

Ghost-gluon coupling, power corrections and $\Lambda_{\overline{MS}}$ from twisted-mass lattice QCD at $N_f = 2$

B. Blossier, Ph. Boucaud, M. Gravina*, O. Pène

Laboratoire de Physique Théorique[†]

Université de Paris-Sud, Bâtiment 210, 91405 Orsay Cedex, France

E-mail:

Benoit.Blossier, Philippe.Boucaud, Mario.Gravina, Olivier.Pene@th.u-psud.fr

F. De soto

Dpto. Sistemas Físicos, Químicos y Naturales,

Universidad Pablo de Olavide, 41013 Sevilla, Spain.

E-mail: fcsotbor@upo.es

V. Morenas

Laboratoire de Physique Corpusculaire,

Université Blaise Pascal, CNRS/IN2P3, France

E-mail: morenas@in2p3.fr

J. Rodríguez-Quintero

Dpto. Física Aplicada, Fac. Ciencias Experimentales,

Universidad de Huelva, 21071 Huelva, Spain

E-mail: jose.rodriguez@dfaie.uhu.es



A non-perturbative calculation of the ghost-gluon running QCD coupling constant is performed using $N_f = 2$ twisted-mass dynamical fermions. The extraction of $\Lambda_{\overline{MS}}$ in the chiral limit reveals the presence of a non-perturbative OPE contribution that is assumed to be dominated by a dimension-two $\langle A^2 \rangle$ condensate. In this context a novel method for calibrating the lattice spacing in lattice simulations is presented.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010

June 14-19, 2010

Villasimius, Italy

*Speaker.

[†]Unité Mixte de Recherche 8627 du Centre National de la Recherche Scientifique

1. Introduction

$\Lambda_{\overline{MS}}$ is the scale of strong interactions. This parameter has to be taken from experiment and can be determined from the running of the QCD coupling constant. This latter can be calculated in a variety of non-perturbative ways on the lattice (see [1, 2, 3, 4, 5] and references therein). In the *quenched* case [6] the comparison between the perturbative and lattice determinations over a large momentum window revealed the presence of a dimension-two $\langle A^2 \rangle$ condensate, signaling that momenta considered in lattice simulation are in a non-perturbative region. Here we extend the strategy of [7] to the case of $N_f = 2$ twisted mass in the sea sector using configurations produced by the ETM Collaboration [7], in order to study the effect of the quark mass.

2. Lattice computation of the coupling in the Taylor scheme

Following [6], we calculate the strong coupling constant from the ghost-gluon vertex. Gluon and ghost propagators in the Landau gauge are defined as

$$\left(G^{(2)}\right)_{\mu\nu}^{ab}(p^2, \Lambda) = \frac{G(p^2, \Lambda)}{p^2} \delta_{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \quad \left(F^{(2)}\right)^{a,b}(p^2, \Lambda) = -\delta_{ab} \frac{F(p^2, \Lambda)}{p^2} \quad (2.1)$$

where $\Lambda = a^{-1}(\beta)$ is the regularisation cut-off. G and F are the gluon and ghost dressing functions which can be determined by a non-perturbative renormalization (MOM). In the Taylor scheme [8], where the incoming ghost momentum vanishes, the ghost-gluon vertex does not renormalize. This allows for a simple determination of the renormalized coupling constant in this scheme as

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \rightarrow \infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2, \Lambda^2) F^2(\mu^2, \Lambda^2); \quad (2.2)$$

in terms of only two-point gluon and ghost dressing function. Here g_0 is the bare strong coupling and μ the renormalization scale. This definition can be used in a lattice determination and is to be compared with a theoretical formula in order to extract Λ_{QCD} . As in the *quenched* case, using the four-loops expression for the coupling constant in the Taylor scheme [9, 10]

$$\begin{aligned} \alpha_T(\mu^2) = & \frac{4\pi}{\beta_0 t} \left(1 - \frac{\beta_1 \log(t)}{\beta_0^2 t} + \frac{\beta_1^2}{\beta_0^4} \frac{1}{t^2} \left(\left(\log(t) - \frac{1}{2} \right)^2 + \frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right) \\ & + \frac{1}{(\beta_0 t)^4} \left(\frac{\tilde{\beta}_3}{2\beta_0} + \frac{1}{2} \left(\frac{\beta_1}{\beta_0} \right)^3 \left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} \right) \log(t) - 1 \right) \right) \end{aligned} \quad (2.3)$$

