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The $B^*B\pi$ Coupling in the Static Limit

ALPHA Collaboration

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We study an accurate method for the lattice calculation of the $B^*B\pi$-coupling in the static limit, paying particular attention to excited state contamination. As this coupling is a parameter of the heavy meson chiral Lagrangian, it is useful for constraining the chiral behaviour of various observables in B-physics. We present a precise study of the continuum limit in the quenched approximation and preliminary results with 2 flavours of improved Wilson quarks (using CLS lattices) for pion masses down to around 250 MeV. With dynamical quarks both the lattice spacing and the light quark mass dependences are found to be very weak. We can quote $g = 0.51(2)$ for the continuum value in the chiral limit, where the error will be reduced when a more complete analysis has been performed.

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1. Introduction

The $B^*B\pi$-coupling is defined via the matrix element associated with the strong decay $B^* \to B\pi$:

$$\langle B^0(p)\pi^+(q)|B^{*+}(p')\rangle \equiv -g_{B^*B\pi}q_\mu\eta^\mu(p')(2\pi)^4\delta(p'-p-q), \tag{1.1}$$

where η^μ is the polarization vector of the B^* meson. With the physical quark masses $m_{B^*} < m_B + m_\pi$, so this decay is kinematically forbidden and direct experimental measurement of the coupling is not possible. The equivalent charmed meson decay, $D^* \to D\pi$, is however the dominant decay mode of the D^* meson, and experimental measurements of the corresponding coupling are available: $g_c = 0.61(7)$ [1].

Much of the interest in this coupling is due to the role it plays in heavy-light meson chiral perturbation theory (HMχPT) [2]. At leading order in the heavy quark and chiral expansions, there is in addition to the pion decay constant f_π a single low-energy constant g which has a simple relation to the $B^*B\pi$-coupling:

$$g = \lim_{m_b\to\infty, m_d\to 0} \frac{g_{B^*B\pi}}{2\sqrt{m_B m_{B^*}}} f_\pi. \tag{1.2}$$

Knowledge of this coupling is thus helpful for constraining the chiral behaviour of various B-physics observables. Since in the ALPHA Collaboration HQET programme we will calculate several such quantities, e.g. f_B and the $B \to \pi l \nu$ form factor, we begin with a precision determination of g.

In lattice calculations of the $B^*B\pi$ coupling[3 – 5], the issue of multihadronic $|B\pi\rangle$ states can be avoided entirely by using a combination of LSZ reduction of the pion and pion dominance in order to relate the coupling to the form factor of the axial current between B and B^* states. This reduction of the pion is analogous to the Golberger-Treiman relation for the nucleon-nucleon-pion coupling. The form factor can then be written:

$$\langle B^0(p)|A_\mu|B^{*+}(p+q)\rangle = \eta_\mu F_1(q^2) + (\eta\cdot q)(2p+q)_\mu F_2(q^2) + (\eta\cdot q)q_\mu F_3(q^2) \tag{1.3}$$

In the static and chiral limits, in which the leading-order HMχ coupling is obtained, only the zero-momentum form factor $F_1(0)$ must be calculated.

2. Method

The lattice determination of a form factor requires the calculation of three-point functions. Particular attention must therefore be paid to the problems of statistical noise (which grows exponentially with the Euclidean time separations in heavy-light correlation functions) and excited state contamination (which is exponentially suppressed with Euclidean time separation).

In a generic lattice three point function:

$$C_3(t,t';\alpha,\beta) = \langle \mathscr{O}_\alpha(t)\mathscr{O}(t')\mathscr{O}^\dagger_\beta(0)\rangle, \tag{2.1}$$

where β and α label the quantum numbers of the interpolating fields at the source and sink respectively, there are 2 time separations t' and $t-t'$ which ought both to be taken as large as

Ensemble parameters	N_{conf}	N_{noise}
$16^3 \times 32$, $\beta = 6.0219$, $a \approx 0.10$ fm	100	200
$24^3 \times 48$, $\beta = 6.2885$, $a \approx 0.08$ fm	100	48
$32^3 \times 64$, $\beta = 6.4956$, $a \approx 0.05$ fm	100	32

Table 1: Details of the quenched lattices and measurements. N_{conf} gives the number of configurations and N_{noise} the number of noise sources per timeslice.

possible if excited state contamination is to be minimized. For example, in the case of a matrix element between states with equal energies, putting the insertion point midway between the source and the sink we have $C_3(t,t/2;\alpha,\beta)$ which has corrections $\mathscr{O}(e^{-(t/2)\Delta E})$ with the energy gap $\Delta E = E_2^{(\alpha)} - E_1^{(\alpha)} = E_2^{(\beta)} - E_1^{(\beta)}$.

