

Gauge-independent “Abelian” dominance and magnetic monopole dominance in SU(3) Yang-Mills theory

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Recently we have proposed a new reformulation of Yang-Mills (YM) theory based on new variables on a lattice by extending the Cho-Faddeev-Niemi-Shabanov decomposition. Our reformulation allows options discriminated by the stability group \tilde{H} of the gauge group G . When \tilde{H} agrees with the maximal torus group \tilde{H} , it reduces to a manifestly gauge-independent reformulation of the conventional Abelian projection in the maximal Abelian gauge. Within this framework, a non-Abelian Stokes theorem enables us to express the Wilson loop operator in the fundamental representation by the “Abelian” variable extracted in association with the stability group in the minimal option, and to rewrite the Wilson loop operator using a non-Abelian magnetic monopole defined in a manifestly gauge-independent way. For $G = SU(3)$, two options are possible: minimal one with $\tilde{H} = U(2)$ and maximal one with $\tilde{H} = U(1) \times U(1)$. In this talk we summarize the results of Monte Carlo simulations for SU(3) in the minimal option. Especially, we compare three Wilson loop averages defined by the “Abelian” variable, the monopole part and the original YM field. We confirm that the quark–antiquark confining potential is reproduced by the “Abelian” variable (“Abelian” dominance), and that the string tension is reproduced by the non-Abelian magnetic monopole (magnetic monopole dominance). Moreover, we mention the behaviors of correlation functions for new variables.

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1. Introduction

The dual superconductivity picture is a promising mechanism for quark confinement. In this picture, the magnetic monopole plays an important role for quark confinement. It is known that the string tension of the Abelian part and the monopole part in Yang-Mills (YM) fields reproduce the original one, which is respectfully called Abelian dominance and monopole dominance in the string tension. In the other approach, the center vortex can explain the string tension. However, such dominances have been observed only in the special gauges such as the maximal Abelian (MA) gauge or the maximal center gauge, whereas this is not the case in other gauges.

We have given a new description of the YM theory on a lattice, which is expected to give an efficient framework for explaining quark confinement based on the dual superconductivity picture in *the gauge independent manner*. In the $SU(2)$ YM theory, it is constructed as a lattice version of the Cho-Faddeev-Niemi-Shabanov (CFNS) decomposition[2] in a continuum theory[3][4][5], and in the $SU(N)$ YM theory ($N \geq 3$) we have the reformulations discriminated by the stability group \tilde{H} of gauge group $G = SU(N)$ [6].

In the $SU(2)$ case, we have a unique option with the stability group $\tilde{H} = U(1)$. While in the $SU(3)$ case, we have two options: (i) the maximal one with the stability subgroup $\tilde{H} = U(1) \times U(1)$, which is the gauge-independent reformulation of the Abelian projection represented by the conventional MA gauge, (ii) the minimal one with the stability group $\tilde{H} = U(2)$, which is a new type of formulation and derives a non-Abelian magnetic monopole [7].

For $SU(2)$ case, in fact, our numerical simulation demonstrated that the gauge-invariant magnetic monopoles constructed reproduce the string tension [4], and that "Abelian" dominance holds, that is, the Wilson loop by using the decomposed "Abelian part" reproduces the string tension in the quark-quark static potential [11]. While in the Landau gauge, we have shown that the infrared "Abelian" dominance in the propagator, i.e., the extracted "Abelian" propagator is dominant in the infrared region[5]. For $SU(3)$ case, applying this framework, we have demonstrated the numerical simulations for the maximal option in the lattice2007 [9], and for the minimal option in the results lattice2008 [10].

In this talk, we apply this method to investigate the dual superconductivity picture. In what follows, restricting to the minimal option for $SU(3)$ YM theory. We summarize the result of the lattice formulation. By combining non-Abelian Stokes' theorem (NAST) with the decomposition, we show the decomposition can extract the dominant degrees of freedom that are relevant to quark confinement in the Wilson criterion in such a way that they reproduce almost all the string tension in the linear inter-quark potential. It should be noticed that for the Wilson loop for quark in the fundamental representation, the relevant part of YM is decomposed by the minimal option, not the maximal option. Then, by using the Hodge decomposition, we can define non-Abelian magnetic monopoles from the decomposed relevant part in the gauge invariant manner.

