

$\mathcal{O}(a_L^2)$ Green's functions of the fermion propagator and of local and extended fermion bilinear operators, using Symanzik and SLiNC actions

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In this work we compute the 1-loop 2-point perturbative bare Green's functions of the fermion propagator and of local and extended fermion bilinear operators on the lattice. **The calculation is carried out up to $\mathcal{O}(a_L^2)$** , where a_L is the lattice spacing. We employ the Symanzik improved gauge actions and Stout Link Clover (SLiNC) fermions. Our results have been obtained for various choices of values for the Symanzik coefficients, c_i . The clover coefficient c_{SW} , the gauge parameter α , the SLiNC parameter ω , the Symanzik coefficients c_i , the fermion masses m_i and the number of colors N are kept as free parameters. The Wilson parameter, r , is set to 1.

Knowledge of these Green's functions allows us to determine renormalization functions for the quark field and each of the fermion bilinear operators.

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1. Introduction

- In this work we compute the one-loop 2-point bare Green's functions (amputated, 1PI) of:

- ▷ The fermion self-energy $\bar{\psi}x\psi$: $\Sigma_{\psi}^L(q, a_L, m)$
- ▷ Local fermion bilinear operators $\mathcal{O} = \bar{\psi}\lambda\Gamma\psi$: $\Sigma_{\Gamma}^L(q, a_L, m)$
- ▷ Extended bilinear operators $\mathcal{O} = \bar{\psi}\lambda\Gamma\overleftrightarrow{D}_{\rho}\psi$: $\Sigma_{\Gamma D_{\rho}}^L(q, a_L, m)$

where q is the external momentum, λ is the flavor matrix, $\Gamma = \hat{1}, \gamma_5, \gamma_{\mu}, \gamma_5\gamma_{\mu}, \gamma_5\sigma_{\mu\nu}$ and $\overleftrightarrow{D} = \frac{1}{2}(\overrightarrow{D} - \overleftarrow{D})$. We have already performed this calculation up to two loops to $\mathcal{O}(a_L^0)$, using the wilson gluon action and the clover fermion action [1].

- The gluon and fermion actions are, in standard notation (see e.g. Ref[2]):

$$S_G = \frac{2}{g^2} \left[c_0 \sum_{\text{plaq}} \text{Re Tr}(1 - U_{\text{plaq}}) + c_1 \sum_{\text{rect}} \text{Re Tr}(1 - U_{\text{rect}}) \right. \\ \left. + c_2 \sum_{\text{chair}} \text{Re Tr}(1 - U_{\text{chair}}) + c_3 \sum_{\text{paral}} \text{Re Tr}(1 - U_{\text{paral}}) \right]$$

$$S_F = \sum_x \left\{ \frac{1}{a_L} (4r + a_L m) \bar{\psi}(x)\psi(x) + a_L \frac{i}{4} c_{\text{SW}} \sum_{\mu, \nu} \bar{\psi}(x) \sigma_{\mu\nu} \hat{F}_{\mu\nu}(x) \psi(x) \right. \\ \left. - \frac{1}{2a_L} \sum_{\mu} \left[\bar{\psi}(x) \tilde{U}_{\mu}(x) (r - \gamma_{\mu}) \psi(x + a_L \mu) + \bar{\psi}(x + a_L \mu) \tilde{U}_{\mu}(x)^{\dagger} (r + \gamma_{\mu}) \psi(x) \right] \right\},$$

where $\sigma_{\mu\nu} = (i/2)[\gamma_{\mu}, \gamma_{\nu}]$. The SLiNC action employs stout links $\tilde{U}_{\mu}(x) = e^{iQ_{\mu}(x)} U_{\mu}(x)$ for the non-clover part of the fermion action, where:

$$Q_{\mu}(x) = \frac{\omega}{2i} \left[V_{\mu} U_{\mu}^{\dagger}(x) - U_{\mu}(x) V_{\mu}^{\dagger}(x) - \frac{1}{3} \text{Tr} \{ V_{\mu} U_{\mu}^{\dagger}(x) - U_{\mu}(x) V_{\mu}^{\dagger}(x) \} \right].$$

$V_{\mu}(x)$ denotes the sum over all staples associated with the link.

