quad (\chi^2/\text{d.o.f.} = 0.60) \quad . \tag{2.6}$$

This value for η is by far incompatible with the $2d$ XY value, $\eta_{XY} = 0.25$. The most extreme consequence of this finding is that the deconfinement transition in the $3d$ $U(1)$ LGT at finite temperature does not belong to the same universality class as $2d$ XY spin model. This would contradict the Svetitsky-Yaffe conjecture, raising a problem in the understanding of the deconfinement mechanism in gauge theories. We will further comment on this issue in the discussion section.

In such a situation, it becomes particularly useful to have another determination of the index η, by an independent approach. Following Ref. [1], we define an *effective* η index, through the 2-point correlator of Polyakov loops, according to

$$\eta_{\text{eff}}(R) \equiv \frac{\log[\Gamma(R)/\Gamma(R_0)]}{\log[R_0/R]} \quad , \tag{2.7}$$

with R_0 chosen equal to 10, as in Ref. [1]. This quantity is constructed in such a way that it exhibits a *plateau* in R if the correlator obeys the law (1.3), valid in the deconfined phase.

The analysis of the behaviour of $\eta_{\text{eff}}(R)$ has been repeated setting β at our estimated value for β_c, *i.e.* $\beta = 3.06$, and increasing the spatial extent of the lattice. It turns out (see Fig. 1) that a plateau develops at small distances when L increases and that the extension of this plateau gets larger with L, consistently with the fact that finite volume effects are becoming less important. The plateau value of η_{eff} can be estimated as $\eta_{\text{eff}}(R = 6)$ on the $256^2 \times 8$ lattice and is equal to 0.4782(25); it agrees with our previous determination of the index η.

3. Discussions

We have studied the critical behaviour of the $3d$ $U(1)$ LGT at finite temperature on isotropic lattice with the temporal extension $N_t = 8$. The pseudo-critical coupling was determined through the peak in the susceptibility of the Polyakov loop; the infinite-volume critical coupling has then been computed assuming the scaling behaviour of the form (2.4), the result being $\beta_c = 3.06(11)$. The deconfinement phase is the phase where $\beta \geq \beta_c$. A thorough investigation of the deconfinement phase was performed in Ref. [8]. However, all β-values used there are smaller than the infinite-volume critical coupling. When the thermodynamic limit is approached the critical coupling increases so that the numerical results of Ref. [8] would refer rather to the confinement phase of the infinite-volume theory.

We found also that the index η turns out to be $\eta \approx 0.496$. This value is essentially larger than expected and requires some discussion. The easiest explanation would be to state that the spatial lattice size used ($L \in [32-256]$) is still too small to exhibit the correct scaling behaviour, hence the wrong values for β_c and η follow. However, if one makes a plot of $\beta_{pc}(L)$ vs L, one can see, by looking at the trend of data, that it is unlikely that β_c is much larger than our estimate. In fact, our fits with the scaling law (2.4) show that β_c decreases when larger lattices are considered. Therefore,