We perform the numerical simulations to investigate, speaking in conventional sense, "Abelian" dominance and magnetic monopole dominance as was carried out in the $SU(2)$ case. Since the relevant part of YM field for confinement corresponds to the stability group $\tilde{H} = U(2)$, we may call restricted $U(2)$ -dominance and non-Abelian magnetic monopole dominance in $SU(3)$ YM theory.

2. New variables on a lattice

We summarize the new description of the $G = SU(N)$ ($N = 3$)YM theory on a lattice. The YM field $\mathbb{A}_{x',\mu}$ is represented as a link variable

$$U_{x,\mu} = \exp \left(-ig \int_x^{x+\hat{\mu}\varepsilon} dx^\mu \mathbf{A}_\mu(x) \right) = \exp(-ig\varepsilon \mathbb{A}_{x',\mu}), \quad (2.1)$$

which is supposed to be decomposed into the product of new variables, $X_{x,\mu}, V_{x,\mu} \in SU(N)$,

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}, \quad V_{x,\mu} = \exp(-ig\varepsilon \nabla_{x',\mu}), \quad X_{x,\mu} = \exp(-ig\varepsilon \mathbb{X}_{x,\mu}), \quad (2.2)$$

such that the decomposed variables are transformed by a full gauge transformation $\Omega_x \in SU(N)$:

$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger, \quad (2.3a)$$

$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger, \quad X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger, \quad (2.3b)$$

where $V_{x,\mu}$ is defined on a link $\langle x, x + \varepsilon \hat{\mu} \rangle$ like $U_{x,\mu}$, and $X_{x,\mu}$ on a site. To define the decomposition for the minimal option, a color field, $\mathbf{h}_x = h_x^k \lambda^k / 2$ ($\in G/\tilde{H}$, $\tilde{H} = U(N-1)$), is introduced as a site variable, where λ^k is the Gell-Mann matrix, and h_x^k ($k = 1, \dots, N^2 - 1$) is a component of a unit vector. The color field is transformed by an independent gauge transformation $\Theta_x \in SU(N)$ as $\mathbf{h}_x \rightarrow \Theta_x \mathbf{h}_x \Theta_x^\dagger$.

The decomposition is determined by solving the defining equations[1],

$$D_\mu^\varepsilon[V] \mathbf{h}_x = \frac{1}{\varepsilon} (V_{x,\mu} \mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0, \quad (2.4a)$$

$$g_x = e^{-2\pi q_x/N} \exp(-ia_x^{(0)} \mathbf{h}_x - i \sum_{i=1}^3 a_x^{(i)} \mathbf{u}_x^{(i)}) = \mathbf{1}, \quad (2.4b)$$

which corresponds to the continuum version of the decomposition $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$:

$$D_\mu[\mathcal{V}] \mathbf{h}(x) = 0, \quad \text{tr}(\mathbf{h}(x) \mathcal{X}_\mu(x)) = 0. \quad (2.5)$$

Note that g_x is a parameter underdetermined from equation (2.4a). The defining equation can be solved exactly, and the solution is given by

$$L_{x,\mu} = \frac{N^2 - 2N + 2}{N} \mathbf{1} + (N-2) \sqrt{\frac{2(N-1)}{N}} (\mathbf{h}_x + U_{x,\mu} \mathbf{h}_{x+\mu} U_{x,\mu}^{-1}) + 4(N-1) \mathbf{h}_x U_{x,\mu} \mathbf{h}_{x+\mu} U_{x,\mu}^{-1}, \quad (2.6a)$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu} L_{x,\mu}^\dagger} \right)^{-1} L_{x,\mu}, \quad (2.6b)$$

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger (\det(\hat{L}_{x,\mu}))^{1/N} g_x^{-1}, \quad (2.6c)$$

$$V_{x,\mu} = X_{x,\mu}^\dagger U_{x,\mu} = g_x \hat{L}_{x,\mu} U_{x,\mu} (\det(\hat{L}_{x,\mu}))^{-1/N} \quad (2.6d)$$

Thus, in order that the theory written in terms of new variables is equipollent to the original YM theory, i.e., the symmetry extended by introducing the color field, $SU(N)_\Omega \times [SU(N)/U(N-1)]_\Theta$ must be reduced to the same symmetry as the original YM theory, i.e., $SU(N)_{\Omega=\Theta}$, we introduce the reduction condition that a set of color fields $\{h_x\}$ is determined by minimizing the functional

$$F_{\text{Red.}} = \sum_{x,\mu} \text{tr} \left((D_\mu^\varepsilon[U] \mathbf{h}_x) (D_\mu^\varepsilon[U] \mathbf{h}_x)^\dagger \right) / \text{tr}(\mathbf{1}), \quad (2.7)$$

which is an extension of the nMAG condition in $SU(2)$ case.

