B-physics with dynamical domain-wall light quarks and relativistic b-quarks

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We report on our progress in calculating the B-meson decay constants and B^0-$\bar{B}^0$ mixing parameters using domain-wall light quarks and relativistic b-quarks. We present our computational method and show some preliminary results obtained on the coarser ($a \approx 0.11$fm) 24^3 lattices. This work is presented on behalf of the RBC and UKQCD collaborations.

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1. Introduction

The study of *B*-meson physics on the lattice is of special phenomenological interest because it allows one to obtain constraints on the CKM unitarity triangle. In the standard global unitarity triangle fit, the apex of the CKM unitarity triangle is constrained using lattice input for neutral *B*-meson mixing. Experimentally, B_q–$\bar{B}_q$ mixing is well measured in terms of mass differences (oscillation frequencies) Δm_q; in the standard model it is parameterized by [1]

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \tag{1.1}$$

where the index q denotes either a *d*- or a *s*-quark, m_{B_q} is the mass of the B_q-meson, and V_{tq}^* and V_{tb} are CKM matrix elements. The Inami-Lim function, S_0 [2], and the QCD coefficient, η_B [1], can be computed perturbatively, whereas the non-perturbative input is given by $f_q^2 B_{B_q}$: the leptonic decay constant f_{B_q} and the *B*-meson bag parameter B_{B_q}. Computing the ratio $\Delta m_s/\Delta m_d$ is particularly advantageous because statistical and systematic uncertainties largely cancel and, moreover, the ratio of CKM matrix elements becomes accessible

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}. \tag{1.2}$$

The non-perturbative input is now solely contained in the $SU(3)$-breaking ratio

$$\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}. \tag{1.3}$$

An alternative way of constraining the CKM triangle has been proposed by Lunghi and Soni [3] that uses the CKM matrix elements V_{ub} or V_{cb}, thereby avoiding the tension between inclusive and exclusive determinations of both V_{ub} (3σ) and V_{cb} (2σ). This alternative method, however, requires precise knowledge of the decay constant f_B as well as $BR(B \to \tau\nu)$ and Δm_s. Moreover, Lunghi and Soni point out [4, 5] that already the current data may show signs of physics beyond the standard model manifesting themselves as deviations of experimental values for $\sin(2\beta)$ (3.3σ) and $BR(B \to \tau l \nu)$ (2.8σ) from the Standard Model. They argue that the most likely sources for new physics are in B_q mixing and $\sin(2\beta)$.

Therefore, it is timely for the lattice community to determine these parameters precisely and prove by using statistically and systematically independent setups that all sources of errors are well under control. Currently, there are three determinations for ξ using 2+1-flavor gauge field configurations: an exploratory study by the RBC-UKQCD collaboration [6] with non-competitive errors and two precise determinations by Fermilab-MILC [7, 8] and HPQCD [9], which however both rely on the same set of MILC gauge field configurations. The latter two collaborations also use the same configurations to obatin the decay constants f_{B_d} and f_{B_s} and find good agreement. Here we present the first results of our project to determine *B*-meson parameters using domain-wall light quarks and relativistic *b*-quarks.

2. Computational setup

Our project is based on using the RBC-UKQCD 2+1-flavor domain wall lattices with the Iwasaki gauge action for the gluons. Our first results presented in these proceedings are obtained

on the coarser $a \approx 0.11$fm lattices with a spatial volume of 24^3 in lattice units [10], whereas the full project will also utilize the finer $a \approx 0.08$fm lattices (32^3) [11] (for details see Tab. 1).

L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.	# time sources
24	≈ 0.11	0.005	0.040	331	1640	1
24	≈ 0.11	0.010	0.040	419	1420	1
24	≈ 0.11	0.020	0.040	558	350	8
32	≈ 0.08	0.004	0.030	307	600	1
32	≈ 0.08	0.006	0.030	366	900	1
32	≈ 0.08	0.008	0.030	418	550	1

Table 1. Overview of the gauge field ensembles to be used for this project.

For the heavy quarks we use a relativistic formulation derived from the "Fermilab action" [12] in which we tune the three relevant parameters of the action non-perturbatively. As demonstraged by Christ, Li and Lin, this requires only one additional experimental input compared to the formulation of Fermilab [13]. The relativistic heavy quark (RHQ) action is given by

$$S = \sum_{n,n'} \bar{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma}\cdot\vec{D} - \frac{a\left(\vec{D}\right)^2}{2} \right] - a\sum_{\mu\nu} \frac{ic_P}{4}\sigma_{\mu\nu}F_{\mu\nu} \right\}_{n,n'} \Psi_{n'}, \quad (2.1)$$

