Status of QCD

Thomas Gehrmann[*]

Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich
E-mail: `thomas.gehrmann@uzh.ch`

In this talk, I review recent developments in perturbative QCD and their applications to collider physics.

---

[*]Speaker.

1. Introduction

QCD is well established as theory of the strong interaction, and its perturbation theory expansion can be used to obtain quantitative predictions for observables in high energy particle collisions. QCD effects are omnipresent in hadronic collisions, and a detailed understanding of QCD is mandatory for the interpretation of collider data, for new physics searches and for precision studies. In this talk, I review the recent progress on applications of QCD at high energy colliders.

2. Jets and Event Shapes

Hadronic jets are the final state signatures of quark or gluon production in particle collisions at high energies. As such, they are important both as tool for precision studies of QCD, and in searches for new physics effects [1]. Jets are defined through a jet algorithm (a measurement and recombination prescription to reconstruct the jet momenta from measured individual hadron momenta). Jet algorithms must fulfill infrared-safety criteria, i.e. the reconstructed jet kinematics must be insensitive to radiation of soft or collinear particles. Historically, two classes of jet algorithms were widely used at high energy colliders: cone-based algorithms and sequential recombination algorithms. Cone-based algorithms allow an intuitive understanding of the jets, and can be formulated in an infrared-safe manner [2]. Recombination algorithms are less intuitive, and their slow performance for a large number of final state particles was overcome only recently with the FastJet implementation [3]. Variants of these algorithms differ in the distance measure used to identify neighboring momenta, it turns out the the so-called anti-k_T recombination algorithm results in perfectly cone-shaped jets [4].

Much recent progress has been made recently in using jets as analysis tools. The concept of the jet catchment area [5] allows to obtain a geometrical interpretation of recombination algorithms, and to identify outside-jet regions, which can be used for underlying event studies. Aiming for the reconstruction of highly boosted massive particles, the study of jet substructure [6] has proven to be very promising. All decay products are first clustered in one fat jet, whose substructure is then resolved by lowering the resolution, resulting in a pronounced discontinuity once the particle decay is resolved. As one of the first results obtained using this procedure, the reconstruction of $t\bar{t}H$ (a reaction that could not be observed with standard cut-based methods due to the large standard model background) final states appears to become feasible [7]. Many more applications are under study.

Closely related to jet observables are event shapes, which characterize the geometrical properties of a hadronic final state. Distributions in several event shape variables were measured very extensively by LEP in view of precision studies of QCD. These results have a wide variety of applications, ranging from precision measurements of α_s, tests of resummation, study of hadronization effects, and tuning of multi-purpose Monte Carlo event generators. At hadron colliders, event shapes were only studied little up to now, and their definition is more involved due to the restricted final state region usually accessible in this environment. If defined properly, they can serve as tools for model-independent searches [8], and may be complementary to jet observables [9]. An extensive classification of event shapes at hadron colliders has been made very recently [8].

3. Multiparticle Production at NLO

The search for new physics signals at the CERN LHC will very often involve multi-particle final states, consisting of numerous jets, leptons, photons and missing energy. Quite in general, massive short-lived particles are detected through their decay signatures, as for example top quark pair production, which was first observed in final states with four jets, a lepton and missing energy.

Meaningful searches for these signatures require not only a very good anticipation of the expected signal, but also of all standard model backgrounds yielding identical final state signatures. Since leading-order calculations are affected by large uncertainties in their normalization and their kinematical dependence, it appears almost mandatory to include NLO corrections, which also allow to quantify the jet algorithm dependence, and effects of extra radiation. For a long time, these corrections were available only for at most three final state particles.

An NLO calculation of a *n*-particle observable consists of two contributions: the virtual one-loop correction to the *n*-particle production process, and the real radiation contribution from the $(n+1)$-particle production process. Both contributions are infrared divergent, and can be evaluated numerically only after extracting the infrared divergent contributions from the real radiation process. Several well-established and widely used methods exist for this task [10–17].

The evaluation of the one-loop multi-leg amplitudes poses a challenge in complexity (due to the large number of diagrams, and large number of different scales present) and stability (due to possible linear dependences among the external momenta). It has been known for long that any one-loop amplitude can be expressed as a linear combination of one-loop integrals with at most four external legs, plus a rational remainder. Enormous progress has been made in recent years in the systematic computation of the one-loop integral coefficients and rational terms. While previously established Feynman-diagram based techniques for tensor reduction and form factor decomposition were successfully extended [18, 19] to multi-leg problems, a new arsenal of techniques was emerging from the use of unitarity and multi-particle cuts [20]. Using these, the one-loop integral coefficients of an amplitude can be inferred [21–24] without evaluation of all individual diagrams. An extension of these ideas is made by performing the reduction at the integrand level in the OPP method [25, 26]. The rational coefficients can be determined in the same framework by extending the unitarity relations from four dimensions to higher-dimensional space-time [27–29].