3. New variables and magnetic monopoles in the view of NAST

Following the papers [7][8], let us consider the Wilson loop in terms of the new variables. By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wilson loop C , $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x) \langle \Lambda, \xi_x|$,

we obtain

$$\begin{aligned} W_C[U] &= \text{tr} \left(\prod_{\langle x, x+\mu \rangle \in C} U_{x, \mu} \right) = \int \prod_{x \in C} d\mu(\xi_x) \prod_{\langle x, x+\mu \rangle \in C} \langle \Lambda, \xi_x | U_{x, \mu} | \xi_{x+\mu}, \Lambda \rangle \\ &= \int \prod_{x \in C} d\mu(\xi_x) \prod_{\langle x, x+\mu \rangle \in C} \langle \Lambda | (\xi_x^\dagger X_{x, \mu} \xi_x) (\xi_x^\dagger V_{x, \mu} \xi_{x+\mu}) | \Lambda \rangle, \end{aligned} \quad (3.1)$$

where we have used $\xi_x \xi_x^\dagger = 1$. For the stability group of \tilde{H} , the 1st defining equation (2.4a) is rewritten as

$$\mathbf{h}_x V_{x, \mu} - V_{x, \mu} \mathbf{h}_{x+\mu} = 0 \iff [\xi_x^\dagger V_{x, \mu} \xi_{x+\mu}, \tilde{H}] \iff \xi_x V_{x, \mu} \xi_{x+\mu}^\dagger \in \tilde{H}, \quad (3.2)$$

that implies that $|\Lambda\rangle$ is eigen state of $\xi_x^\dagger V_{x, \mu} \xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x, \mu} \xi_{x+\mu}) |\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle \Lambda | \xi_x^\dagger V_{x, \mu} \xi_{x+\mu} | \Lambda \rangle = \langle \Lambda, \xi_x | V_{x, \mu} | \xi_{x+\mu}, \Lambda \rangle. \quad (3.3)$$

Then, $W_C[U]$ is rewritten to

$$W_C[U] = \int \prod_{\langle x \rangle \in C} d\mu(\xi_x) \rho[X; \xi] \prod_{\langle x, x+\mu \rangle \in C} \langle \Lambda, \xi_x | V_{x, \mu} | \xi_{x+\mu}, \Lambda \rangle, \quad (3.4)$$

$$\rho[X; \xi] := \prod_{\langle x \rangle \in C} \langle \Lambda, \xi_x | X_{x, \mu} | \xi_x, \Lambda \rangle \quad (3.5)$$

By substituting the $X_{x, \mu} = 1 - ig\varepsilon \mathcal{X}_\mu(x) + O(\varepsilon^2)$ we obtain $\rho[X; \xi] = 1 + O(\varepsilon^2)$, since

$$\begin{aligned} \langle \Lambda, \xi_x | X_{x, \mu} | \xi_x, \Lambda \rangle &= \text{tr}(X_{x, \mu}) / \text{tr}(\mathbf{1}) + 2\text{tr}(X_{x, \mu} \mathbf{h}_x) = 1 + 2ig\varepsilon \text{tr}(\mathcal{X}_\mu(x) \mathbf{h}(x)) + O(\varepsilon^2) \\ &= 1 + O(\varepsilon^2), \end{aligned} \quad (3.6)$$

where we have used the 2nd defining equation in the continuum version, $\text{tr}(\mathcal{X}_\mu(x) \mathbf{h}(x)) = 0$. Therefore, we obtain

$$W_C[U] = \int \prod_{x \in C} d\mu(\xi_x) \prod_{\langle x, x+\mu \rangle \in C} \langle \Lambda, \xi_x | V_{x, \mu} | \xi_{x+\mu}, \Lambda \rangle = W_C[V]. \quad (3.7)$$