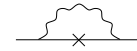
2. Feynman Diagrams

A total of 7 one-loop Feynman diagrams contribute to the present calculation. In the figures that follow, a wavy (solid) line represents a gluon (fermion) field and a cross denotes the operator insertion.

- Feynman diagrams contributing to the fermion self energy:



- Feynman diagram contributing to local fermion bilinears:



- Feynman diagrams contributing to extended fermion bilinears:



The evaluation and algebraic manipulation of Feynman diagrams, leading to a code for numerical loop integration, is performed automatically using our software for Lattice Perturbation Theory, written in Mathematica.

3. Computation

• The most laborious aspect of the procedure is the extraction of the dependence on $a_L q$, $a_L m$. This is a delicate task even at one-loop level, since we are interested in $\mathcal{O}(a_L^2)$ improvement; for this purpose, we cast algebraic expressions (typically involving thousands of summands) into terms which can be naively Taylor expanded in a_L to the required order, plus a smaller set of terms containing superficial divergences. The latter can be evaluated via analytical continuation to $D > 4$ or even $D > 6$ dimensions, and splitting each expression into a UV-finite part (which can thus be calculated in the continuum), and a part which is polynomial in a_L . A list of the divergent integrals appearing in this calculation can be found in Ref.[3].

- Dealing with “strong” IR divergent terms

A typical example:
$$I(a_L, q) = \int_{-\pi}^{+\pi} \frac{d^4 p}{(2\pi)^4} \frac{1}{\hat{p}^2 \widehat{p + a_L q}^2}, \text{ needed up to } \mathcal{O}(a_L^2).$$

We can express the integrand in the following form: $f(p, q) = [f(p, q) - f_1(p, q)] + f_1(p, q)$, where:

$$f_1(p, q) = \frac{1}{p^2 (p + a_L q)^2} + \frac{1}{12} \left[\frac{(p + a_L q)^4}{p^2 ((p + a_L q)^2)^2} + \frac{p^4}{(p^2)^2 (p + a_L q)^2} \right]$$

By construction, the quantity $f(p, q) - f_1(p, q)$ does not contain IR divergences, up to the desired order \implies We can evaluate it by a naive Taylor expansion in a_L ; the expansion coefficients will be (a_L, m, q) -independent lattice integrals, which are evaluated numerically. For the expression $f_1(p, q)$ we split the integration region into a hypersphere of radius μ and the hypercube minus the hypersphere:

$$\int_{-\pi}^{+\pi} = \int_{|p| \leq \mu} + \left(\int_{-\pi}^{+\pi} - \int_{|p| \leq \mu} \right)$$

Outside the hypersphere no IR divergences exist \implies As before, a naive Taylor expansion in a_L is performed; the expansion coefficients are (a_L, m, q) -independent lattice integrals. The integration within the hypersphere, since it contains only continuum expressions, can be performed in closed form, and leads to nontrivial dependence on a_L , m and q . The result will be μ -independent.

4. Propagator Results

- Below, we present our results for the fermion self energy, up to $\mathcal{O}(a_L^2)$. For the sake of simplicity, we only give results for the Tree-level Symanzik action ($c_0 = 5/3$, $c_1 = -1/12$ and $c_2 = c_3 = 0$):

$$\begin{aligned} \Sigma_{\psi}^L(q, a_L, m) = & i \not{q} + m + g^2 \frac{N^2 - 1}{N} \left[\frac{1}{a_L} \left(-0.1280549051(3) + 1.4422876785(4) \omega \right. \right. \\ & \left. \left. - 5.337085154(1) \omega^2 + \left(0.0378314931(1) - 0.118106866(1) \omega + 0.0147633581(1) \omega^2 \right) c_{\text{sw}} \right) \right. \\ & \left. + \frac{1}{32\pi^2} \left[\log \left(1 + \frac{q^2}{m^2} \right) \left(4 \frac{m^3}{q^2} - \alpha \frac{m^3}{q^2} - i \frac{m^4 \not{q}}{q^4} \right) \right. \right. \\ & \left. \left. + \log(a_L^2 (m^2 + q^2)) (4m + i \not{q} - \alpha m - i \alpha \not{q}) + i \frac{m^2 \not{q}}{q^2} (1 - \alpha) \right] \right] \end{aligned}$$