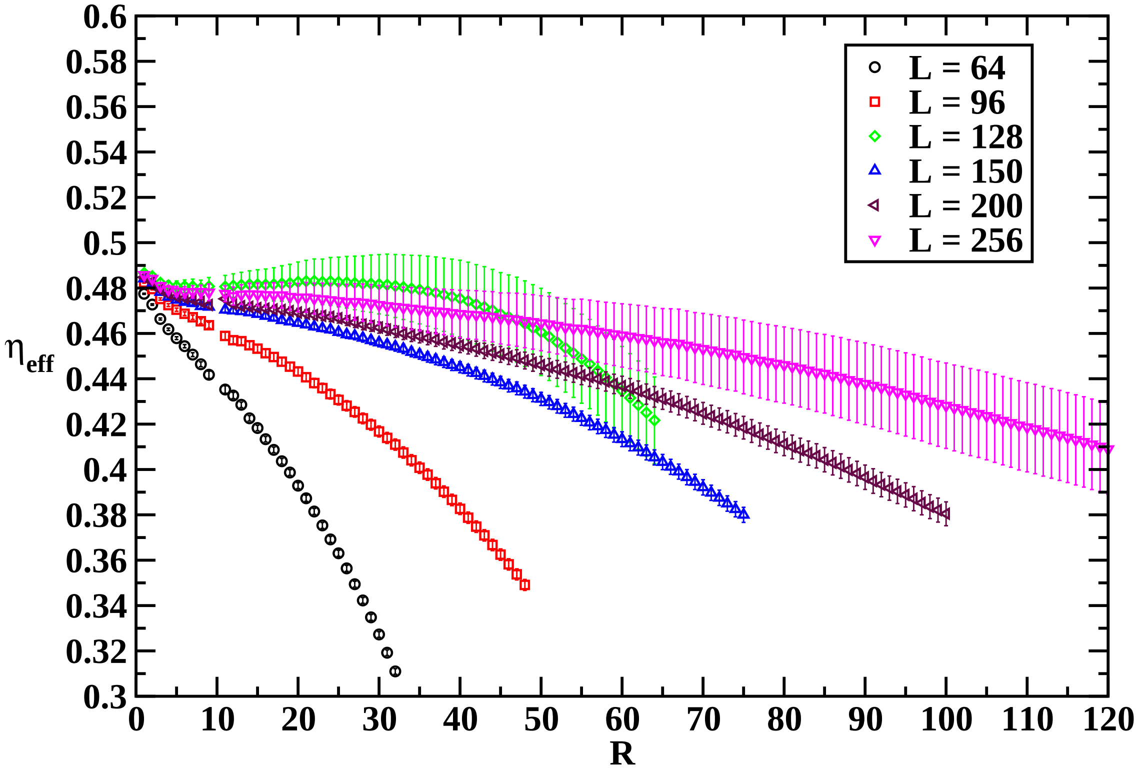

Figure 1: $\eta_{\rm eff}$ *vs* R at β_c on lattices with several values of L.

our result is most likely an overestimation. This implies that the true η is likely even larger than what we found.

Moreover, neglecting logarithmic corrections to the scaling law (2.5) cannot have such a strong impact to decrease η to half of the value we found.

Let us give a simple argument why the index η can be different from its XY value. Consider the anisotropic lattice, in the limit of large $\beta_s = \infty$. Here the spatial plaquettes are frozen to unity and the ground state is a state where all spatial fields are pure gauge, i.e. $U_n(x) = V_x V^*_{x+e_n}$, $n = 1,2$. Under the change of variables $U_0(x) \to V_x U_0(x) V^*_{x+e_0}$, in the leading order of the large-β_s expansion the partition function factorizes into the product of N_t independent $2d$ XY models. Since the Polyakov loop is the product of gauge fields in the temporal direction, the correlation function factorizes, too, and becomes a product of independent XY correlations, i.e.

$$\Gamma_{U(1)}(\beta_s = \infty, \beta_t) \; = \; [\Gamma_{XY}(\beta_t)]^{N_t} \; . \tag{3.1}$$

Hence, for asymptotically large $R \gg 1$, we get

$$\Gamma_{U(1)}(\beta_s = \infty, \beta_t \geq \beta_t^{cr}) \; \asymp \; \left[\frac{1}{R^{\eta_{XY}}} \right]^{N_t} \; . \tag{3.2}$$

This leads to a simple relation

$$\eta(\beta_s = \infty, \beta_t^{cr}) \; = \; N_t \; \eta_{XY} \; . \tag{3.3}$$

Some conclusions could now be drawn. The critical behaviour of the $3d$ $U(1)$ LGT in the limit $\beta_s \to \infty$ is also governed by the $2d$ XY model. Nevertheless, the effective index η appears to be N_t

times of its XY value. Now, for $\beta_s = 0$ we have $\eta(\beta_s = 0, \beta_t^{cr}) = \eta_{XY}$. This relation and formula (3.3) allow to conjecture that

$$\eta_{XY} \leq \eta(\beta_s, \beta_t^{cr}) \leq N_t\, \eta_{XY} . \tag{3.4}$$

$\beta_s = 0$ corresponds to the lower limit while $\beta_s = \infty$ corresponds to the upper limit. In general, η could interpolate between two limits with β_s. Whether this interpolation is monotonic or there exists critical value β_s^{cr}, such that $\eta(\beta_s \leq \beta_s^{cr}, \beta_t^{cr}) = \eta_{XY}$ and η changes monotonically above β_s^{cr}, cannot be answered with data we have and requires simulations on the anisotropic lattices. In the paper [12] a renormalization group study of $3d\ U(1)$ model at small β_s will be presented and computations of the leading correction to the large β_s behaviour will be given. The results of our computations support the scenario that the index η depends on the ratio β_s/β_t. Recently, we have obtained the results of simulations for $N_t = 2, 4$ performed by A. Bazavov [13]. His results also point in the direction of our scenario.

Finally, it is worth mentioning that the factorization in the large β_s limit does not affect the index ν. It follows from its definition (1.4) that in this limit $\nu = 1/2$ as in the XY model. We expect therefore that ν equals $1/2$ for all β_s and is thus universal.

In view of our results it might be worth to perform numerical simulations for small but nonvanishing β_s and for larger volumes.

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