Given the large number of different multi-particle final states of potential interest to new physics searches, an automation of NLO calculations is highly desirable. Based on existing multipurpose leading order matrix element generators, the implementation of the real radiation contributions and their infrared subtraction terms is straightforward, and has been accomplished in the Sherpa [30], MadGraph [31–33] and Helac/Phegas [34] frameworks, as well as in the form of independent libraries [35, 36], which complement already existing libraries in the MCFM [37, 38] and NLOJET++ [39] packages. The automation of the virtual corrections is a much larger challenge, which is currently being accomplished in several program packages based on the various available methods. A semi-numerical form factor decomposition is automated in the Golem package [40]. Unitarity and multi-particle cuts are used in the BlackHat package [41], and the OPP method is automated in CutTools [42]. Numerical *D*-dimensional unitarity is applied in the Rocket package [43] and the Samurai package [44]; it also forms the basis of several currently ongoing implementations [45, 46].
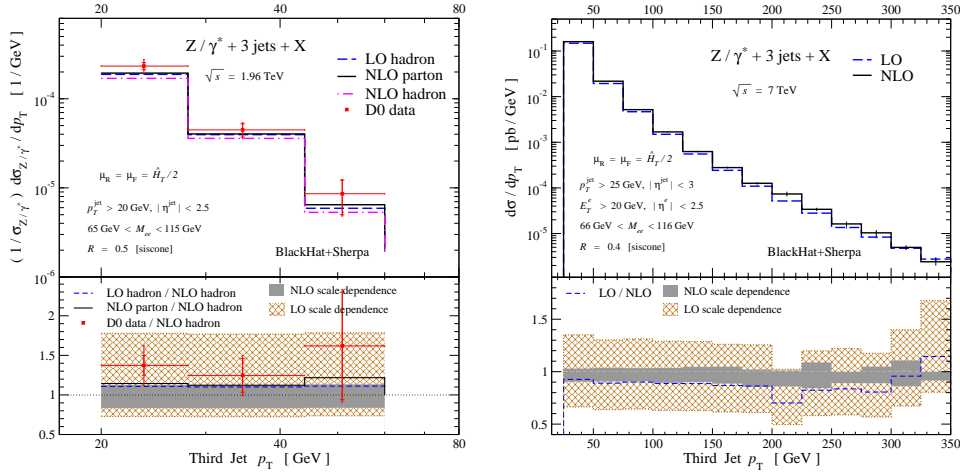
Figure 1: Transverse momentum distribution of the third-hardest jet in $Z^0 + 3j$ events at the Tevatron (left) and the LHC (right) to LO and NLO. Figure taken from [73].

Several NLO calculations of $2 \to 3$ reactions at hadron colliders were completed recently. These include the production of two vector bosons and one jet [47–52], of a Higgs boson and two jets [53–56], of $t\bar{t}Z$ [57], and of three vector bosons [58–62]. Of a similar kinematical type are vector boson fusion processes, which are computed to NLO accuracy in the VBFNLO package [63]. The current frontier of complexity are NLO calculations of $2 \to 4$ reactions. Several very important processes of this type have been computed most recently.

An important channel for Higgs boson searches, and for subsequent determinations of Yukawa couplings, is the associated production of a Higgs with a heavy quark-antiquark pair, with the Higgs boson decaying into $b\bar{b}$. The QCD background processes yielding $t\bar{t}b\bar{b}$ final states were computed recently to NLO [64–67], displaying moderate but non-constant QCD corrections, which show a non-trivial dependence on the event selection cuts. A natural extension of these calculations are $t\bar{t} + 2j$ final states [68]. Extended Higgs sectors predict a sizable rate of associated production with bottom quark pairs, and the calculation of $b\bar{b}b\bar{b}$ final states is in progress [69].

The final state signature of a vector boson and three hadronic jets is often relevant in generic new physics searches. NLO corrections of $W + 3j$ were obtained by two groups in the Rocket [70] and in the Blackhat+Sherpa [71,72] framework. The corrections to $Z^0 + 3j$ were also obtained with Blackhat+Sherpa [73]. For both observables, corrections are moderate, and stabilize the QCD prediction to the ten per cent level, required for precision phenomenology, as can be seen in Figure 1 from [73]. Knowledge of the NLO corrections to these processes allows many phenomenological studies, such as for example the stability of final state correlations [72] under perturbative corrections, and the optimal choice of scales in multi-scale processes [72–75]. A crossing of $Z^0 + 3j$ is the process $e^+e^- \to 5j$, which was measured at LEP. The NLO calculation of it is in progress.

4. Precision Observables at NNLO

Few benchmark observables (e.g. jet cross sections, vector boson production, heavy quark production) are measured experimentally to an accuracy of one per cent or below. For a theoretical

interpretation of these observables, an NLO description (which has a typical residual uncertainty around ten per cent) is insufficient: extractions of fundamental parameters from these observables would be limited by the theory uncertainty. For a meaningful interpretation of these observables, NNLO corrections are mandatory. Likewise, NNLO corrections are required for a reliable description of observables with potentially large perturbative corrections, like Higgs boson production.