$$\begin{aligned}
& - 0.020186563(1)m + 0.01833916458(9)m\alpha + 0.0260625405(7)iq + 0.0151728776(3)iq\alpha \\
& - \left[0.0389466188(1)m + 0.4830619180(7)iq \right] \omega \\
& + \left[0.1900400001(5)m + 1.71416609309(3)iq \right] \omega^2 \\
& - c_{\text{SW}} \left(0.02802387855(7)m + 0.00638141407(3)iq \right) \\
& - \left[0.07249134896(3)m + 0.01479744282(4)iq \right] \omega \\
& + c_{\text{SW}}^2 \left(0.00039645466(2)m - 0.00393317032(3)iq \right) \Big] + \mathcal{O}(a_L, a_L^2)
\end{aligned}$$

- $\mathcal{O}(a_L, a_L^2)$ propagator results ($\alpha = 1, c_{\text{SW}} = 1, \omega = 0.1$):

$$\begin{aligned}
\Sigma'_{\psi}{}^L(q, a_L, m) &= \frac{g^2}{16\pi^2} \frac{N^2 - 1}{N} \left[a_L \left(\log \left(1 + \frac{q^2}{m^2} \right) \left(-\frac{9}{4} \frac{m^4}{q^2} - \frac{3}{2} im^3 \frac{q}{q^2} \right) \right. \right. \\
& - \log(a_L^2(m^2 + q^2)) \left(\frac{3}{4} m^2 + \frac{9}{4} \frac{m^4}{q^2} \right) + 1.6305381(2)m^2 \\
& + 0.1951559(2)q^2 + 2.3381261(2)imq \Big) \\
& + a_L^2 \left(\log(a_L^2(m^2 + q^2)) \left[-\frac{101}{120} m^3 + \frac{1}{q^2} \left(\frac{157}{148} m^5 + \frac{347}{120} im^4 q \right) \right. \right. \\
& + \frac{1}{(q^2)^2} \left(\sum_{\mu} q_{\mu}^4 \left[-\frac{1}{12} im^2 q + \frac{5}{36} m^3 \right] - \frac{1}{4} im^4 q^3 - \frac{71}{120} im^6 q + \frac{1}{24} m^7 \right) \\
& + \frac{1}{(q^2)^3} \left(\sum_{\mu} q_{\mu}^4 \left[-\frac{1}{24} im^4 q - \frac{1}{12} im^5 \right] - \frac{5}{36} im^6 q^3 + \frac{5}{48} im^8 q + \frac{3}{16} m^9 \right) \\
& + \frac{1}{(q^2)^4} \left(\sum_{\mu} q_{\mu}^4 \left[\frac{3}{8} im^6 q - \frac{1}{12} m^7 \right] - \frac{5}{24} im^8 q^3 + \frac{7}{80} im^{10} q \right) \\
& + \frac{1}{(q^2)^5} \left(-\frac{3}{8} im^9 \sum_{\mu} q_{\mu}^4 + \frac{7}{60} im^{10} q^3 \right) - \frac{7}{24} im^{10} \frac{q}{(q^2)^6} \sum_{\mu} q_{\mu}^4 \Big] \\
& + \log(a_L^2(m^2 + q^2)) \left[-\frac{5}{8} m q^2 + \frac{157}{120} im^2 q - \frac{143}{720} iq^2 q + \frac{121}{120} m^3 + \frac{157}{360} iq^3 \right] \\
& + \frac{1}{q^2} \left(\sum_{\mu} q_{\mu}^4 \left[\frac{71}{288} m - \frac{1}{240} iq \right] + \frac{67}{240} im^2 q^3 + \frac{251}{288} im^4 q + \frac{5}{96} m^5 \right) \\
& + \frac{1}{(q^2)^2} \left(\sum_{\mu} q_{\mu}^4 \left[\frac{5}{32} im^2 q + \frac{1}{6} m^3 \right] - \frac{1}{240} im^4 q^3 - \frac{29}{480} im^6 q - \frac{3}{16} m^7 \right) \\
& + \frac{1}{(q^2)^3} \left(\sum_{\mu} q_{\mu}^4 \left[-\frac{5}{18} im^4 q - \frac{5}{48} m^5 \right] + \frac{4}{15} im^6 q^3 - \frac{7}{80} im^8 q \right) \\
& + \frac{1}{(q^2)^4} \left(\sum_{\mu} q_{\mu}^4 \left[\frac{7}{48} im^6 q + \frac{3}{8} m^7 \right] 2 - \frac{7}{60} im^8 q^3 \right) + \frac{7}{24} im^8 \frac{q}{(q^2)^5} \sum_{\mu} q_{\mu}^4 + 0.8081488(5)m q^2 \\
& - 1.8549207(2)im^2 q + 0.4012352(4)iq^2 q - 2.0536382(2)m^3 - 0.9750041(2)iq^3 \Big) \Big]
\end{aligned}$$