In this work we investigate a method in which we calculate instead, for a given source-sink separation, the sum of the correlation functions with the operator inserted on each timeslice. [6, 7]. This helps to reduce excited state contamination. Defining the summed correlator by:

$$D(t;\alpha,\beta) \equiv a \sum_{t'} C_3(t,t';\alpha,\beta), \tag{2.2}$$

it can be shown that the form of the corrections in the effective matrix element is:

$$R(t) \equiv \partial_t \frac{D(t;\alpha,\beta)}{\sqrt{C_2(t;\beta)C_2(t;\alpha)}} = \langle\beta|\mathscr{O}|\alpha\rangle + \mathscr{O}(te^{-t\Delta E}), \tag{2.3}$$

where the two-point function is given by $C_2(t;\alpha) \equiv \langle \mathscr{O}_\alpha(t)\mathscr{O}_\alpha^\dagger(0)\rangle$.

In the calculation of the bare matrix element we use both the HYP1 and HYP2 discretizations of the static action in order to mitigate the exponential decay of signal-to-noise in Euclidean time [8, 9] and also to get an additional handle on discretization effects. We also use noisy estimation of the all-to-all light quark propagator[10], with U(1) noise and full time-dilution. Since we use a sequential propagator for the light quark, we obtain easily the insertion summation over all timeslices t' but must invert again for each operator used (in this case we perform the insertion for each spatial component of the axial current).

Additionally, we use a variational basis of 8 interpolating operators for the B and B^* mesons, consisting of Gaussian smeared operators with an APE-smeared gauge covariant Laplacian Δ:

$$\psi_l^{(k)}(x) = \left(1+\kappa_G a^2\Delta\right)^{R_k} \psi_l(x) \tag{2.4}$$

with widths

$$r_{phys,k} \approx 2\sqrt{\kappa_G R_k}a \tag{2.5}$$

varied over the range 0 - 0.7 fm. In order to extract the desired matrix element from the matrices of correlation functions obtained in this way, we use a Generalized Eigenvalue Problem (GEVP) analysis in order to construct optimized interpolating operators[11].

3. Quenched Tests and Results

We have performed a test of our method using the lattices and valence strange quark parameter κ_s of the ALPHA Collaboration quenched HQET programme[12 – 14]. The Wilson gauge action

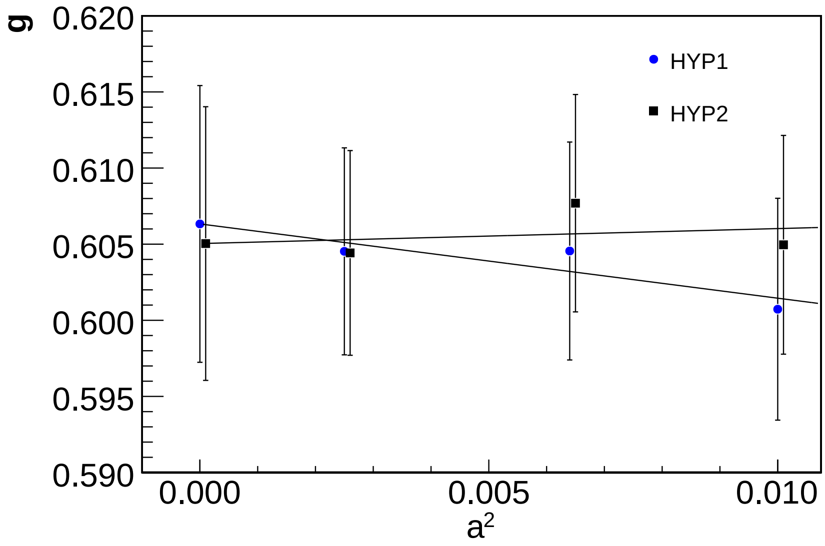


Figure 1: Quenched continuum extrapolation. A horizontal offset has been added to the HYP2 points for clarity.

is used and there are 3 lattice spacings with a fixed physical volume of $L \approx 1.5$ fm, and temporal extent $T = 2L$. The details of the lattices and of the measurements performed on them are given in Table 1.