where $t = \ln \frac{\mu^2}{\Lambda_T^2}$ and coefficients are

$$\begin{aligned} \beta_0 &= 11 - \frac{2}{3} N_f, \quad \beta_1 = \bar{\beta}_1 = 102 - \frac{38}{3} N_f \\ \tilde{\beta}_2 &= 3040.48 - 625.387 N_f + 19.3833 N_f^2 \\ \tilde{\beta}_3 &= 100541 - 24423.3 N_f + 1625.4 N_f^2 - 27.493 N_f^3, \end{aligned} \quad (2.4)$$

Extracting Λ_T from the lattice data at each μ^2 using this perturbative formula does not lead to a constant value. To understand the mismatch between lattice and perturbative determination, a

non-perturbative OPE correction to the perturbative formula is to be considered. This accounts for the minimal power correction associated to the presence of a dimension-two $\langle A^2 \rangle$ condensate:

$$\alpha_T(\mu^2) = \alpha_T^{\text{pert}}(\mu^2) \left(1 + \frac{9}{\mu^2} \frac{g_T^2(q_0^2) \langle A^2 \rangle_{R, q_0^2}}{4(N_C^2 - 1)} \right), \quad (2.5)$$

where $q_0^2 \gg \Lambda_{\text{QCD}}$ is some perturbative scale. This will cure the mismatch and lead to a good determination for Λ_T in the Taylor scheme, which eventually can be related to the value of the scale in the \overline{MS} scheme through

$$\frac{\Lambda_{\overline{MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507 - 40N_f}{792 - 48N_f}} = 0.541449. \quad (2.6)$$

3. Artefacts

We exploited data from ETMC configurations at maximal twist for a variety of run parameters (tab. 1) in order to study physical and systematic effects in our determinations. This has the main advantage of reducing the discretization artefacts to $\mathcal{O}(a^2)$, where a is the lattice spacing. Nevertheless, artefacts are expected to come at different levels. A first kind of artefacts that can be systematically cured [11, 12] are those due to the breaking of the rotational symmetry of the euclidean space-time when using a hypercubic lattice, where this symmetry is restricted to the discrete $H(4)$ isometry group. It is convenient to compute first the average of any dimensionless lattice quantity $Q(ap_\mu)$ over every orbit of the group $H(4)$. In general several orbits of $H(4)$ correspond to one value of p^2 . Defining the $H(4)$ invariants $p^{[n]} = \sum_{\mu=1}^4 p_\mu^n$, if the lattice spacing is small enough such that $\varepsilon = a^2 p^{[4]}/p^2 \ll 1$, the dimensionless lattice correlation function can be expanded in powers of ε :

$$Q(a^2 p^2, a^4 p^{[4]}, a^6 p^{[6]}, a^2 \Lambda_{\text{QCD}}^2) = Q(a^2 p^2, a^2 \Lambda_{\text{QCD}}^2) + \left. \frac{dQ}{d\varepsilon} \right|_{\varepsilon=0} a^2 \frac{p^{[4]}}{p^2} + \dots \quad (3.1)$$

$H(4)$ methods are based on the appearance of $\mathcal{O}(a^2)$ corrections driven by a $p^{[4]}$ term. The basic method is to fit from the whole set of orbits sharing the same p^2 the coefficient $dQ/d\varepsilon$ and get the extrapolated value of Q , free from $H(4)$ artefacts.

A second kind of artefact could come from dynamical quark masses. We will argue that this is a $\mathcal{O}(a^2 \mu_q^2)$ effect and therefore that it is a lattice artefact. We have calculated the $H(4)$ -free ghost and gluon dressing functions and combined in order to calculate the $H(4)$ -free lattice coupling through eq. (2.2). In Fig. 2 one can see the Taylor coupling after hypercubic extrapolation for different μ_q at fixed $\beta = 3.9$ and 4.05. Indeed, a dependence in μ_q is clearly seen. If it is an artefact the dependence should be in $a^2 \mu_q^2$. If it is an effect in the continuum it should be some unknown function of the physical mass μ_q . Trying an $\mathcal{O}(a^2 \mu_q^2)$ dependence, we write the expansion :

$$\widehat{\alpha}_T(a^2 p^2, a^2 \mu_q^2) = \alpha_T(p^2) + R_0(a^2 p^2) a^2 \mu_q^2, \quad R_0(a^2 p^2) \equiv \frac{\partial \widehat{\alpha}_T}{\partial (a^2 \mu_q^2)} \quad (3.2)$$