where the covariant derivative is denoted by D, $F_{\mu\nu}$ is the field strength tensor and we need to tune the three parameters $m_0 a$, c_P and ζ. Exploratory studies on how to tune these parameters have been performed by Li and Peng [14–16]. The idea is to use experimental values for the spin averaged meson mass ($\overline{m} = (m_{B_s} + 3m_{B_s^*})/4$) and the hyperfine splitting ($\Delta_m = m_{B_s^*} - m_{B_s}$) as inputs together with the constraint from the dispersion relation that the rest mass m_1 equal the kinetic mass m_2. These three quantities are computed for a set of seven trial parameters determined by making an initial guess for $m_0 a$, c_P, and ζ and then varying it by a chosen uncertainty $\pm\sigma_{\{m_0 a, c_P, \zeta\}}$:

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}, \begin{bmatrix} m_0 a - \sigma_{m_0 a} \\ c_P \\ \zeta \end{bmatrix}, \begin{bmatrix} m_0 a + \sigma_{m_0 a} \\ c_P \\ \zeta \end{bmatrix}, \begin{bmatrix} m_0 a \\ c_P - \sigma_{c_P} \\ \zeta \end{bmatrix}, \begin{bmatrix} m_0 a \\ c_P + \sigma_{c_P} \\ \zeta \end{bmatrix}, \begin{bmatrix} m_0 a \\ c_P \\ \zeta - \sigma_\zeta \end{bmatrix}, \begin{bmatrix} m_0 a \\ c_P \\ \zeta + \sigma_\zeta \end{bmatrix} \quad (2.2)$$

(see Fig. 1). We iterate over the parameters $\{m_0 a, c_P, \zeta\}$ until we determine the values that reproduce the known experimental measurements for $\overline{m}, \Delta_m, m_1/m_2$:

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\text{RHQ}} = J^{-1} \times \left(\begin{bmatrix} \overline{m} \\ \Delta_m \\ \frac{m_1}{m_2} \end{bmatrix}^{\text{PDG}} - A \right) \quad (2.3)$$

with

$$J = \left[\frac{Y_3 - Y_2}{2\sigma_{m_0 a}}, \frac{Y_5 - Y_4}{2\sigma_{c_P}}, \frac{Y_7 - Y_6}{2\sigma_\zeta} \right] \quad \text{and} \quad A = Y_1 - J \times [m_0 a, c_P, \zeta]^t. \quad (2.4)$$

In (2.4) we use the vectors Y_i as shorthand notation for $[\overline{m}, \Delta_m, m_1/m_2]_i^t$ obtained for the input parameters as given in Eq. (2.2), labeled 1-7 from left to right. Eq. (2.3) assumes a linear dependence of the meson masses on the parameters of the action; therefore we must be in a linear regime to reliably extract $\{m_0a, c_P, \zeta\}$. We stop the iteration once the tuned values lie within our variation range.
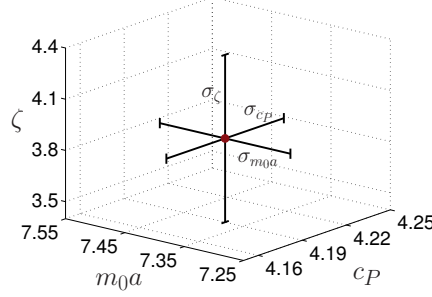
Figure 1. Initial guess for the RHQ parameters and their uncertainties.

In order to finally compute *B*-meson decay constants and mixing parameters as precisely as possible we repeated the original tuning presented at Lattice 2008 [14] with increased statistics and optimized smeared wavefunction parameters used for generating the heavy quark propagators. Moreover, performing the tuning on the same configurations we intend to use for computing weak matrix elements such as f_B will allow for an improved error analysis in which the correlations among the three RHQ parameters can be fully taken into account.

Our method for computing the $\Delta B = 2$ four-quark operators requires the light quarks to be generated with a point source and sink, but for the heavy quarks we are free to explore different smearing choices. In addition to point sources and sinks, we tried Gaussian smeared sources/sinks and also varied the radius of the Gaussian smearing. As a first guess we chose the radius of the Gaussian source to be the rms radius of the $b\bar{b}$- and $c\bar{c}$ states [17]. Later we extended the radius and found that $r_{\rm rms} = 0.634$fm gives the best signal as can be seen in Fig. 2.

Using this setup we obtain the RHQ parameters on the three 24^3 ensembles as given in Table 2. In contrast to earlier work [14] the determination uses only quantities from the heavy-light system. We expect these values to be close to our final determination. Moreover, we observe that within statistical uncertainties there is no dependence on the light sea quark mass ($m_{\rm sea}^l$).

