PROCEEDINGS OF SCIENCE

Hadron properties at finite temperature and density with two-flavor Wilson fermions

Hideaki Iida*
Mathematical Physics Lab., RIKEN Nishina Center, Wako, Saitama 351-0198, Japan
E-mail: hiida@riken.jp

Yu Maezawa
Mathematical Physics Lab., RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

Koichi Yazaki
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan, and
Mathematical Physics Lab., RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

Meson properties at finite temperature and density are studied in lattice QCD simulations with two-flavor Wilson fermions. For this purpose, we investigate screening masses of mesons in pseudo-scalar (PS) and vector (V) channels. The simulations are performed on $16^3 \times 4$ lattice along the lines of constant physics at $m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$ and 0.80, where $m_{\rm PS}/m_{\rm V}|_{T=0}$ is a ratio of meson masses in PS and V channels at $T = 0$. A temperature range is $T/T_{\rm pc} = (0.8-4.0)$, where $T_{\rm pc}$ is the pseudo-critical temperature. We find that the temperature dependence of the screening masses normalized by temperature, M_0/T, shows notable structure around $T_{\rm pc}$, and approach 2π at high temperature in both channels, which is consistent with twice the thermal mass of a free quark in high temperature limit. The screening masses at low density are also investigated by using the Taylor expansion method with respect to the quark chemical potential. We find that the expansion coefficients in the leading order become positive in the temperature range, and thermal and density effect on the meson screening-masses becomes apparent in the quark-gluon plasma phase. The meson screening-masses are also compared with the gluon (Debye) screening masses at finite temperature and density.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010
June 14-19, 2010
Villasimius, Italy

*Speaker.

PoS(Lattice 2010)189

1. Introduction

Study of hadron properties at finite temperature and density is important to understand behavior of quarks and gluons in hot and/or dense QCD medium. In particular, mesons in the medium are expected to have abundant information about characteristic properties of QCD, such as the deconfinement phase transition at finite temperature and the partial chiral symmetry restoration at finite density [1]. In this article, we focus on screening masses of mesons in pseudo-scalar (PS) and vector (V) channels calculated from the spatial correlation functions, and present current results of our group in lattice QCD simulations with two flavors of the Wilson-type quark action. Using gauge configurations generated by WHOT-QCD Collaboration [2, 3], we calculate the meson correlation functions along the line of constant physics and extract temperature dependence of the meson screening-masses from long spatial-distance behavior of the correlators.

We find that the screening masses in both channels show notable behavior around pseudo-critical temperature $T_{\rm pc}$, where the transition occurs from the hadronic phase to the quark-gluon plasma phase. On the other hand, these approach $2\pi T$ at high temperature, which corresponds to twice the thermal mass of a non-interacting quark in high temperature limit. We also investigate the meson screening-masses at low density by using the Taylor expansion method with respect to the quark chemical potential. We find that the leading order expansion coefficients become positive in the temperature range, and thermal and density effect on the meson screening-masses becomes apparent in the quark-gluon plasma phase. We also compare the meson screening-masses with the gluon (Debye) screening-masses obtained from Polyakov-line correlation functions, and find characteristic difference in medium contributions between mesons and gluons.

The paper is organized as follows. In Sec. 2, we show the formalism to calculate the screening masses on the lattice. In Sec. 3, numerical results of the screening masses at finite temperature and zero density are discussed, and results of the screening masses at finite density in Sec. 4 using the Taylor expansion method. In Sec. 5, we compare the meson screening-masses with the gluon (Debye) screening masses. Section 6 is devoted to summarize the paper.

2. Meson screening masses on lattice

In order to extract the meson screening masses at finite temperature and density, we calculate an expectation value of the spatial correlation functions $G(x)$ of the mesons,

$$\langle G(x)\rangle \equiv \sum_{y,z,t} \langle M(x,y,z,t)M(0,0,0,0)^{\dagger}\rangle, \tag{2.1}$$

where $M(x,y,z,t) \equiv \bar{q}(x,y,z,t)\Gamma q(x,y,z,t)$ is the meson operator with Γ denoting the gamma matrix, i.e., $\Gamma = \gamma_5$ for a pseudo-scalar (PS) meson and $\Gamma = \gamma_\mu$ for a vector (V) meson. The correlator is summed over y,z,t, which means the zero-momentum projection for y and z directions and zero-energy projection for temporal direction.