The calculation of NNLO corrections to an *n*-particle final state requires three ingredients: the two-loop matrix elements for the *n*-particle production, the one-loop matrix elements for the $(n+1)$-particle production and the tree-level matrix elements for $(n+2)$-particle production. The latter two contributions develop infrared singularities if one or two particles become soft or collinear, requiring a subtraction method to extract these infrared poles, which are then combined with the virtual corrections to yield a finite prediction. The two major challenges of NNLO calculations are the two-loop matrix elements and the handling of the real radiation at NNLO. Up to now, two types of approaches to real radiation have been applied in NNLO calculations of exclusive observables. The sector decomposition method [76–78] is based on a systematic expansion in distributions, followed by numerical integration over many different small phase space sectors. Subtraction methods search to approximate the full real radiation contribution by subtraction terms in all unresolved limits; these terms are then integrated analytically. While many subtraction methods have been worked out at NLO, only two methods have so far yielded results at NNLO: the antenna subtraction method [79] for processes in e^+e^- annihilation, and the q_T-subtraction [80] for hadron collider processes in specific kinematic configurations. Alternative approaches are under intensive development [81–83]. A combination of subtraction with sector decomposition [84] may hold the potential to become a general multi-purpose method.

The dominant Higgs boson production process is gluon fusion, mediated through a top quark loop. This process has been computed (in the infinite top mass limit) to NNLO accuracy in a fully exclusive form including the Higgs boson decay, i.e. allowing for arbitrary infrared-safe final state cuts, both using sector decomposition [85–87] and using q_T-subtraction [88, 89]. These results can be directly applied to the Higgs boson search at the Tevatron, based on a neural network combination of many different kinematical distributions [90]. Finite top mass effects at NNLO were derived most recently [91–93] for the inclusive gluon fusion cross section. At this level of precision, mixed QCD and electroweak corrections [94] become equally important. The gluon fusion reaction can be mediated through loops involving any type of massive color-charged particles, thereby offering an indirect constraint on physics beyond the standard model, such as supersymmetric particles [95–99], extra heavy quark families [100] or color-octet scalars [101].

Another very promising Higgs discovery channel is vector boson fusion. The factorizable NNLO corrections to the inclusive cross section for this process are closely related to inclusive deep inelastic scattering. They were computed very recently [102], and turn out to be rather small, resulting in a high theoretical stability of the prediction. This channel can be equally sensitive on supersymmetric contributions [103].

Fully exclusive NNLO corrections to vector boson production have equally been derived using sector decomposition [104, 105] and with q_T-subtraction [106], including the leptonic vector boson decay. Observables derived from vector boson production are very important for precision studies of the electroweak interaction, and for the determination of the quark distributions in the proton. Using the newly obtained results, the NNLO corrections (and their uncertainty) to the lepton charge
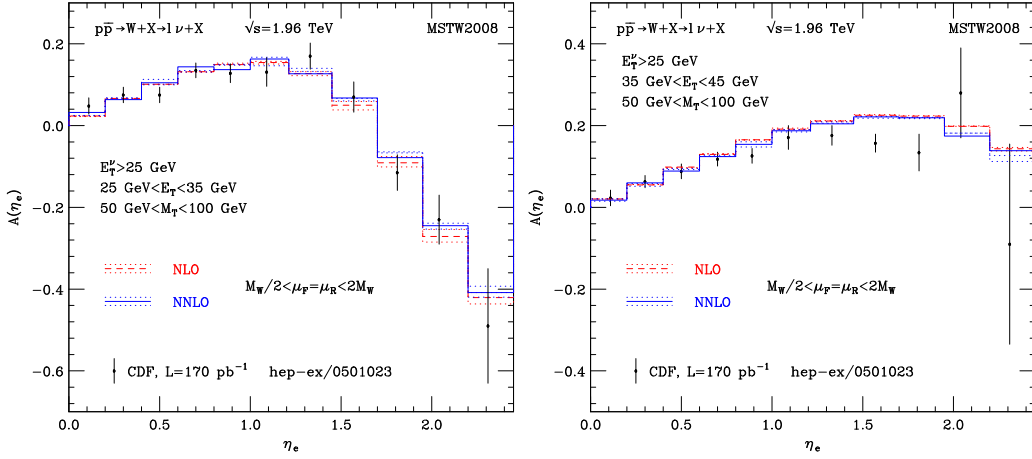
Figure 2: Lepton charge asymmetry at the Tevatron at NLO and NNLO, compared to CDF data. Figure taken from [107].

asymmetry [107] can be quantified, see Figure 2, and this observable can be consistently included into NNLO fits of parton distributions.