In each case, it was possible to determine the value of the bare matrix element with sub-percent precision. The renormalized axial current is given however by:

$$(A_k)_R = Z_A(g_0^2)(1 + b_A(g_0^2)am_q)(A_k + ac_A(g_0^2)\partial_k P), \tag{3.1}$$

where P represents the pseudoscalar density. At zero momentum transfer $\vec{q} = 0$, the c_A term does not contribute. We use the values of κ_c and Z_A given in Ref. [15] and the one-loop perturbative estimate of b_A[16]. The overall uncertainty in the calculation is therefore dominated by the 1% error on Z_A.

As can be seen in the continuum extrapolation shown in Figure 1, there is no discernible lattice spacing dependence at this precision. Fitting linearly in a^2, we obtain the continuum limit results:

$$g_{\mathrm{HYP1}} = 0.606(9), \qquad g_{\mathrm{HYP2}} = 0.605(9). \tag{3.2}$$

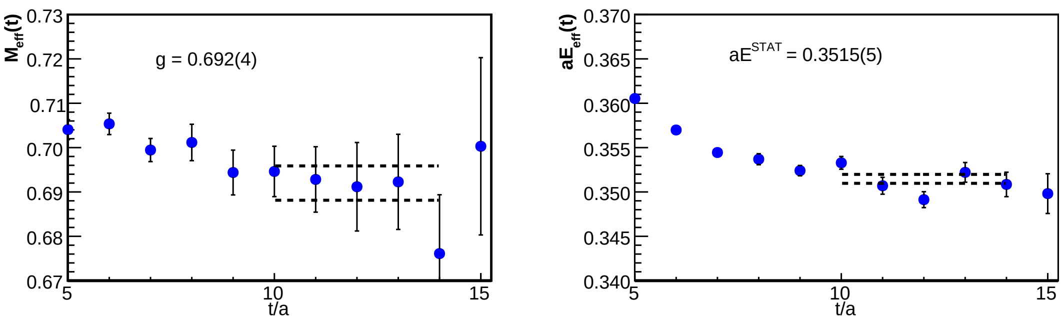
4. $N_f = 2$ Results

For the $N_f = 2$ calculation we use gauge configurations generated and shared within the CLS ("Coordinated Lattice Simulations") community effort[1]. The action is non-perturbatively $O(a)-$improved $N_f = 2$ Wilson QCD, and the algorithm used is deflation-accelerated DD-HMC [17, 18]. Details of the lattices used and of the measurements performed so far are given in Table 2.

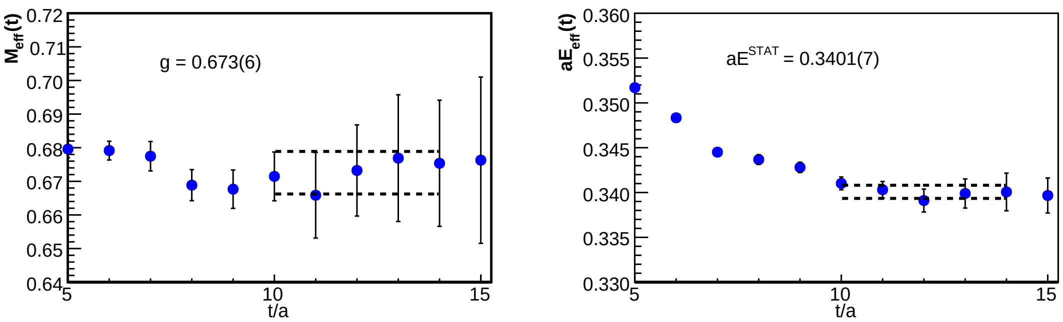
[1] `https://twiki.cern.ch/twiki/bin/view/CLS/WebHome`

	Name	L (fm)	m_π (MeV)	$m_\pi L$	N_{conf}	N_{noise}
$\beta = 5.2$	A1	2.6	700	9.0	288	8
$a = 0.08$ fm	A4	2.6	360	4.8	370	8
	E4	2.3	540	6.2	157	16
$\beta = 5.3$	E5	2.3	410	4.7	2000	4
$a = 0.07$ fm	F6	3.4	280	5.0	339	4
	F7	3.4	250	4.2	301	4
$\beta = 5.5$	M5	1.6	430	3.0	258	12
$a = 0.05$ fm	N5	2.4	430	5.3	477	2

Table 2: Details of the $N_f = 2$ lattices and measurements. The scale setting is based on f_K and is preliminary.