In order to extract response to finite density, we apply the Taylor expansion method with respect to the quark chemical potential, $\mu \equiv (\mu_u + \mu_d)/2$, where μ_u (μ_d) is the chemical potential for the u (d) quark. The Taylor expansion enables us to investigate meson properties at $\mu \neq 0$ from the expectation values at $\mu = 0$. Following Ref. [4], we expand the expectation value of the operator $\mathscr{O}$ in powers of $\tilde{\mu} \equiv \mu/T$ as,

$$
\begin{aligned}
\langle \mathscr{O} \rangle_\mu &= \frac{\int \mathscr{D}U\ e^{-S_{\rm gluon}} (\det D(\mu))^2 \mathscr{O}(\mu)}{\int \mathscr{D}U\ e^{-S_{\rm gluon}} (\det D(\mu))^2} \\
&= \frac{\int \mathscr{D}U\ e^{-S_{\rm gluon}} (\Delta + \dot{\Delta}\tilde{\mu} + \frac{1}{2}\ddot{\Delta}\tilde{\mu}^2 + O(\tilde{\mu}^3))(\mathscr{O} + \dot{\mathscr{O}}\tilde{\mu} + \frac{1}{2}\ddot{\mathscr{O}}\tilde{\mu}^2 + O(\tilde{\mu}^3))}{\int \mathscr{D}U\ e^{-S_{\rm gluon}} (\Delta + \dot{\Delta}\tilde{\mu} + \frac{1}{2}\ddot{\Delta}\tilde{\mu}^2 + O(\tilde{\mu}^3))} \\
&= \frac{\langle (\mathscr{O} + \dot{\mathscr{O}}\tilde{\mu} + \frac{1}{2}\ddot{\mathscr{O}}\tilde{\mu}^2 + O(\tilde{\mu}^3))(1 + \frac{\dot{\Delta}}{\Delta}\tilde{\mu} + \frac{\ddot{\Delta}}{\Delta}\tilde{\mu}^2 + O(\tilde{\mu}^3)) \rangle}{1 + \langle \frac{\dot{\Delta}}{\Delta} \rangle \tilde{\mu} + \frac{1}{2}\langle \frac{\ddot{\Delta}}{\Delta} \rangle \tilde{\mu}^2 + O(\tilde{\mu}^3)} \\
&= \langle \mathscr{O} \rangle + \left(\langle \mathscr{O}\frac{\dot{\Delta}}{\Delta} \rangle + \langle \dot{\mathscr{O}} \rangle \right) \tilde{\mu} + \left(\langle \dot{\mathscr{O}}\frac{\dot{\Delta}}{\Delta} \rangle + \frac{1}{2}\langle \ddot{\mathscr{O}} \rangle + \frac{1}{2}\langle \mathscr{O}\frac{\ddot{\Delta}}{\Delta} \rangle - \frac{1}{2}\langle \mathscr{O} \rangle \langle \frac{\ddot{\Delta}}{\Delta} \rangle \right) \tilde{\mu}^2 + O(\tilde{\mu}^3), \quad (2.2)
\end{aligned}
$$

where $\Delta \equiv (\det D(\mu))^2$ with $D(\mu)$ denoting the Dirac operator, and the dots on operators denote the derivatives with respect to $\tilde{\mu}$. Note that the expectation values in the right-hand-side of Eq. (2.2) are calculated at $\mu = 0$. We take $\mathscr{O}$ to be the meson correlator $G \equiv {\rm tr}(D^{-1}_{x0}(\mu_u)\Gamma D^{-1}_{0x}(\mu_d)\Gamma^\dagger)$ where tr denotes the trace with respect to color, spinor, and flavor indices. Then the Taylor expansion of the meson correlator is given by,

$$
\langle G \rangle_\mu = \langle G \rangle_0 + \langle G \rangle_1 \tilde{\mu} + \langle G \rangle_2 \tilde{\mu}^2 + O(\tilde{\mu}^3), \quad (2.3)
$$