Jet production observables have been computed to NNLO only for e^+e^- annihilation up to now. Two implementations of the NNLO corrections to $e^+e^- \to 3j$ and related observables are available [108–115], both based on antenna subtraction. The magnitude of the NNLO corrections differs substantially between different event shape observables; including these new NNLO corrections, LEP data on event shapes and jet cross sections were reanalyzed in view of an improved determination of the strong coupling constant. In general, an improved consistency among different observables was observed. To use measurements over an extended kinematical range, resummation of large logarithmic corrections in the two-jet limit is needed. This is available to next-to-leading logarithmic accuracy (NLLA) for all shape variables [116, 117], and to N^{3}LLA for thrust [118, 119] and heavy jet mass [120] distributions. The by-now limiting factor in precision physics with event shape observables in e^+e^- annihilation is the description of the parton-to-hadron transition (hadronization), which was previously modeled from parton shower based event generators. Substantial differences are observed between the LEP-era programs and more modern generators, and to analytic approaches to hadronization, based on the shape function formalism [118–120] and on a dispersive model [121–123]. The recent determinations of the strong coupling constant from event shapes and jet cross sections at NNLO [118–120, 123–127] are summarized in Figure 3. Electroweak NLO corrections to jet observables [128–130] are potentially of the same numerical importance as NNLO QCD corrections, and could be included in future studies.

In view of the very precise jet production data from HERA [131, 132] and the Tevatron [133, 134], the derivation of NNLO corrections to jet cross sections in hadronic collisions is of high priority. The relevant two-loop matrix elements for hadronic collisions and for deep inelastic scattering [135] are known for some time already, and substantial progress is being made to extend the antenna subtraction method to include hadrons in the initial state. The proper functioning of this method on the $gg \to 4g$ subprocess to hadronic dijet production has been demonstrated [136] most recently. The integrated forms of all antenna functions have been derived for one parton in
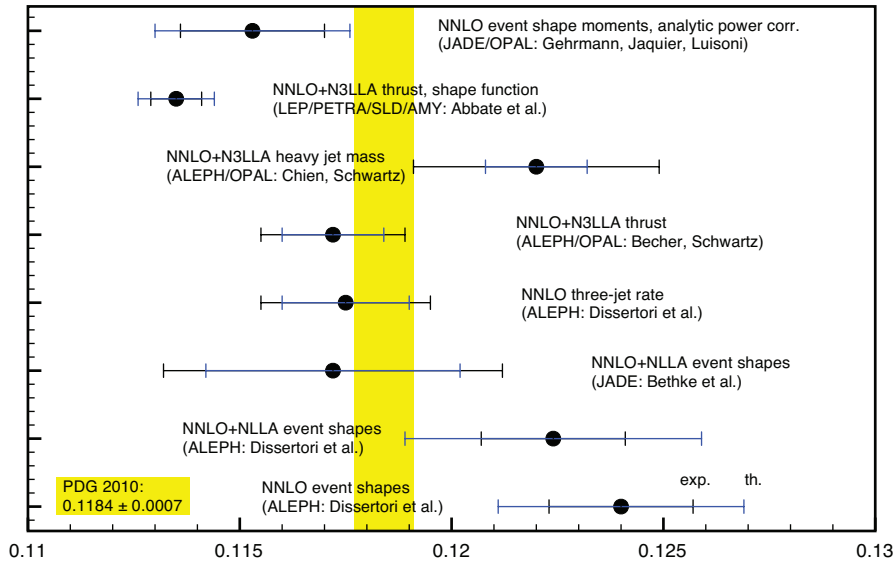
Figure 3: Determinations of α_s from event shapes and jet cross sections in e^+e^- annihilation at NNLO, compared to the Particle Data Group world average. Experimental errors are indicated in black, theoretical errors in blue.

the initial state [137], the case of two initial state partons [138] is work in progress.

The large number of top quark pairs expected to be produced at the LHC will allow for precision top quark studies, requiring NNLO accuracy on the theoretical side. The relevant two-loop matrix elements were first derived in the high energy limit [139,140]. The exact $q\bar{q} \rightarrow t\bar{t}$ matrix element is known numerically [141], substantial parts of it have been confirmed by an analytic calculation [142,143]. The one-loop self-interference contributions are equally known [144–146] The matrix elements with one and two extra partons form part of the $t\bar{t} + j$ production at NLO [147–149]. Methods to handle real radiation at NNLO in the presence of massive top quarks are currently under intensive development. Generalizing the subtraction method of [10] to NNLO and numerically integrating the relevant subtraction terms using sector decomposition [84] may provide a powerful method by combining the virtues of both approaches.

5. Parton Distributions

The parton distribution functions in the proton are a crucial ingredient in all hadron collider cross sections. They are determined (at LO, NLO or NNLO) from global fits [150] to a variety of data sets from fixed target experiments, from HERA and from the Tevatron. On the theory side, the parton distribution fits require the DGLAP splitting functions (which govern the scale evolution of the parton distributions, and are known to NNLO [151, 152]) and coefficient functions for each process considered in the global fit. These coefficient functions are known to NNLO for inclusive DIS [153], the Drell-Yan process [105] and for heavy quark production in DIS [154], but

only to NLO for jet production observables. The fit procedure must incorporate experimental and theoretical errors in a consistent manner.

