PROCEEDINGS OF SCIENCE

The Decay Constants f_{D_s}, f_{D^+}, f_{B_s} and f_B from Lattice QCD

Fermilab Lattice and MILC Collaborations

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In this report we summarize results for the leptonic decay constants f_{D_s}, f_{D^+}, f_{B_s} and f_B, obtained at the three lattice spacings $a \approx 0.09$, 0.125 and 0.15 fm, from the now concluded first computational phase of our on-going project. The decay constants are computed on the MILC collaboration's $2+1$ flavor asqtad gauge ensembles. We use clover heavy quarks in the Fermilab interpretation and asqtad improved staggered light quarks. For each of the D and B meson systems, a simultaneous chiral and continuum extrapolation is performed with partially quenched lattice results. We show how improvements being implemented now, such as, higher statistics, additional finer lattice spacings and better determinations of inputs to the calculation, are expected to reduce future statistical and systematic uncertainties.

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a [fm]	am_h	am_l	β	r_1/a	configs
0.09	0.031	0.0031	7.08	3.69	906
		0.0062	7.09	3.70	557
		0.0124	7.11	3.72	518
0.125	0.05	0.005	6.76	2.64	678
		0.007	6.76	2.63	833
		0.01	6.76	2.62	592
		0.02	6.79	2.65	460
		0.03	6.81	2.66	549
0.15	0.0484	0.0097	6.572	2.13	631
		0.0194	6.586	2.13	631
		0.029	6.600	2.13	576

Table 1: Properties of the MILC three-flavor ensembles used in this study [3].

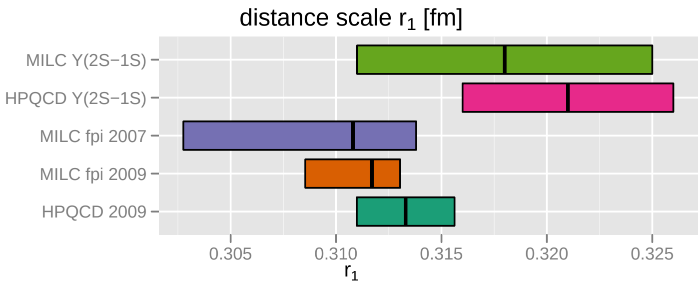

Figure 1: The distance r_1 in Fermi units. "HPQCD $\Upsilon(2S\text{-}1S)$" uses Υ spectrum results to set r_1 [5]. "MILC $\Upsilon(2S\text{-}1S)$" derives from essentially the same spectrum analysis [6]. Recent calculation of f_π lead to more precise r_1 values: "MILC f_π 2007" [7] and "MILC f_π 2009" [8]. "HPQCD 2009" uses several physical quantities, including recent Υ results, as inputs [9].

1. Introduction and overview of the calculation

We summarize results for the D and B decay constants obtained the Fermilab Lattice and MILC collaborations. These results, at three lattice spacings, represent the conclusion of the first numerical phase of our study. Results for the form factors of B meson semileptonic decays with similar statistics and at these lattice spacings and have already been finalized [1, 2].

The details of the decay constant calculation have been presented previously [4], hence, we summarize the salient features. This calculation was performed at lattice spacings of $a \approx 0.09$, 0.125 and 0.15 fm, on the eleven MILC asqtad ensembles listed in Table 1. On each ensemble, asqtad valence quarks were computed for at least eight masses generally yielding partially quenched decay constant results. Included among the results are the "full QCD" points where the light valence and sea quark masses are equal.

A separate chiral fit is done for each of the D and B systems combining results from different lattice spacings. The fit function begins with the NLO expression for $\phi = f\sqrt{M}$ from partially-quenched heavy-light staggered chiral perturbation theory [10]. We extend the function to include some next order effects by adding the NNLO analytic (quadratic in quark mass) terms. Hyperfine

source	f_{D_s}	f_{D^+}	f_{D_s}/f_{D^+}	f_{B_s}	f_B	f_{B_s}/f_B
statistics and discretization effects	2.9	3.6	1.1	2.3	2.9	1.1
chiral extrapolation	0.8	1.4	1.2	1.3	1.9	1.2
inputs r_1, m_s, m_d and m_u	0.7	0.8	0.1	0.7	0.8	0.1
input m_c or m_b	1.2	1.0	0.2	1.1	1.1	0.1
Z_V^{hh} and Z_V^{qq}	1.0	1.0	0	1.0	1.0	0
higher-order ρ_{A_4}	0.3	0.3	0.2	0.4	0.4	0.1
finite volume	0.2	0.4	0.4	0.2	0.4	0.4
total	3.5	4.2	1.7	3.1	3.9	1.7

Table 2: Uncertainties as a percentage of the decay constants and their ratio. The total combines all of the errors in quadrature.

and flavor splitting effects, formally appearing at order $1/M_H$ in heavy meson mass, are also included in the model function. Staggered chiral perturbation theory accounts for the leading light quark discretization (taste) effects. We account for leading order clover heavy quark discretization effects (in the Fermilab interpretation) with the addition of model terms (and priors) [11].