Provided that the first-order expansion in eq. (3.2) is reliable, a linear behaviour on $a^2 \mu_q^2$ has to be expected for the lattice estimates of $\widehat{\alpha}_T$ for any fixed lattice momentum computed from simulations at any given β and several values of μ_q . We explicitly check this linear behaviour to occur

for the results from our $\beta = 4.05$ and $\beta = 3.9$ simulations and show in Fig. 3 some plots of $\hat{\alpha}_T$ computed at $\beta = 4.05$ (where four different quark masses are available) for some representatives lattice momenta in terms of $a^2\mu_q^2$. In fig. 4, we plot $R_0(a^2p^2)$ as a function of ap computed for the four lattices simulations at $\beta = 4.05$ with different quark masses and for the three ones at $\beta = 3.9$. Indeed, it can be seen that a constant behaviour appears to be achieved for $p \geq p_{\min} \simeq 2.8$ GeV. We will not risk an interpretation of the data below $(ap)_{\min}$. The striking observation here is that above p_{\min} both lattice spacings exhibit a fairly constant $R_0(a^2p^2)$ and a good enough scaling between both β 's. The fact that R_0 with our present data goes to the same constant for both β 's, leads us to consider that the μ dependence of α is mainly a lattice artefact (else it should be a function of μ and not of $a\mu$).

The main result of this work is taking into account the effects due to dynamical quarks in a global analysis of the lattice determinations. This lead to a proper extrapolation to the continuum limit, which can be compared with continuous formula in order to extract $\Lambda_{\overline{\text{MS}}}$.

β	$a\mu_q$	Volume	Number of confs.
3.9	0.004	$24^3 \times 48$	120
	0.0064		20
	0.010		20
4.05	0.003	$32^3 \times 64$	20
	0.006		20
	0.008		20
	0.012		20
4.2	0.0065	$32^3 \times 64$	20

	This paper	String tension
$a(3.9)/a(4.05)$	1.224(23)	1.255(42)
$a(3.9)/a(4.2)$	1.510(32)	1.558(52)
$a(4.05)/a(4.2)$	1.233(25)	1.241(39)
$\Lambda_{\overline{\text{MS}}}a(3.9)$	0.134(7)	
$g^2\langle A^2 \rangle a^2(3.9)$	0.70(23)	

Figure 1: Left: Run parameters of the exploited data from ETMC collaboration. Right: Best-fit parameters for the ratios of lattice spacings, $\Lambda_{\overline{\text{MS}}}$ and the gluon condensate (for which $a(3.9)q_0 = 4.5$ is chosen). For the sake of comparison, we also quote the results from [13] that were obtained by computing the hadronic quantity, $r_0/a(\beta)$, and applying to it a chiral extrapolation.

4. $\Lambda_{\overline{\text{MS}}}$ and the gluon condensate

The running of α_T given by the combination of Green functions in eq. (2.2) and the extrapolation through eq. (3.2), provided that we are not far from the continuum limit and discretization errors are treated properly, depend only on the momentum (except, maybe, finite volume errors at low momenta). The supposed scaling of the Taylor coupling implies for the three curves plotted in fig. 4 to match to each other after the appropriate conversion of the momentum (in x-axis) from lattice to physical units, with the multiplication by the lattice spacing at each β . Thus, we can apply the ‘‘plateau’’-method described in [6] for the three β 's all at once by requiring the minimisation of the total χ^2 :

$$\chi^2 \left(a(\beta_0)\Lambda_{\overline{\text{MS}}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)} \right) = \sum_{j=0}^2 \sum_i \frac{\left(\Lambda_i(\beta_j) - \frac{a(\beta_j)}{a(\beta_0)} a(\beta_0)\Lambda_{\overline{\text{MS}}} \right)^2}{\delta^2(\Lambda_i)}; \quad (4.1)$$