From the non-Abelian Stokes theorem, Wilson loop along the path C is written to area integral on $\Sigma : C = \partial\Sigma$ (in the continuum limit)[7]:

$$W_C[\mathcal{A}] := \text{tr} \left[P \exp \left(-ig \oint_C dx^\mu \mathcal{A}_\mu(x) \right) \right] / \text{tr}(\mathbf{1}) = \int d\mu_\Sigma(\xi) \exp \left(-ig \int_{S: C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathcal{V}] \right), \quad (3.8)$$

where $\mathcal{V}_\mu(x)$ is defined by

$$\mathcal{V}_\mu(x) = \mathcal{A}_\mu(x) - \frac{2(N-1)}{N} [\mathbf{h}(x), [\mathbf{h}(x), \mathcal{A}_\mu(x)]] + ig^{-1} \frac{2(N-1)}{N} [\mathbf{h}(x), \partial_\mu \mathbf{h}(x)]. \quad (3.9)$$

Therefore, it turns out that the defining equation (2.4a) and (2.4b) give the decomposition which reproduces "Abelian" (V) dominance for the Wilson loop operator on a lattice even with a finite lattice spacing ε , $W_C[U] \cong \text{const.} W_C[V]$, and we can identify the $\mathbb{V}_{x, \mu}$ in eq(2.2) with $\mathcal{V}_\mu(x)$, eq(3.8).

Thus, by using the Hodge decomposition, eq(3.8) is further rewritten into

$$W_C[\mathcal{A}] = \int d\mu_\Sigma(\xi) \exp \left[ig \sqrt{\frac{N-1}{2N}} (k, \Xi_\Sigma) + ig \sqrt{\frac{N-1}{2N}} (j, N_\Sigma) \right] \quad (3.10)$$

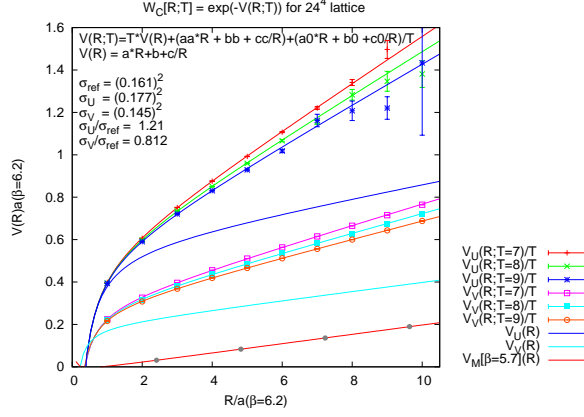


Figure 1: The combinational plot of the quark–anti-quark inter potential. (from above to below) $V(R, T)$ and $V(R)$ of the Wilson loop for $SU(3)$ -YM field $\langle W_{(R,T)}[U_{x,\mu}] \rangle$, and ones for restricted $U(2)$ -part $\langle W_{(R,T)}[V_{x,\mu}] \rangle$, and $V(R)$ for the magnetic monopole part $\langle W_{(R,T)}[k_{x,\mu}] \rangle$, respectively. The static potential $V(R)$ is represented by the bold lines.

where k and j are gauge invariant and conserved current $\delta k = 0 = \delta j$ defined by

$$k := \delta^* F = *dF, \quad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}, \quad (3.11a)$$

$$j := \delta F, \quad N_\Sigma := \delta \Theta_{\Sigma(x)} \Delta^{-1}, \quad (3.11b)$$

with $\Delta := d\delta + \delta d$ and $\Theta_\Sigma^{\mu\nu}(x) := \int_\Sigma d^2 S^{\mu\nu}(x(\sigma)) \delta^D(x - x(\sigma))$. Thus, magnetic monopole current on a lattice is calculated from the decomposed variable as

$$V_{x,\mu} V_{x+\mu,\nu} V_{x+\nu,\mu}^\dagger V_{x,\nu}^\dagger = \exp(-ig \mathcal{F}[\mathbf{V}_\mu(x)]_{\mu\nu}) = \exp(-ig \Theta_{\mu\nu}^8 \mathbf{h}_{x'}), \quad (3.12)$$

$$\Theta_{\mu\nu}^8 = -\arg \text{Tr} \left[\left(\frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} \mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\nu} V_{x+\nu,\mu}^\dagger V_{x,\nu}^\dagger \right], \quad (3.13)$$

$$k_{x,\mu} := \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} \partial_\nu \Theta_{\alpha\beta}^8. \quad (3.14)$$

It should be notice that k_μ is a gauge invariant non-Abelian magnetic monopole, since it is defined from the plaquet of the field $V_{x,\mu}$, which is an element of the non-Abelian stability group $\tilde{H} = U(2)$. Thus, on a lattice the Wilson loop by magnetic-monopole part is given by $W_C[k_{x,\mu}] = \exp\left(-ig\varepsilon\sqrt{\frac{1}{3}}\sum_{x,\mu} k_{x,\mu}\Xi_{x,\mu}\right)$.