where the expansion coefficients become,

$$
\begin{aligned}
\langle G \rangle_0 &= \langle {\rm tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \rangle, \quad (2.4) \\
\langle G \rangle_1 &= 0, \quad (2.5) \\
\langle G \rangle_2 &= \langle G_{\rm opr} \rangle_2 + \langle G_{\rm det} \rangle_2, \quad (2.6) \\
\langle G_{\rm opr} \rangle_2 &\equiv \langle \dot{G}\frac{\dot{\Delta}}{\Delta} \rangle + \frac{1}{2}\langle \ddot{G} \rangle \\
&= 2\langle {\rm Retr}[(D^{-1}\dot{D}D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \rangle \\
&\quad - \langle {\rm Retr}[(D^{-1}\ddot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \rangle \\
&\quad - \langle {\rm Retr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1}\dot{D}D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \rangle \\
&\quad + 4\langle {\rm Imtr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \cdot {\rm ImTr}(D^{-1}\dot{D}) \rangle, \\
\langle G_{\rm det} \rangle_2 &\equiv \frac{1}{2}\langle G\frac{\ddot{\Delta}}{\Delta} \rangle - \frac{1}{2}\langle G \rangle \langle \frac{\ddot{\Delta}}{\Delta} \rangle \\
&= {\rm Re}\{ \langle {\rm tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger](2({\rm Tr}(D^{-1}\dot{D}))^2 - {\rm Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + {\rm Tr}(D^{-1}\ddot{D})) \rangle \\
&\quad - \langle {\rm tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger] \rangle \langle 2({\rm Tr}(D^{-1}\dot{D}))^2 - {\rm Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + {\rm Tr}(D^{-1}\ddot{D}) \rangle \},
\end{aligned}
$$

where Tr denotes the trace including space-time indices in addition to those for tr. We have divided the second derivatives into two parts: $\langle G_{\rm opr} \rangle_2$ and $\langle G_{\rm det} \rangle_2$, where the former includes the derivatives of the operator G, and the latter consists only of the derivatives of the quark determinant. Note that the meson correlator does not have the odd orders in the Taylor expansion since it is symmetric under $\mu \to -\mu$, namely the meson correlator is invariant under the charge conjugation. For the calculation of the trace with respect to spatial indices, we apply the random noise method with 100 sets of U(1) random numbers.

In order to study the screening effect, we fit the meson correlator by the following form,

$$\langle G(x)\rangle_{(\mu,T)} = A(\mu,T)\left[e^{-M(\mu,T)x} + e^{-M(\mu,T)(L-x)}\right], \tag{2.7}$$

where L is the spatial lattice size, and we assume that contributions of finite μ appear only in the coupling factor (A) and the meson screening mass (M). We also assume that $A(\mu,T)$ and $M(\mu,T)$ are also expressed as power series in $\tilde{\mu}$,

$$A(\mu,T) = A_0 + A_2\tilde{\mu}^2 + O(\tilde{\mu}^4), \tag{2.8}$$

$$M(\mu,T) = M_0 + M_2\tilde{\mu}^2 + O(\tilde{\mu}^4). \tag{2.9}$$

By comparing both sides of Eq. (2.8) at each order of $\tilde{\mu}$, we obtain,

$$\langle G(x)\rangle_0 = A_0\left(e^{-M_0x} + e^{-M_0(L-x)}\right), \tag{2.10}$$

$$\frac{\langle G(x)\rangle_2}{\langle G(x)\rangle_0} = \frac{A_2}{A_0} + M_2\left\{\left(x-\frac{L}{2}\right)\tanh\left[M_0\left(x-\frac{L}{2}\right)\right] - \frac{L}{2}\right\}. \tag{2.11}$$

We extract the screening masses by fitting the meson correlators for each temperature with the expressions Eqs. (2.10) and (2.11) at large distance.

3. Numerical simulations

Simulation setup is the following. We utilize the gauge configurations generated by WHOT-QCD Collaboration on $16^3 \times 4$ lattice with the renormalization-group improved Iwasaki gauge action and $N_f = 2$ clover-improved Wilson quark action [2, 3]. The simulations have been performed along the line of constant physics corresponding to the PS and V meson mass ratio, $m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$ and 0.80 at $T = 0$. The temperature range for $m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$ (0.80) is $T/T_{\rm pc}$ =0.82–4.0 (0.76–3.0), where $T_{\rm pc}$ is the pseudo-critical temperature for the transition from hadronic phase to quark-gluon plasma phase. The number of configurations we use is 100 for each temperature and quark mass.