(a) E5 ensemble.



(b) F6 ensemble.

Figure 2: Example plateaus and bare results. The left panels show the ratio $R(t)$ defined in Eqn. 2.3 as well as fits to the bare matrix element. The right panels show the corresponding two-point function effective mass (the static energy). All plateau ranges are chosen by examination of the two-point function effective mass. The GEVP initial timeslice t_0 is chosen to be at 0.3 fm.

Example plateaus for the E5 and F6 ensembles with the HYP2 static action are shown in Fig. 2. For now, we do not explicitly estimate the autocorrelation time but take 50 jackknife bins in all cases, which gives a reasonable estimate of the error judging by the investigation in Ref. [19]. As in the quenched case, we are able to determine the bare matrix elements with sub-percent precision in most cases, and the overall uncertainty is dominated by that on Z_A[20]. We use the value of κ_c

given in Ref. [21] and for b_A we again take the one-loop perturbative estimate from Ref. [16].

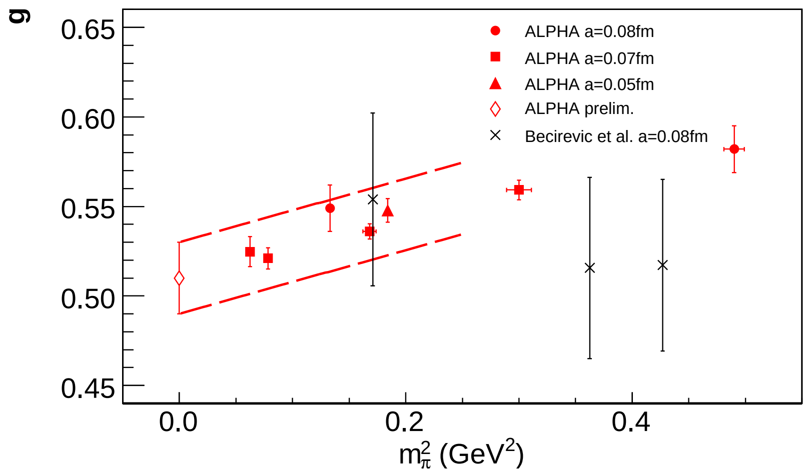


Figure 3: The current status of our renormalized $N_f = 2$ results, together with those of Ref. [5] for comparison.

In Fig. 3, we summarize our renormalized results for the various ensembles and compare them to those of the recent publication Ref. [5]. The results shown use the HYP2 static action but those for HYP1 look essentially the same. Our results at $\beta = 5.2$ have large errors relative to those at finer lattice spacings due to the $\approx 2\%$ uncertainty in Z_A at that value of the bare coupling (compared to $\approx 0.6\%$ at $\beta = 5.3$, for example). At the time of the conference, we had a single result at $\beta = 5.5$ corresponding to the small volume M5 ensemble which suffers from finite volume effects, and which appeared significantly lower than those at the coarser lattice spacings. This point was 10% smaller than the results for the N5 ensemble that are now available and is therefore excluded from Fig. 3.

5. Summary

We have performed a lattice study of the $B^*B\pi$ coupling in the static limit, from which the leading-order HMχ coupling can be obtained. Our method combines a summed-insertion three-point function with a GEVP analysis in order to minimize excited state contamination. In addition, more standard techniques such as the HYP2 static action and noisy all-to-all light-quark propagators with timeslice sources are used, in order to deal efficiently with statistical noise.

We have studied the continuum limit of this quantity in the quenched approximation with $< 2\%$ precision, and see no evidence of lattice spacing dependence over the range $0.05\,\mathrm{fm} < a < 0.10\,\mathrm{fm}$. In the $N_f = 2$ theory we obtain a comparable precision, surpassing by far that of previous lattice calculations of this quantity, and have made substantial progress towards controlling all sources of systematic uncertainty. It is already safe to quote, in the chiral and continuum limits:

$$g = 0.51(2). \tag{5.1}$$

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