4. Lattice data

We have performed the numerical simulation. The sets of configurations $\{U_{x,\mu}\}$ for the standard Wilson action are generated by using the standard pseudo heat bath method. For a given configuration $\{U_{x,\mu}\}$, the color field $\{\mathbf{h}_x\}$ is determined by solving the reduction condition minimizing the functional eq(2.7). Thus, new variables $\{X_{x,\mu}, V_{x,\mu}\}$ are obtained as the decomposition by using eqs(2.6a)-(2.6d).

First, we study the quark–anti-quark potential from the Wilson loop average. Here, we use the fitting of the Wilson loop ($R \times T$ rectangle) average with the two-variable function $V(R, T)$;

$$\langle W_{(R,T)}[V] \rangle = \exp(-V(R, T)), \quad (4.1a)$$

$$V(R, T) := T \times V(R) + (a_1 R + b_1 + c_1/R) + (a_2 R + b_2 + c_2/R)/T, \quad (4.1b)$$

$$V(R) = \sigma R + b + c/R. \quad (4.1c)$$

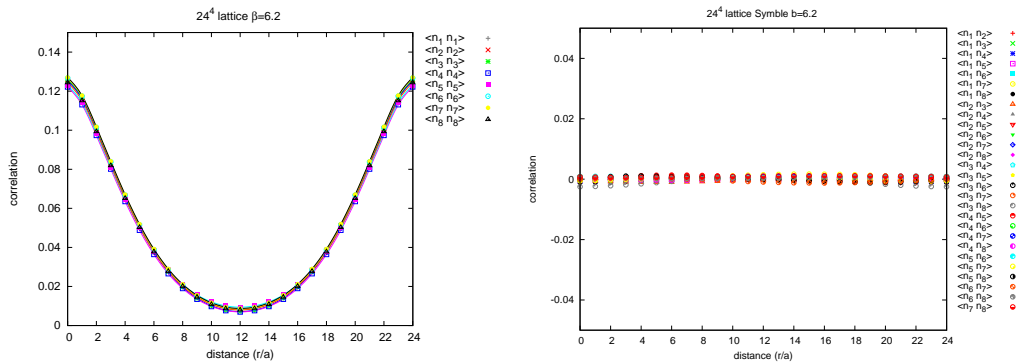


Figure 2: Two point-correlation functions $\langle h^A(x)h^B(y) \rangle$ ($A, B = 1, 2, \dots, 8$) measured at $\beta = 6.2$ on 24^4 lattice, using 500 configurations in the Landau gauge. The left panel shows ($A = B$) and the right ($A \neq B$).

and the static potential is extracted by the extrapolated function: $V(R) = -\lim_{T \rightarrow \infty} \frac{1}{T} \log \langle W_{(R,T)}[U_{x,\mu}] \rangle$. Fig.1 shows the combinational plot of the quark–anti-quark potential, i.e., from upper to lower, $V_U(R, T)$ ($T = 7, 8, 9$) and $V_U(R)$ from the $SU(3)$ Wilson loop average $\langle W_{(R,T)}[U_{x,\mu}] \rangle$, $V_V(R, T)$ ($T = 7, 8, 9$) and $V_V(R)$ from the restricted $U(2)$ -part $\langle W_{(R,T)}[V_{x,\mu}] \rangle$, and $V_M(R)$ from the magnetic-monopole part $\langle W_{(R,T)}[k_{x,\mu}] \rangle$, respectively. The potentials, $V_U(R)$ (blue solid line), $V_U(R)$ (sian solid line) and $V_U(R)$ (red solid line) show the good agreement of the string tension, i.e., “Abelian”-dominance (or restricted $U(2)$ -dominance) of 85-90%, and non-Abelian $U(2)$ magnetic-monopole dominance of 75%