5. Fermion Bilinear Results

• In what follows, we present the bare Green's functions up to $\mathcal{O}(a_L^2)$, for the Tree-level Symanzik action, in the case of the scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) fermion bilinear operators. The results for the tensor (T) operator have not been presented in this proceeding, but are available from the authors upon request. For these results we have set all fermion masses to zero, but we also have computed the corresponding expressions for generic masses $m_i \neq m_j$ for each flavor.

$$\begin{aligned} \Sigma_S^L(q, a_L, m) = & 1 + \frac{g^2}{16\pi^2} \frac{N^2 - 1}{N} \left[\log(a_L^2 q^2) \left(\frac{\alpha}{2} - 2 \right) + 3.1877338(2) - 2.8960046(1) \alpha \right. \\ & + 6.15020431(1) \omega - 30.009913380(5) \omega^2 + \left(4.42535334(4) - 11.44737523(2) \omega \right) c_{\text{SW}} \\ & - 0.062605579(4) c_{\text{SW}}^2 \\ & + i \not{q} a_L \left(\log(a_L^2 q^2) \left(\frac{5}{4} - \frac{1}{2} \alpha \right) - 1.798179789(2) + 1.967879641 \alpha - 7.33824069(5) \omega \right. \\ & + 26.045917929(7) \omega^2 + \left(\frac{3}{4} \log(a_L^2 q^2) - 1.881768497 + 4.223375220(7) \omega \right) c_{\text{SW}} \\ & \left. - 0.5750299734(4) c_{\text{SW}}^2 \right) \\ & + q^2 a_L^2 \left(\log(a_L^2 q^2) \left(\frac{1}{4} - \frac{3}{8} \alpha \right) + 0.040566522 + 1.136796962 \alpha + \frac{\sum_{\mu} q_{\mu}^4}{(q^2)^2} \left(\frac{3}{16} + \frac{1}{16} \alpha \right) \right. \\ & - 5.72421009(4) \omega + 13.684251761(4) \omega^2 \\ & \left. + \left(\frac{3}{4} \log(a_L^2 q^2) - 1.92638916 + 2.64878107(3) \omega \right) c_{\text{SW}} + 0.098230963(7) c_{\text{SW}}^2 \right) \Big] \end{aligned}$$