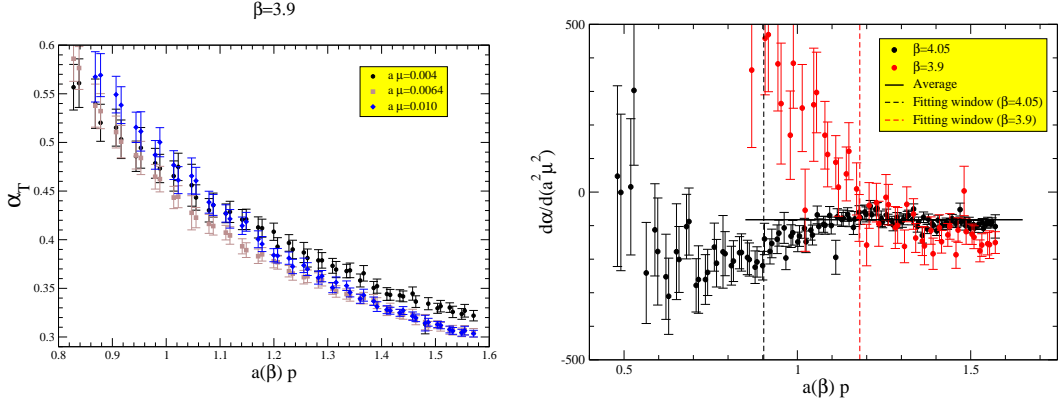


Figure 2: Left: The Taylor couplings estimates, after $H(4)$ -extrapolation, at $\beta = 3.9$ for $\mu_q = 0.004, 0.0064, 0.010$. Right: The slopes for the mass squared extrapolation in terms of ap computed for the four lattices simulations at $\beta = 4.05$ ($32^3 \times 64$) with $a\mu_q = 0.003, 0.006, 0.008, 0.012$ and for the three ones at $\beta = 3.9$ ($24^3 \times 48$) with $a\mu_q = 0.004, 0.0064, 0.010$.

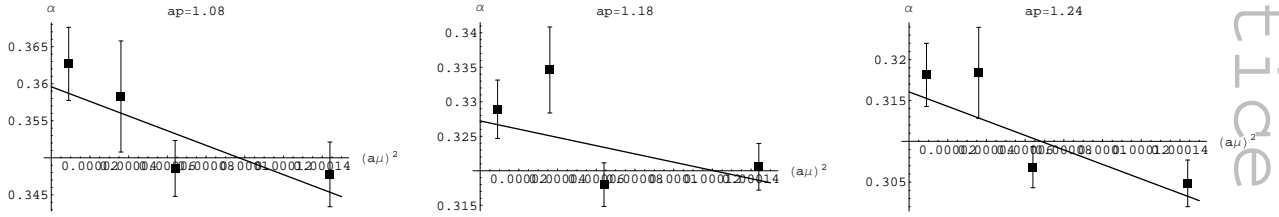


Figure 3: We plot the values of the Taylor coupling at $\beta = 4.05$, computed for some representative values of the lattice momentum, $a(4.05)p = 1.08, 1.18, 1.24, 1.36, 1.45, 1.52$, in terms of $a^2(4.05)\mu_q^2$ and show the suggested linear extrapolation at $a^2\mu_q^2 = 0$.

where the sum over j covers the sets of coupling estimates for the three β 's ($\beta_0 = 3.9, \beta_1 = 4.05, \beta_2 = 4.2$), the index i runs to cover the fitting window of momenta to be contained in a region in which the slope $R_0 \sim -90$ was found to be constant. $\Lambda_i(\beta_j)$ is obtained for any β_j by requiring the best-fit to a constant; c results from the best-fit: it is the Wilson coefficient of the gluon condensate in eq. (2.5), where the leading logarithm correction is now taken into account, where $a(\beta_0)q_0 = 4.5$ (this means $q_0 \approx 10$ GeV) was chosen. The function χ^2 is minimised over the functional space defined by the four parameters that are explicitly put in arguments for eq. (4.1)'s l.h.s.: $a(\beta_0)\Lambda_{\overline{MS}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)}$. Thus we obtain all at once $\Lambda_{\overline{MS}}$ and the gluon condensate, in units of the lattice spacing for $\beta_0 = 3.9$, and the ratios of lattice spacings for our three simulations after the extrapolation to the limit $\mu_q \rightarrow 0$ (see tab. 1). The errors are calculated again by jackknife analysis. The ratios of lattice spacings can be applied to express the momenta for all the three sets of coupling estimates plotted in fig. 4 (left) in units of the lattice spacing at $\beta = 3.9$. Thus they indeed match each other and fit pretty well to the analytical prediction with the best-fit parameters for $\Lambda_{\overline{MS}}$ and the gluon condensate, in units of $1/a(3.9)$ (see tab. 1), as can be seen in the plot of fig. 4. A detailed