3.1 Screening masses at finite temperature and zero density

Figure 1(a) shows the meson screening-masses normalized by temperature, M_0/T, in PS channel as a function of $T/T_{\rm pc}$. The circle (triangle) points correspond to the results at $m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$ (0.80). The same figure in V channel is shown in Fig. 1(b). We can see a concave structure around $T_{\rm pc}$, that is, when temperature increases, M_0/T decreases below $T_{\rm pc}$, whereas it increases above $T_{\rm pc}$. This implies that the screening masses M_0 stay constant below $T_{\rm pc}$, whereas they monotonically increase above $T_{\rm pc}$. Namely, thermal effect on the meson screening-masses becomes apparent in the quark-gluon plasma phase. At high temperature, M_0/T converges to a constant value of 2π, which implies that the meson becomes a weakly interacting pair of a quark and an anti quark, each carrying the thermal mass πT. We also find clear quark-mass dependence, i.e., magnitude of the screening masses with lighter quark mass ($m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$) becomes smaller than that with heavier quark mass ($m_{\rm PS}/m_{\rm V}|_{T=0} = 0.80$) in both channels, similarly to the ordinary meson mass measured by temporal correlation. This implies that the screening of the meson spatial correlation becomes weak when the quark mass becomes small.

The temperature dependence of the meson screening masses is consistent with that calculated in the staggered-type quark action [5].

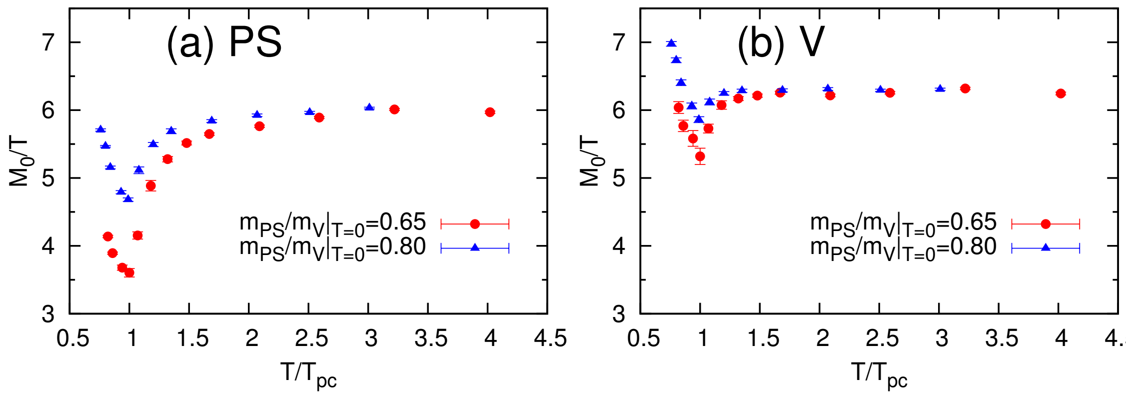

Figure 1: Meson screening masses M_0/T in the PS channel (a) and V channel (b) as a function of temperature.

3.2 Screening masses at finite density

In this section, we investigate properties of the screening masses at finite density by calculating the second response of the screening masses M_2/T to the quark chemical potential μ via the Taylor expansion method. Figure 2(a) and 2(b) show temperature dependence of M_2/T in PS and V channels, respectively, with $m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$ (circle points) and 0.80 (triangle points). We find that M_2/T is always positive in the temperature range we have explored, which implies that the screening masses increase in the leading order contribution of μ. M_2/T increases rapidly at $T_{\rm pc}$ which means that density effect on the meson screening-masses becomes significant in the quark-gluon plasma phase. We also found from the simulations that main contribution to the rapid increases of M_2/T comes from the $\langle G_{\rm opr}\rangle_2$ in Eq. (2.6) which consists of the derivatives of the operator. We can also see the quark-mass dependence in the PS channel; the magnitude of M_2/T with lighter quark mass ($m_{\rm PS}/m_{\rm V}|_{T=0} = 0.65$) is slightly smaller than that with heavier quark mass ($m_{\rm PS}/m_{\rm V}|_{T=0} = 0.80$). This is similar to the quark-mass dependence of M_0/T, but different from the results of M_2 calculated in the staggered-type quark action [4]. This difference should be further investigated in the simulations with smaller quark mass and larger spacial volume.