Global fits of parton distributions are performed by various collaborations, with slight differences in the methodology [150]. Recent sets of parton distributions are from MSTW [155], CTEQ [156], JR [157], NNPDF [158] and ABKM [159]. A comparison of them shows that the quark distributions are known rather precisely at large x, while the gluon distribution is uncertain to within ten per cent at large x, and systematic differences between the fits exist within errors. For small values of $x < 10^{-3}$, uncertainties on the distributions become very large.

## 6. Infrared Structure and Resummation

The perturbative expansion of QCD observables in the strong coupling constant is reliable if only a single hard scale is present, it becomes problematic for observables depending on several hard scales, leading to large logarithmic corrections at all orders. In these cases, a rearrangement of the perturbative series by means of a resummation of large logarithmic corrections often appears more suitable.

Resummation of leading logarithmic corrections is accomplished by event generators [160] based on parton showers, initially based on leading order calculations. Parton showers can be combined with fixed order NLO calculations in the MC@NLO [161] or the POWHEG [162] approach. The MC@NLO event generator already covers a large number of different processes, with $W^{\pm}t$ production [163] and $H^{\pm}t$ production [164] among the most recent additions. Within POWHEG, single top production [165] and Higgs production in vector boson fusion [166] were accomplished most recently. The POWHEG box [167] provides users with a framework for implementing existing NLO calculations in this framework.

A detailed understanding of the infrared structure of QCD can be gained from the observation that infrared poles in loop amplitudes translate into large logarithms in real radiation processes and vice versa. This relation can be applied successfully in both directions: for example to predict infrared poles at two loops from resummation [168, 169] and to extract large-x resummation constants [170] from the poles of the QCD form factors. By relating the infrared poles in QCD to ultraviolet poles in soft-collinear effective theory (SCET) [171], it becomes possible to express the infrared pole structure of QCD amplitudes by a multiplicative renormalization in SCET. Based on constraints [172, 173] and symmetry arguments, it becomes possible to conjecture that the infrared pole structure of massless QCD multi-loop amplitudes is uniquely determined [172–175] by the cusp anomalous dimension and the collinear anomalous dimensions of the external particles.

The resummed description of an observable consists [176, 177] of a hard coefficient, a soft function, jet functions containing final state collinear radiation and parton distributions containing initial state collinear radiation. In SCET [171], each of these elements is identified with an operator or a non-local function. The resummation [178, 179] then proceeds by computing their anomalous dimensions and solving the respective evolution equations. First applications of SCET-based resummation are the thrust [118, 119] and heavy jet mass [120] distributions in e^+e^- annihilation, the inclusive Drell-Yan and Higgs production [178, 180] and inclusive photon production [181]. This topic is currently under fast development, any many yet open issues, like jet production and radiation off incoming partons [182, 183] are being addressed.
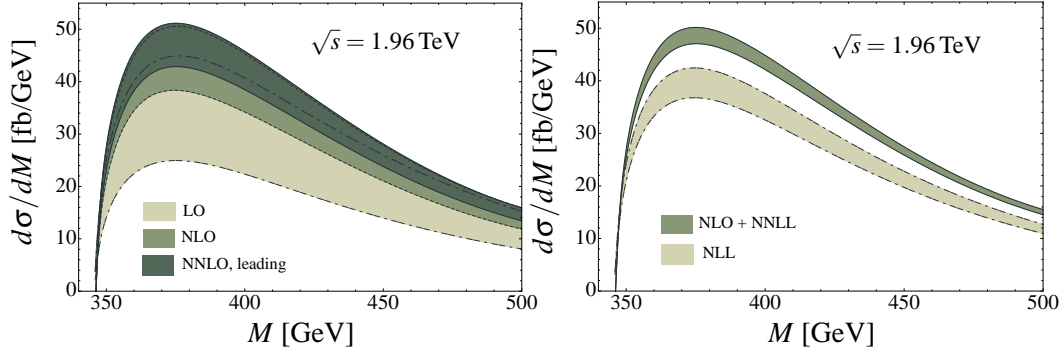
Figure 4: Top quark invariant mass distribution at the Tevatron in fixed order expansion (left) and resummation (right). Figure taken from [190].

Many of the constraints used to obtain the all-order conjecture for massless QCD amplitudes do not apply in the presence of particle masses. Consequently, the pole structure of massive amplitudes is more involved; in particular, it contains multi-particle correlations [184], which were absent in the massless case. Only recently, a prediction of the infrared poles to two-loop order has been accomplished [184, 185]. With these results, the resummation of the top quark pair production cross section to third logarithmic order (NNLL) could be completed. While dominant contributions at this order were known for some time [186, 187] the full corrections have been obtained now in two approaches: based massive soft anomalous dimensions [188, 189] and by using SCET [190]. The $t\bar{t}$ invariant mass distribution is compared in fixed order and resummed expansion in Figure 4, taken from [190]. It can be seen that the resummation has only moderate numerical impact on the central value, but results in a substantial reduction of the scale uncertainty. By expanding the resummed results to fixed order, one can in turn approximate the NNLO corrections to the top quark production cross section [186, 191, 192].