$$\begin{aligned} \Sigma_P^L(q, a_L, m) = & \gamma_5 + \gamma_5 g^2 \frac{N^2 - 1}{N} \frac{1}{16\pi^2} \left[\log(a_L^2 q^2) \left(\frac{\alpha}{2} - 2 \right) + 7.251046195(3) \right. \\ & - 2.8960046(1) \alpha - 24.31708203(1) \omega + 54.24328996(6) \omega^2 + 1.493527516(9) c_{\text{SW}}^2 \\ & + q^2 a_L^2 \left(-\frac{\alpha}{8} \log(a_L^2 q^2) - 0.065851587 + 0.419052855 \alpha + \frac{\sum_{\mu} q_{\mu}^4}{(q^2)^2} \left(\frac{3}{16} + \frac{\alpha}{16} \right) \right. \\ & \left. - 3.25296064(4) \omega + 14.626567330(5) \omega^2 - 0.137781205(8) c_{\text{SW}}^2 \right) \Big] \end{aligned}$$

$$\begin{aligned}
\Sigma_V^L(q, a_L, m) = & \gamma_\mu + g^2 \frac{N^2 - 1}{N} \frac{1}{16\pi^2} \left[\gamma_\mu \left[\log(a_L^2 q^2) \frac{1}{2} (\alpha - 1) + 4.1858116(2) \right. \right. \\
& - 2.3960046(1) \alpha - 7.849554(7) \omega + 21.73261375(6) \omega^2 \\
& + \left. \left. \left(-1.106338412(2) + 2.86184377(1) \omega \right) c_{\text{SW}} + 0.389033269(2) c_{\text{SW}}^2 \right] + \frac{q_\mu \not{q}}{q^2} (\alpha - 1) \right. \\
& + i q_\mu a_L \left(-\log(a_L^2 q^2) \left(\frac{\alpha}{2} + 1 \right) + 0.580838937(2) + 0.467879641 \alpha - 11.60673207(1) \omega \right. \\
& + 19.231631786(2) \omega^2 + \left. \left. \left(\frac{3}{2} \log(a_L^2 q^2) - 0.759386031 + 3.09273999(3) \omega \right) c_{\text{SW}} - 0.192563629 c_{\text{SW}}^2 \right) \right. \\
& + a_L^2 \left(\log(a_L^2 q^2) \left[\left(\frac{77}{720} - \frac{\alpha}{16} + \frac{\omega}{2} - \frac{5}{24} c_{\text{SW}} + \frac{1}{8} c_{\text{SW}}^2 \right) q^2 \gamma_\mu \right. \right. \\
& \quad \left. \left. + \left(-\frac{163}{360} - \frac{\alpha}{8} + \omega + \frac{1}{12} c_{\text{SW}} + \frac{1}{4} c_{\text{SW}}^2 \right) q_\mu \not{q} - \frac{4}{15} q_\mu^2 \gamma_\mu \right] \right. \\
& + \left[-0.39481322 + 0.405520353 \alpha + \frac{\sum_v q_v^4}{(q^2)^2} \left(\frac{1}{15} - \frac{5\alpha}{96} \right) - 4.068918572(7) \omega + 12.250720808(7) \omega^2 \right. \\
& \quad \left. + \left(0.417132989 - 0.47165346(2) \omega \right) c_{\text{SW}} - 0.143680812(2) c_{\text{SW}}^2 \right] q^2 \gamma_\mu \\
& + \left[0.900644155 - 0.062497180 \alpha - 1.503952230 \omega - 3.718679573 \omega^2 \right. \\
& \quad \left. + \left(0.113009992 - 0.810026586 \omega \right) c_{\text{SW}} + 0.014114849 c_{\text{SW}}^2 \right] q_\mu^2 \gamma_\mu \\
& + \left[0.640691982 - 0.121823715 \alpha + \frac{\sum_v q_v^4}{(q^2)^2} \left(-\frac{2}{15} + \frac{5\alpha}{48} \right) - 4.376037464 \omega + 7.9429434074(8) \omega^2 \right. \\
& \quad \left. + \left(0.144847295 + 0.047859450 \omega \right) c_{\text{SW}} - 0.151408345 c_{\text{SW}}^2 \right] q_\mu \not{q} \\
& + \left. \left. \left(\frac{1}{60} - \frac{\alpha}{24} \right) \frac{q_\mu^2 \not{q}}{q^2} - \left(\frac{127}{180} + \frac{\alpha}{60} \right) \frac{q_\mu \not{q}^3}{q^2} \right] \right)
\end{aligned}$$