4. Comparison with gluon screening-masses

Let us compare the screening masses of the PS mesons with these of the gluons. The response to the quark chemical potential μ should be different for fermionic objects and gluonic ones, because the former (such as the meson screening-mass) have direct coupling with μ described by $\langle G_{\rm opr}\rangle_2$ while the latter (such as the gluon screening-mass) have only indirect coupling via the dynamical quark loops in the medium. Therefore, it is important to compare these two screening masses. The gluon screening-mass (so called Debye screening mass) has been studied from the Polyakov-line correlator based on the Taylor expansion method using the same gauge configurations [2, 3]. The Debye screening mass is also expressed as power series of $\tilde{\mu}$,

$$m_D(\mu) = m_{D,0} + m_{D,2}\tilde{\mu}^2 + O(\tilde{\mu}^4). \quad (4.1)$$

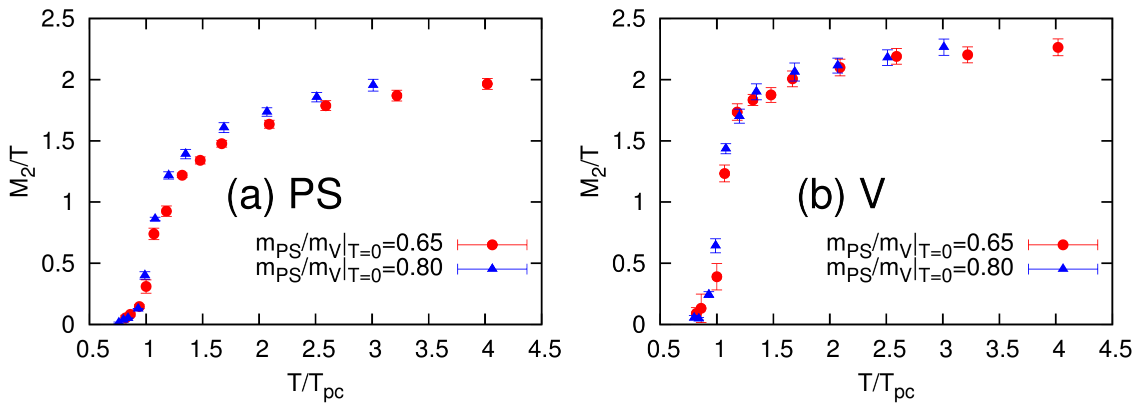

Figure 2: Second response of the screening masses M_2/T with respect to $\tilde{\mu}$ as a function of $T/T_{\rm pc}$ for PS channel (a) and V channel (b).

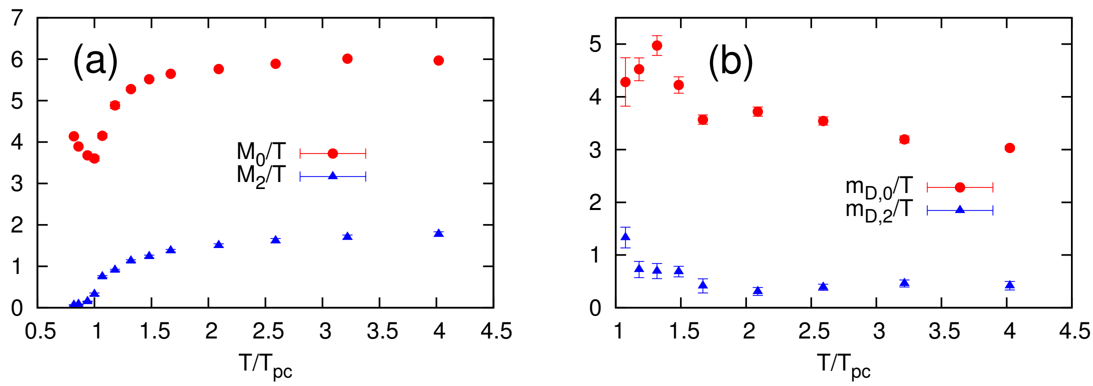

Figure 3: (a) meson screening-masses in PS channel, (b) gluon (Debye) screening-masses at $m_{PS}/m_V|_{T=0} = 0.65$ as a function of $T/T_{\rm pc}$.