$m_{\rm sea}^l$	m_0a	c_P	ζ
0.005	8.41(9)	5.7(2)	3.1(2)
0.010	8.4(1)	5.8(2)	3.1(1)
0.020	8.4(1)	5.6(2)	3.1(1)

Table 2. Preliminary determination of the RHQ parameters on the three 24^3 ensembles with $a \approx 0.11$fm.

3. Computation of decay constants

Using these newly determined RHQ parameters we compute as a first non-trivial test the decay
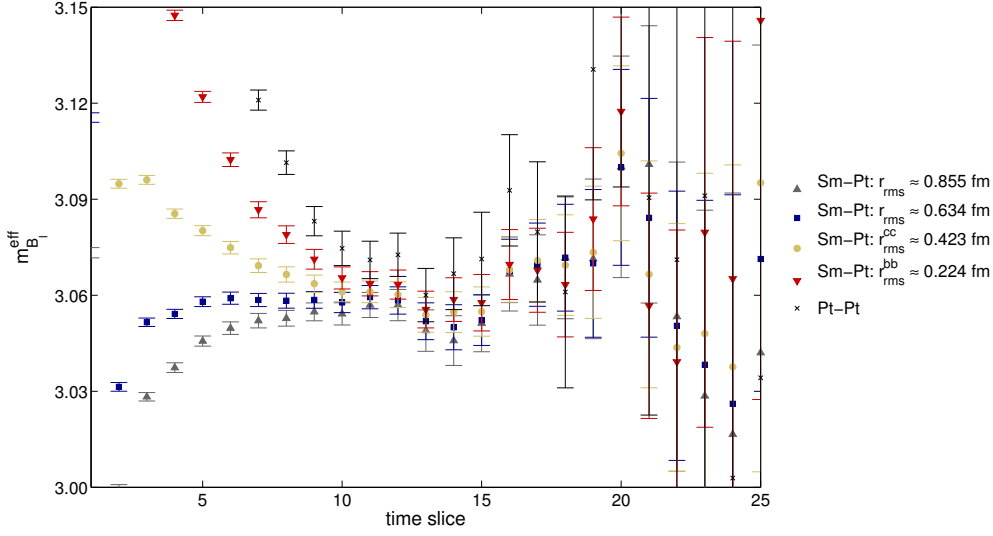
Figure 2. B_l-meson effective masses for different b-quark spatial wavefunctions; in each case the light quark has a point source.

constants of the unitary (B_l) and strange (B_s) mesons via the relation

$$f_{B_q} = Z_\Phi \, \Phi_{B_q} \, a^{-3/2} / \sqrt{m_{B_q}}, \tag{3.1}$$

where Φ_B is the lattice decay amplitude, Z_Φ is the renormalization factor. For the B_s meson (domain-wall valence quark has mass of physical s-quark) we generated data for the set of seven different RHQ input parameters. Therefore we can use similar equations to (2.3) and (2.4) in order to determine the decay amplitude Φ_{B_s} at the tuned RHQ parameters:

$$\Phi^{\text{RHQ}} = J_\Phi^{(1\times3)} \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\text{RHQ}} + A_\Phi \tag{3.2}$$

with

$$J_\Phi = \left[\frac{\Phi_3 - \Phi_2}{2\sigma_{m_0 a}}, \frac{\Phi_5 - \Phi_4}{2\sigma_{c_P}}, \frac{\Phi_7 - \Phi_6}{2\sigma_\zeta} \right] \quad \text{and} \quad A_\Phi = \Phi_1 - J_\Phi \times [m_0 a, c_P, \zeta]^t. \tag{3.3}$$

In Fig. 3 we show the results for Φ_{B_s} obtained on the ensemble with $m^l_{\text{sea}} = 0.005$. To test the linearity with respect to the input parameters $\{m_0 a, c_P, \zeta\}$ we used three different sets of seven RHQ parameters all centered around the same point. The vertical black line with the gray error band indicates the tuned value of $m_0 a$, c_P or ζ and allows for a simple estimate of the error in Φ_{B_s} due to the uncertainty in each of the three parameters. These plots show that the effect of the uncertainty in $m_0 a$ and c_P is negligible, whereas ζ contributes an error of about 1%. Moreover, we see that Φ_{B_s} depends linearly on $m_0 a$, c_P and ζ in the range of interest.