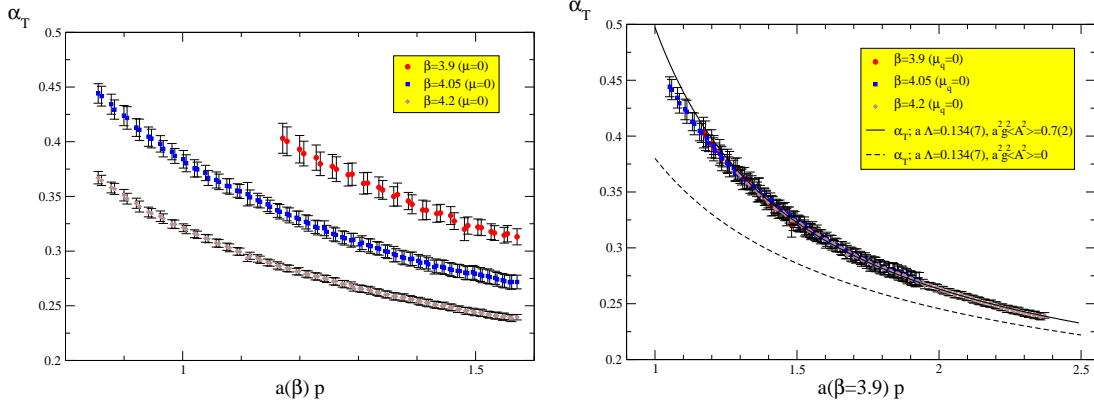


Figure 4: Left: The Taylor coupling, free of $H(4)$ and mass-quarks artefacts, for the three $\beta = 3.9, 4.05, 4.2$ and plotted in terms of the lattice momentum $a(\beta)p$. Right: The scaling of the Taylor coupling computed by for the three $\beta = 3.9, 4.05, 4.2$ is shown. The lattice momentum, $a(\beta)p$ in the x-axis, is converted to a physical momentum in units (the same for the three β 's) of $a(3.9)^{-1}$.

discussion about systematics can be found in [14] indicating that main sources of errors are under control. Assuming the value $a(3.9) = 0.0801(14)$ fm [13], we quote our result as

$$\Lambda_{\overline{\text{MS}}} = (330 \pm 23) \times \frac{0.0801 \text{ fm}}{a(3.9)} \text{ MeV}, \quad g^2(q_0^2)\langle A^2 \rangle_{q_0} = (2.4 \pm 0.8) \times \left(\frac{0.0801 \text{ fm}}{a(3.9)} \right)^2 \text{ GeV}^2.$$

5. Conclusions and outlooks

We computed the renormalized strong coupling constant analyzing a variety of $N_f = 2$ gauge configurations generated in the ETM Collaboration. We performed an elaborated treatment of the lattice artefacts and a precise estimate of the couplings at the infinite cut-off limit. The coupling estimates for lattices at different β 's were seen to match pretty well, as should happen if the cut-off limit is properly taken, when plotted in terms of the renormalization momenta converted to the same units by applying the appropriate lattice spacings ratios. These ratios could be either taken from independent computations or obtained by requiring the best matching with pretty compatible results. Thus, once we are left with the estimates of the coupling constant extrapolated at vanishing dynamical mass μ_q , for every value of the renormalization momentum, μ , they were converted via a fit with a four loops formula into the value of $\Lambda_{\overline{\text{MS}}}$. As in the *quenched* case a condensate $\langle A^2 \rangle$ is needed in order to get a constant $\Lambda_{\overline{\text{MS}}}$. As an outlook, we want to apply the same analysis to the case of lattice QCD with $N_f = 2 + 1 + 1$ and $N + f = 4$ dynamical flavors. This will lead to give a reliable lattice prediction for the coupling constant, say at M_Z , to be compared with available experimental determinations.

Acknowledgements

We thank the IN2P3 Computing Center (Lyon) and the apeNEXT computing laboratory (Rome) where part of our simulations have been done. J. R-Q is indebted to the Spanish MICINN for the support by the research project FPA2009-10773 and to ‘‘Junta de Andalucia’’ by P07FQM02962.

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