7. Multi-loop Results

Several reasons motivate the derivation of perturbative corrections even beyond NNLO. Such theoretical accuracy is required to describe very precisely measured quantities (like sum rules or total decay rates) or for processes with a very slowly converging perturbative series. Moreover, these corrections allow insight into the infrared structure of QCD at high orders, and determine resummation coefficients.

The current frontier of complexity for QCD loop amplitudes are $2 \to 4$ processes at one loop, $2 \to 2$ processes at two loops, $1 \to 2$ processes at three loops and $1 \to 1$ processes at four loops. Various innovative techniques have allowed substantial progress on multi-loop QCD calculations in recent times: a substantial reduction of complexity is achieved by reducing the large number of integrals appearing in a calculation to a small number of master integrals by exploiting linear relations among the integrals [193]. Various techniques have proven successful in the derivation of these master integrals: for example the Mellin-Barnes transformation [194, 195], differential equations [196–199] in masses and momenta, and the sector decomposition technique [76, 200]. The reduction to master integrals is usually based on a lexicographic ordering (the Laporta algorithm [201]). Implementations of this algorithm are available in several computer algebra frame-

works: the AIR package [202] in Maple, the FIRE package [203] in Mathematica, and the Reduze package [204] as a stand-alone C++ implementation based on Ginac and CLN.

The Mellin-Barnes method allows to express master integrals in a systematic manner [205] in a form suitable for analytical or numerical evaluation. Recent results obtained with this method include the massless three-loop QCD form factor integrals [206–208]. Another very successful method is the glue-and-cut technique, which exploits relations among topologically different master integrals. It recently led to the derivation of the four-loop propagator master integrals [209]. Many of these results were validated independently [210] using the sector decomposition technique.

The quark and gluon form factors are the simplest infrared-divergent objects in QCD. They allow the determination of resummation coefficients and enter benchmark reactions like Drell-Yan and Higgs production. They were recently computed to three loops [211, 212]. The static QCD potential formed by an infinitely heavy quark-antiquark pair is an important ingredient in the determination of heavy quark masses from sum rules, and in the description of top quark pair production at threshold. It was computed at three-loop accuracy most recently [213, 214].

Many important observables can be expressed as two-point functions: total decay rates, sum rules and moments of structure functions. Massless two-point functions were obtained recently at four loops. Among the results derived in this context are the hadronic R-ratio and the τ-decay rate [215], which allow some of the most accurate determinations of the strong coupling constant: $\alpha_s(M_Z)_R = 0.1190 \pm 0.0026$ and $\alpha_s(M_Z)_\tau = 0.1202 \pm 0.0019$. Most recent results [216] are the polarized Bjorken sum rule, the Adler function and the Crewther relation.

8. Conclusions

QCD is crucial for the success of the LHC physics programme in understanding signals and backgrounds, knowing parton distribution functions, and using jets and event shapes as analysis tools. Particle theory is getting ready for this challenge on many frontiers: with improved jet algorithms and event shape definitions, with an enormous progress on NLO calculations for multi-leg final states, with first NNLO results for precision observables, with an emerging understanding of the all-order structure of infrared singularities, and with landmark results at three loops and beyond.

Acknowledgments

References

[1] G. P. Salam, Eur. Phys. J. C **67** (2010) 637 [0906.1833].

[2] G. P. Salam and G. Soyez, JHEP **0705** (2007) 086 [0704.0292].

[3] M. Cacciari and G. P. Salam, Phys. Lett. B **641** (2006) 57 [hep-ph/0512210].

[4] M. Cacciari, G. P. Salam and G. Soyez, JHEP **0804** (2008) 063 [0802.1189].

[5] M. Cacciari, G. P. Salam and G. Soyez, JHEP **0804** (2008) 005 [0802.1188].

[6] J. M. Butterworth, A. R. Davison, M. Rubin and G. P. Salam, Phys. Rev. Lett. **100** (2008) 242001 [0802.2470].

[7] T. Plehn, G. P. Salam and M. Spannowsky, Phys. Rev. Lett. **104** (2010) 111801 [0910.5472].

[8] A. Banfi, G. P. Salam and G. Zanderighi, JHEP **1006** (2010) 038 [1001.4082].

[9] I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, 1004.2489.

[10] S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B **467** (1996) 399 [hep-ph/9512328].

[11] S. Catani and M. H. Seymour, Nucl. Phys. B **485** (1997) 291 [hep-ph/9605323].

[12] S. Catani, S. Dittmaier, M. H. Seymour and Z. Trocsanyi, Nucl. Phys. B **627** (2002) 189 [hep-ph/0201036].

[13] D. A. Kosower, Phys. Rev. D **57** (1998) 5410 [hep-ph/9710213].

[14] J. M. Campbell, M. A. Cullen and E. W. N. Glover, Eur. Phys. J. C **9** (1999) 245 [hep-ph/9809429].

[15] A. Daleo, T. Gehrmann and D. Maître, JHEP **0704** (2007) 016 [hep-ph/0612257].

[16] A. Gehrmann-De Ridder and M. Ritzmann, JHEP **0907** (2009) 041 [0904.3297].