    Alternatively, one may simply use the tuned RHQ values to compute the needed correlation functions. We followed this procedure to calculate the B_l meson (domain-wall valence quark mass equals the light sea quark mass) the decay amplitude Φ_{B_l}.
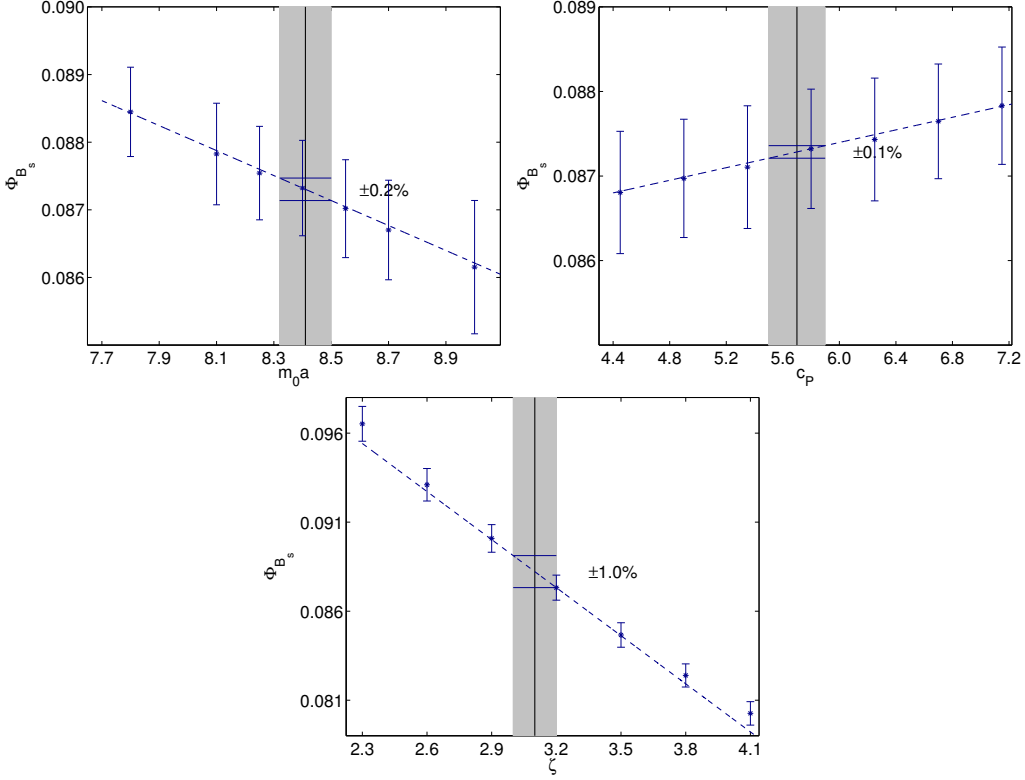
Figure 3. Dependence of Φ_{B_s} on the three RHQ parameters $\{m_0 a, c_P, \zeta\}$; results are shown for the lightest sea quark mass $m^l_{\text{sea}} = 0.005$ on the 24^3 ensembles.

Atfer renormalizing Φ_B multiplicatively at 1-loop [18] we obtain the decay constants which are given in Tab. 3 and shown in Fig. 4. As mentioned above, the values for f_{B_l} are obtained by simulating directly at the tuned RHQ parameters, whereas the f_{B_s} values are extracted using Eq. (3.2). The statistical errors on f_{B_s} are therefore larger because they take into account the statistical uncertainties in the three RHQ parameters. For a better comparison, we increase the errors in f_{B_l} in the Fig. 4 by the error due to the statistical uncertainty in the RHQ parameters estimated in Fig. 3. Moreover, we emphasize that this computation is performed without $O(a)$ improvement.

4. Conclusion

We presented our first results computing *B*-meson decay constants using domain-wall light and relativistic *b*-quarks with all parameters of the RHQ action tuned non-perturbatively. Currently, our results still need to be $\mathscr{O}(a)$ improved. For f_{B_s} we expect only a mild chiral extrapolation but of course need to perform an extrapolation to the continuum and estimate other systematic errors. Despite these caveats the small statistical errors and the fact that our central values lie in the same ballpark as results of other collaborations indicates the promise of our method.

m^l_{sea}	f_{B_l}(MeV)	f_{B_s}(Mev)
0.005	188(2)	215(3)
0.010	194(2)	214(4)
0.020	—	221(2)

Table 3. Preliminary results for the decay constants using 1-loop multiplicative renormalization, but without $O(a)$ improvement of the axial-current operator.
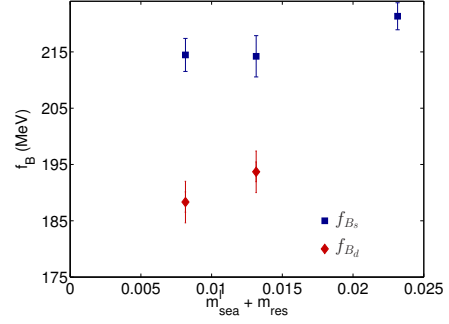
Figure 4. Preliminary decay constants f_{B_l} and f_{B_s} computed on our three 24^3 ensembles.

Acknowledgments

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