The distance scale r_1, the physical quark masses m_s, m_d and m_u and the $O(a^2)$ LEC inputs are determined by the MILC collaboration [4]. We have adopted an r_1 value from the MILC f_π determination which agree well with the recent HPQCD collaboration value shown in Figure 1. The newer r_1 values contribute to an upward shift in the recent lattice decay constant determinations, although the subsequent retuning of the charm and bottom quark masses partially compensates for this shift.

The lattice heavy-light axial current requires renormalization. The bulk of the matching to the continuum is captured nonperturbatively by relating it to the normalization (factors Z_V^{hh} and Z_V^{ll}) of the heavy-to-heavy and light-to-light vector currents. A remaining (small) correction from one, ρ_{A4}, is known to one loop in perturbation theory.

In each of Figure 2 (the D system) and Fig. 3 (the B system) we show a fit to the lattice points, plotted for each ensemble, and the subsequent simultaneous continuum and chiral extrapolation. The fit shown for the B system includes only results at lattice spacings $a = 0.09$ and $0.125\,\mathrm{fm}$. We quote central results for each of the B and D system decay constants from a fit to the two finest lattice spacings. Fits including the coarsest $0.15\,\mathrm{fm}$ results have somewhat lower confidence levels and are used in the analysis of systematic effects.

2. Results and future improvements

Our preliminary results are:

$$f_{D_s} = 261 \pm 8 \pm 5\ \mathrm{MeV} \qquad f_{B_s} = 256 \pm 6 \pm 6\ \mathrm{MeV}$$
$$f_{D^+} = 220 \pm 8 \pm 5\ \mathrm{MeV} \qquad f_B = 212 \pm 6 \pm 6\ \mathrm{MeV}$$
$$f_{D_s}/f_{D^+} = 1.19 \pm 0.01 \pm 0.02 \qquad f_{B_s}/f_B = 1.21 \pm 0.01 \pm 0.02$$

Each of the the first errors include the uncertainty from both light and heavy quark discretization effects, modeled in the fit function, as well as the statistical error. Each of the second errors combine