[17] G. Somogyi, JHEP **0905** (2009) 016 [0903.1218].

[18] A. Denner and S. Dittmaier, Nucl. Phys. B **734** (2006) 62 [hep-ph/0509141].

[19] T. Binoth, J. P. Guillet, G. Heinrich, E. Pilon and C. Schubert, JHEP **0510** (2005) 015 [hep-ph/0504267].

[20] Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425** (1994) 217 [hep-ph/9403226].

[21] R. Britto, F. Cachazo and B. Feng, Nucl. Phys. B **725** (2005) 275 [hep-th/0412103].

[22] R. Britto, B. Feng and P. Mastrolia, Phys. Rev. D **73** (2006) 105004 [hep-ph/0602178].

[23] P. Mastrolia, Phys. Lett. B **644** (2007) 272 [hep-th/0611091].

[24] D. Forde, Phys. Rev. D **75** (2007) 125019 [0704.1835].

[25] G. Ossola, C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763** (2007) 147 [hep-ph/0609007].

[26] G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0805** (2008) 004 [0802.1876].

[27] R. K. Ellis, W. T. Giele and Z. Kunszt, JHEP **0803** (2008) 003 [0708.2398].

[28] W. T. Giele, Z. Kunszt and K. Melnikov, JHEP **0804** (2008) 049 [0801.2237].

[29] R. K. Ellis, W. T. Giele, Z. Kunszt and K. Melnikov, Nucl. Phys. B **822** (2009) 270 [0806.3467].

[30] T. Gleisberg and F. Krauss, Eur. Phys. J. C **53** (2008) 501 [0709.2881].

[31] R. Frederix, T. Gehrmann and N. Greiner, JHEP **0809** (2008) 122 [0808.2128].

[32] R. Frederix, T. Gehrmann and N. Greiner, JHEP **1006** (2010) 086 [1004.2905].

[33] R. Frederix, S. Frixione, F. Maltoni and T. Stelzer, JHEP **0910** (2009) 003 [0908.4272].

[34] M. Czakon, C. G. Papadopoulos and M. Worek, JHEP **0908** (2009) 085 [0905.0883].

[35] M. H. Seymour and C. Tevlin, 0803.2231.

[36] K. Hasegawa, S. Moch and P. Uwer, 0911.4371.

[37] J. M. Campbell and R. K. Ellis, Phys. Rev. D **60** (1999) 113006 [hep-ph/9905386].

[38] J. M. Campbell and R. K. Ellis, Phys. Rev. D **65** (2002) 113007 [hep-ph/0202176].

[39] Z. Nagy, Phys. Rev. D **68** (2003) 094002 [hep-ph/0307268].

[40] T. Binoth, J. P. Guillet, G. Heinrich, E. Pilon and T. Reiter, Comput. Phys. Commun. **180** (2009) 2317 [0810.0992].

[41] C. F. Berger *et al.*, Phys. Rev. D **78** (2008) 036003 [0803.4180].

[42] G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0803** (2008) 042 [0711.3596].

[43] W. T. Giele and G. Zanderighi, JHEP **0806** (2008) 038 [0805.2152].

[44] P. Mastrolia, G. Ossola, T. Reiter and F. Tramontano, 1006.0710.

[45] W. Giele, Z. Kunszt and J. Winter, 0911.1962.

[46] A. Lazopoulos, 0911.5241.

[47] S. Dittmaier, S. Kallweit and P. Uwer, Phys. Rev. Lett. **100** (2008) 062003 [0710.1577].

[48] S. Dittmaier, S. Kallweit and P. Uwer, Nucl. Phys. B **826** (2010) 18 [0908.4124].

[49] J. M. Campbell, R. Keith Ellis and G. Zanderighi, JHEP **0712** (2007) 056 [0710.1832].

[50] T. Binoth, T. Gleisberg, S. Karg, N. Kauer and G. Sanguinetti, Phys. Lett. B **683** (2010) 154 [0911.3181].

[51] F. Campanario, C. Englert, M. Spannowsky and D. Zeppenfeld, Europhys. Lett. **88** (2009) 11001 [0908.1638].

[52] F. Campanario, C. Englert, S. Kallweit, M. Spannowsky and D. Zeppenfeld, JHEP **1007** (2010) 076 [1006.0390].

[53] J. M. Campbell, R. K. Ellis and G. Zanderighi, JHEP **0610** (2006) 028 [hep-ph/0608194].

[54] S. Badger, E. W. Nigel Glover, P. Mastrolia and C. Williams, JHEP **1001** (2010) 036 [0909.4475].

[55] S. Badger, J. M. Campbell, R. K. Ellis and C. Williams, JHEP **0912** (2009) 035 [0910.4481].

[56] J. M. Campbell, R. K. Ellis and C. Williams, Phys. Rev. D **81** (2010) 074023 [1001.4495].

[57] A. Lazopoulos, T. McElmurry, K. Melnikov and F. Petriello, Phys. Lett. B **666** (2008) 62 [0804.2220].