Figure 3 shows the meson (a) and Debye (b) screening-masses at $m_{PS}/m_V|_{T=0} = 0.65$, where the circle (triangle) plots show M_0/T and $m_{D,0}/T$ (M_2/T and $m_{D,2}/T$), respectively. We find characteristic difference in the temperature dependence, i.e. the meson (Debye) screening-masses increase (decrease) when temperature increases above $T_{\rm pc}$. This is related to the fact that, at high temperature limit, M_0 goes to $2\pi T$ which is twice the thermal mass of a free quark, whereas $m_{D,0}$ is proportional to the running coupling and goes to zero according to the prediction of the thermal perturbation theory. We also find that the ratio of the meson screening-masses, M_2/M_0, is larger than that of the Debye screening-masses, $m_{D,2}/m_{D,0}$. This means that the response of the fermionic object to μ is larger than that of the gluonic object due to the difference in their ways of coupling with μ mentioned above.

5. Summary

We have studied the meson screening-masses in PS and V channel at finite temperature and density in lattice QCD simulations with two-flavor Wilson fermions. The simulations have been performed along the line of constant physics at $m_{PS}/m_V|_{T=0} = 0.65$ and 0.80 with the temperature range of $T/T_{\rm pc} = 0.82$–4.0 and 0.76–3.0, respectively. On the basis of the Taylor expansion method, we have calculated temperature dependence of the leading order term of the screening masses (M_0) and the second response to the quark chemical potential μ (M_2). We have found that M_0/T shows a concave structure around $T_{\rm pc}$ and goes to 2π at high temperature, which corresponds to twice the thermal mass of a free quark. From the quark-mass dependence, we have seen that the screening effect on the meson correlator becomes weak when the quark mass becomes small.

The second response M_2/T is always positive in the temperature range we have explored, which implies that the screening masses increase in the leading order contribution of μ. We have also found that thermal and density effect on the meson screening-masses becomes significant in the quark-gluon plasma phase.

We have also compared the meson screening-masses with the gluon (Debye) screening-masses, and found characteristic difference of the temperature dependence, i.e. the meson (Debye) screening-masses increase (decrease) when temperature increases. We have also seen that the response to μ is different between the meson and Debye screening-masses, which reflects the fact that the fermionic object has direct coupling with μ, whereas the gluonic object has only indirect coupling via the dynamical quark loops in the medium.

Acknowledgments

We thank WHOT-QCD Collaboration for providing us with the gauge configurations. The calculations were performed by using the RIKEN Integrated Cluster of Clusters (RICC) facility.

References

[1] K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma*, Cambridge University Press, Cambridge.

[2] **WHOT-QCD** Collaboration, Y. Maezawa *et. al.*, *Heavy-Quark Free Energy, Debye Mass, and Spatial String Tension at Finite Temperature in Two Flavor Lattice QCD with Wilson Quark Action*, *Phys. Rev.* **D75** (2007) 074501 [hep-lat/0702004].

[3] **WHOT-QCD** Collaboration, S. Ejiri *et. al.*, *Equation of State and Heavy-Quark Free Energy at Finite Temperature and Density in Two Flavor Lattice QCD with Wilson Quark Action*, *Phys. Rev.* **D82** (2010) 014508 [0909.2121].

[4] S. Choe *et. al.*, *Responses of hadrons to the chemical potential at finite temperature*, *Phys. Rev.* **D65** (2002) 054501; **QCD-TARO** Collaboration, I. Pushkina *et. al.*, *Properties of hadron screening masses at finite baryonic density*, *Phys. Lett.* **B609** (2005) 265–270 [hep-lat/0410017].

[5] M. Cheng *et. al.*, *Meson screening masses from lattice QCD with two light and the strange quark*, 1010.1216.