$$\begin{aligned}
\Sigma_A^L(q, a_L, m) = & \gamma_5 + \gamma_5 g^2 \frac{N^2 - 1}{N} \frac{1}{16\pi^2} \left[\gamma_\mu \left(\log(a_L^2 q^2) \frac{1}{2} (\alpha - 1) + 2.1541554(2) \right. \right. \\
& - 2.396004636(1) \alpha + 7.384089183(7) \omega - 20.39398792(4) \omega^2 \\
& + \left. \left. \left(1.10633835(4) - 2.86184377(5) \right) c_{\text{SW}} - 0.389033279(5) c_{\text{SW}}^2 \right) + \frac{q_\mu \not{q}}{q^2} (\alpha - 1) \right. \\
& + \left. \left(i q_\mu - i \not{q} \gamma_\mu \right) a_L \left(\log(a_L^2 q^2) \frac{1}{2} (\alpha - 1) + 1.005173559 - 1.467879641 \alpha + 8.761071178 \omega \right. \right. \\
& - 23.774489198(8) \omega^2 + \left. \left. \left(0.773024234 - 1.799580559(4) \omega \right) c_{\text{SW}} - 0.064187875(4) c_{\text{SW}}^2 \right) \right. \\
& + a_L^2 \left(\log(a_L^2 q^2) \left[\left(\frac{77}{720} - \frac{5\alpha}{16} + \frac{\omega}{2} + \frac{7}{24} c_{\text{SW}} - \frac{1}{8} c_{\text{SW}}^2 \right) q^2 \gamma_\mu \right. \right. \\
& \quad \left. \left. + \left(\frac{17}{360} + \frac{3\alpha}{8} + \omega - \frac{1}{24} c_{\text{SW}} - \frac{1}{4} c_{\text{SW}}^2 \right) q_\mu \not{q} - \frac{4}{15} q_\mu^2 \gamma_\mu \right] \right. \\
& + \left. \left(-0.222823436 + 0.873264461 \alpha + \frac{\sum_v q_v^4}{(q^2)^2} \left(\frac{1}{15} - \frac{5\alpha}{96} \right) - 7.20446927(4) \omega + 12.6725228447(9) \omega^2 \right. \right. \\
& - \left. \left. \left(0.74380978(2) - 1.33545335(2) \omega \right) c_{\text{SW}} + 0.15708628(2) c_{\text{SW}}^2 \right) q^2 \gamma_\mu \right. \\
& + \left. \left(0.210583804 - 0.062497173 \alpha - 0.514408403 \omega + 5.223852331 \omega^2 \right. \right. \\
& \quad \left. \left. + \left(0.881481731 - 2.567109739 \omega \right) c_{\text{SW}} - 0.055681736 c_{\text{SW}}^2 \right) q_\mu^2 \gamma_\mu \right. \\
& + \left. \left(0.855629413 - 0.557311936 \alpha + \frac{\sum_v q_v^4}{(q^2)^2} \left(-\frac{2}{15} + \frac{5\alpha}{48} \right) + 2.234122616 \omega - 4.571427781 \omega^2 \right. \right. \\
& \quad \left. \left. + \left(0.167368530 - 0.125922540 \omega \right) c_{\text{SW}} + 0.139353381 c_{\text{SW}}^2 \right) q_\mu \not{q} \right. \\
& \left. + \left. \left(\frac{1}{60} - \frac{\alpha}{24} \right) \frac{q_\mu^2 \not{q}}{q^2} - \left(\frac{127}{180} + \frac{\alpha}{60} \right) \frac{q_\mu \not{q}^3}{q^2} \right] \right]
\end{aligned}$$

Results regarding all extended bilinear operators will be presented in a following extensive paper.

References

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