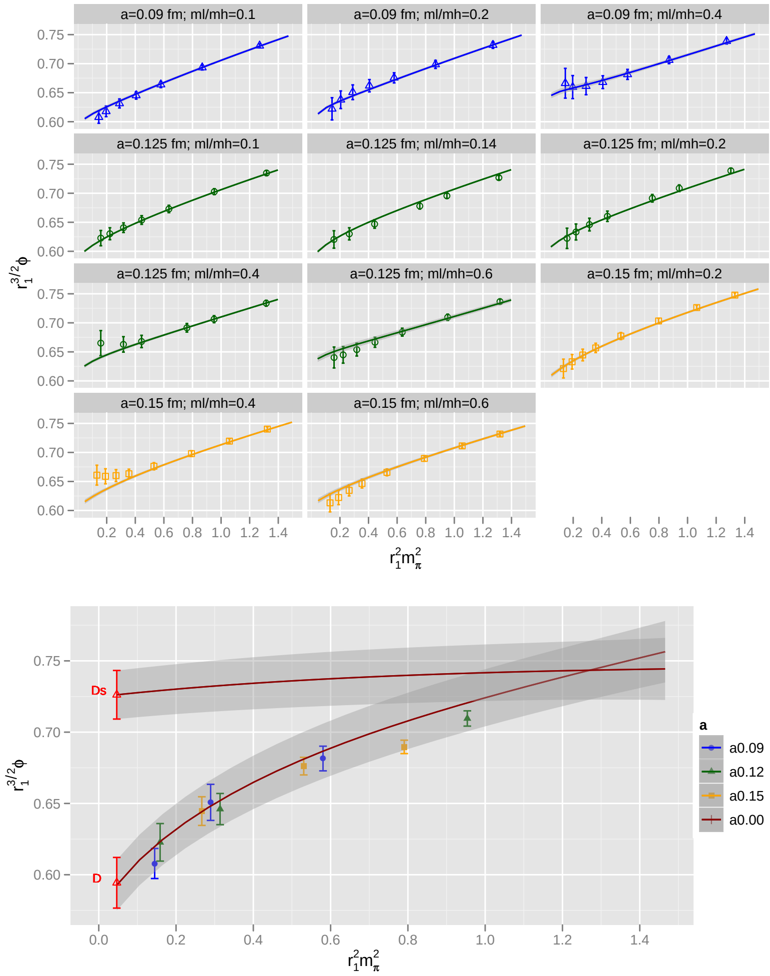

Figure 2: Example fit at three lattice spacings and extrapolations for the D system. The eleven panels at top show the fit to the lattice points from each MILC ensemble in Table 1. The larger plot below shows the extrapolations to $a \to 0$ and the physical quark masses for f_{D_s} and f_{D^+}. Only the subset of "full QCD" points (equal sea and valence quark masses) are visible in the bottom plot.

the rest of the systematic effects in quadrature. Detailed error budgets are presented in Table 2. The total uncertainty for the decay constants is about 3 to 4% while the ratios of the decay constants have an uncertainty of 1.7%.

In Figure 4 we depict total errors (in quadrature) for f_B and f_{D^+}. The colors within each stacked bar indicate the importance of each source of uncertainty. The leftmost bar ("now") depict the present uncertainties, while subsequent bars depict projected errors roughly one, two and five years hence. Errors for f_{D^+} are presently dominated by heavy quark discretization effects and the tuning of input m_c. For f_B, heavy quark discretization effects and chiral extrapolation uncertainties dominate.

Our future plans are designed to reduce both statistical errors and systematic effects in our present analysis. Within a year ("+1 Yr' ') we will have completed the analysis of three lattices spacings used in this study but with four times the statistics and better tunings of the input charm

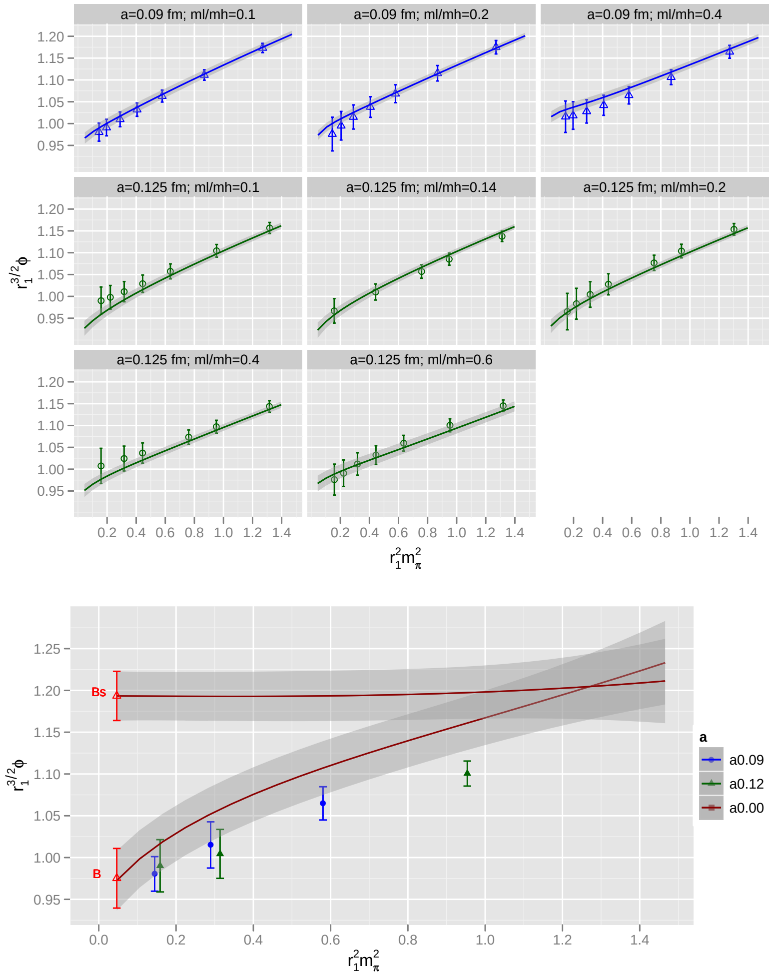

Figure 3: Example fit and extrapolation for the B system. The panels at top show the fit to the lattice points from the eight ensembles at the two finest lattice spacings in Table 1. Fits to all three lattice spacings are used to estimate systematic errors.

and bottom masses. We will also extend the analysis to a finer $a = 0.06\,\mathrm{fm}$ lattice spacing. Within two years ("+2 Yr"), we add $a = 0.045\,\mathrm{fm}$ results and results from ensemble(s) having a second value of the strange sea-quark mass. Finer lattice spacings will reduce both heavy and light quark discretization errors while better statistics and results at more combinations of the sea and valence quark masses should improve the chiral extrapolations. We are also implementing technical improvements that will reduce statistical errors for the nonperturbative current renormalization.