Second, we study the correlation functions for new variables, where we have adopted the Landau gauge for the original YM field. We have checked the one-point function vanishes $\langle h_x^A \rangle = \pm 0.002 \cong 0$ ($A = 1, 2, \dots, 8$). Fig.2 shows two-point correlation function of color field. The global $SU(3)$ color symmetry is indicated, $\langle h^A(x)h^B(y) \rangle = D(x-y)\delta^{AB}$. The left panel of Fig.3 shows the two-point correlations of new variables $\langle \mathcal{V}_\mu(0)\mathcal{V}_\mu(x) \rangle$, $\langle \mathcal{X}_\mu(0)\mathcal{X}_\mu(x) \rangle$ and that of the original YM field $\langle \mathcal{A}_\mu(0)\mathcal{A}_\mu(x) \rangle$. This indicates the restricted variable $\mathcal{V}_\mu(x)$ -dominance in the sense that the correlation function of $\mathcal{V}_\mu(x)$ behaving just as that of the YM field $\mathcal{A}_\mu(x)$, dominates in the long-range. While the correlation function of $\mathcal{X}_\mu(x)$, $SU(3)/U(2)$ variable, dumps quickly. For field $\mathcal{X}_\mu(x)$, we can introduce a mass term $\mathcal{L}_{M_X} = -\frac{1}{2}M_X^2 \text{tr} \mathcal{X}_\mu^A(x)\mathcal{X}_\mu^A(x)$, since $X_{x,\mu}$ transforms adjointly under gauge transformation, see eq(2.3b). The right panel of Fig.3 shows rescaled propagator for $\mathcal{X}_\mu(x)$: $r^{3/2}D_{\mu\nu}^{XX}(r)$, $r = |x|$. The gauge boson propagator $D_{\mu\nu}^{XX}(x-y)$ is related to the Fourier transform of the massive propagator, $D_{\mu\nu}^{XX}(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} D_{\mu\nu}^{XX}(k)$, and the scalar type of propagator as function r should behave for large $M_X|x-y|$ as

$$D^{XX}(x-y) = \langle X_\mu(x)X_\mu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{3}{k^2 + M_X^2} \simeq \frac{3\sqrt{M}}{2(2\pi)^{3/2}} \frac{e^{-M_X|x-y|}}{|x-y|^{3/2}}. \quad (4.2)$$

Therefore, the mass parameter M_X can be measured as the dumping factor of $r^{3/2}D_{\mu\nu}^{XX}(r)$, and we obtain $M_X = 2.4\sqrt{\sigma_{phys}} = 1.1\text{GeV}$. This value should be compared with the result in MA gauge.

5. Summary and discussions

We have given a new description of the YM theory on a lattice, which gives an efficient framework to explain quark confinement based on the dual superconductivity picture in the gauge independent manner. In the implication of the non-Abelian Stokes’ theorem, we have shown the minimal option of change of variables can extract the relevant fields (the restricted part) the quark–antiquark confining potential of the fundamental

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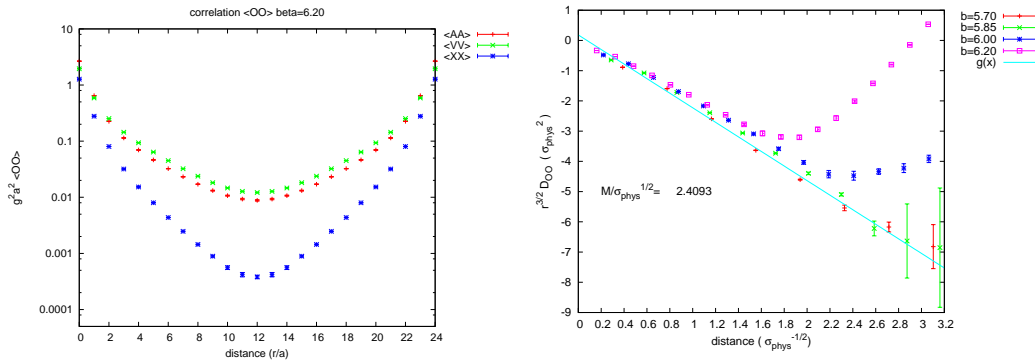


Figure 3: (Left) The correlators as function of distans r . measured at $\beta = 6.2$ on 24^4 lattice, using 500 cofigurations in the Landau gauge.: (from above to blelow) $\langle \mathcal{V}_\mu(0)\mathcal{V}(x) \rangle$, $\langle \mathcal{A}_\mu(0)\mathcal{A}(x) \rangle$, $\langle \mathcal{X}_\mu(0)\mathcal{X}(x) \rangle$. (Right) The rescaled two-point correlator for $\mathcal{X}_\mu : r^{3/2} D_{\mu\nu}^{XX}(r)$.

representation. By performing the numerical simulation, we have compare the confining potential from three types of Wilson loop averages, original YM field, restricted $U(2)$ -part and non-Abelian magnetic monopole part, and confirmed restricted $U(2)$ -dominance and magnetic monopole dominance.

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