In the next few years ("+5 Yr"), the analysis will transition from the MILC asqtad lattices to four-flavor HISQ quark lattices now being generated [12]. Our error projections are based upon switching to HISQ charm quarks, while continuing with clover-type bottom quarks. The projection assumes simulations at five lattice spacings, comparable to the asqtad lattice spacings, and continual improvements in determining the input parameters. We also expect to have results simulated near the physical quark masses. We project a total uncertainty around 1% for both f_{D^+} and f_B.

Figure 5 compares D system results of this study to other lattice results and to recent exper-

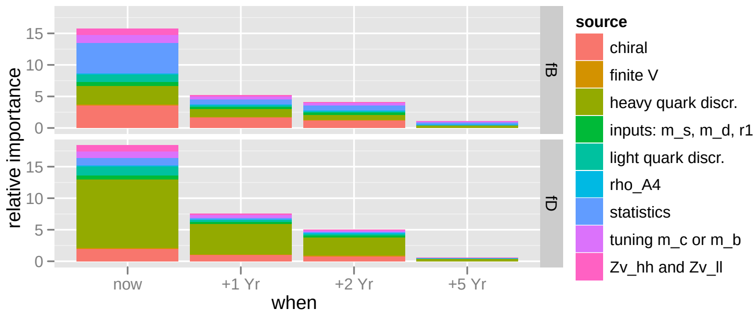

Figure 4: Current and projected errors in the next few years given anticipated improvements to the decay constant analysis. Individual colors within each stack indicate the relative importance of the sources of uncertainty when combined in quadrature. The total bar height correspond to the total percent error. Computations with asqtad quarks in the next two years will lead to errors at the 5% level. HISQ calculations in the next five years will reduce errors to the percent level.

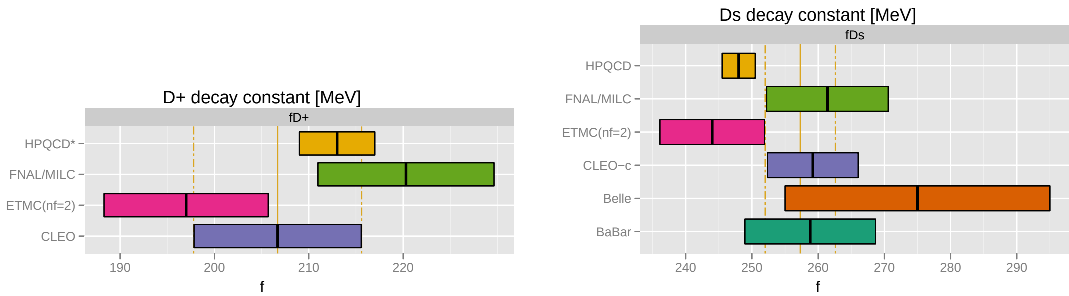

Figure 5: Comparison of lattice results and recent experiment for the D system. Experimental world averages are denoted by the gold colored vertical solid lines bounded by the dashed lines. Experimental values and averages from Refs. [13, 14]. Lattices results from this study and Refs. [15, 16]. The ETMC results are computed in two-flavor QCD neglecting effects of a dynamical strange quark.

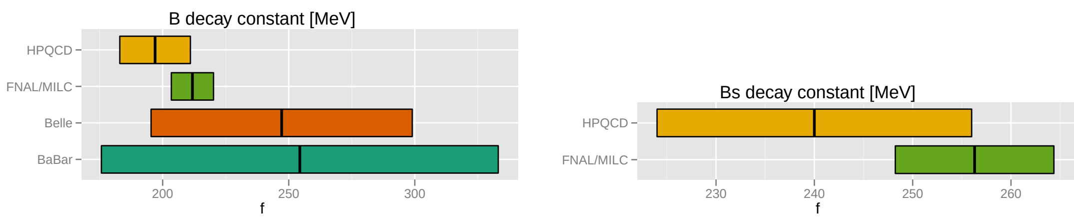

Figure 6: Comparison of lattice results and experiment. Experimental results for f_B are inferred from Ref. [14]. The HPQCD results are from Ref. [17].

imental results. Lattice three-flavor results and experiment agree well for f_{D^+}. The HPQCD f_{D_s} result and our result agree at the 1.4σ level. Figure 6 shows comparisons for the B system. For f_B experiment and lattice agree, though uncertainties are larger than in the D system. For f_{B_s} our result and the HPQCD result agree, again with large errors.

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