[58] A. Lazopoulos, K. Melnikov and F. Petriello, Phys. Rev. D **76** (2007) 014001 [hep-ph/0703273].

[59] T. Binoth, G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0806** (2008) 082 [0804.0350].

[60] V. Hankele and D. Zeppenfeld, Phys. Lett. B **661** (2008) 103 [0712.3544].

[61] F. Campanario, V. Hankele, C. Oleari, S. Prestel and D. Zeppenfeld, Phys. Rev. D **78** (2008) 094012 [0809.0790].

[62] G. Bozzi, F. Campanario, V. Hankele and D. Zeppenfeld, Phys. Rev. D **81** (2010) 094030 [0911.0438].

[63] K. Arnold *et al.*, Comput. Phys. Commun. **180** (2009) 1661 [0811.4559].

[64] A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, JHEP **0808** (2008) 108 [0807.1248].

[65] A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, Phys. Rev. Lett. **103** (2009) 012002 [0905.0110].

[66] A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, JHEP **1003** (2010) 021 [1001.4006].

[67] G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau and M. Worek, JHEP **0909** (2009) 109 [0907.4723].

[68] G. Bevilacqua, M. Czakon, C. G. Papadopoulos and M. Worek, Phys. Rev. Lett. **104** (2010) 162002 [1002.4009].

[69] T. Binoth, N. Greiner, A. Guffanti, J. Reuter, J. P. Guillet and T. Reiter, Phys. Lett. B **685** (2010) 293 [0910.4379].

[70] R. Keith Ellis, K. Melnikov and G. Zanderighi, Phys. Rev. D **80** (2009) 094002 [0906.1445].

[71] C. F. Berger *et al.*, Phys. Rev. Lett. **102** (2009) 222001 [0902.2760].

[72] C. F. Berger *et al.*, Phys. Rev. D **80** (2009) 074036 [0907.1984].

[73] C. F. Berger *et al.*, 1004.1659.

[74] K. Melnikov and G. Zanderighi, Phys. Rev. D **81** (2010) 074025 [0910.3671].

[75] C. W. Bauer and B. O. Lange, 0905.4739.

[76] T. Binoth and G. Heinrich, Nucl. Phys. B **585** (2000) 741 [hep-ph/0004013].

[77] T. Binoth and G. Heinrich, Nucl. Phys. B **693** (2004) 134 [hep-ph/0402265].

[78] C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. D **69** (2004) 076010 [hep-ph/0311311].

[79] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, JHEP **0509** (2005) 056 [hep-ph/0505111].

[80] S. Catani and M. Grazzini, Phys. Rev. Lett. **98** (2007) 222002 [hep-ph/0703012].

[81] G. Somogyi and Z. Trocsanyi, JHEP **0808** (2008) 042 [0807.0509].

[82] U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi and Z. Trocsanyi, JHEP **0809** (2008) 107 [0807.0514].

[83] P. Bolzoni, S. O. Moch, G. Somogyi and Z. Trocsanyi, JHEP **0908** (2009) 079 [0905.4390].

[84] M. Czakon, 1005.0274.

[85] C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. Lett. **93** (2004) 262002 [hep-ph/0409088].

[86] C. Anastasiou, K. Melnikov and F. Petriello, Nucl. Phys. B **724** (2005) 197 [hep-ph/0501130].

[87] C. Anastasiou, G. Dissertori and F. Stöckli, JHEP **0709** (2007) 018 [0707.2373].

[88] M. Grazzini, JHEP **0802** (2008) 043 [0801.3232].

[89] D. de Florian and M. Grazzini, Phys. Lett. B **674** (2009) 291 [0901.2427].

[90] C. Anastasiou, G. Dissertori, M. Grazzini, F. Stockli and B. R. Webber, JHEP **0908** (2009) 099 [0905.3529].

[91] R. V. Harlander and K. J. Ozeren, JHEP **0911** (2009) 088 [0909.3420].

[92] R. V. Harlander, H. Mantler, S. Marzani and K. J. Ozeren, Eur. Phys. J. C **66** (2010) 359 [0912.2104].

[93] A. Pak, M. Rogal and M. Steinhauser, JHEP **1002** (2010) 025 [0911.4662].

13

[94] C. Anastasiou, R. Boughezal and F. Petriello, JHEP **0904** (2009) 003 [0811.3458].

[95] C. Anastasiou, S. Beerli and A. Daleo, Phys. Rev. Lett. **100** (2008) 241806 [0803.3065].

[96] R. V. Harlander and M. Steinhauser, Phys. Lett. B **574** (2003) 258 [hep-ph/0307346].

[97] R. V. Harlander and M. Steinhauser, JHEP **0409** (2004) 066 [hep-ph/0409010].

[98] G. Degrassi and P. Slavich, Nucl. Phys. B **805** (2008) 267 [0806.1495].

[99] M. Muhlleitner, H. Rzehak and M. Spira, JHEP **0904** (2009) 023 [0812.3815].

[100] C. Anastasiou, R. Boughezal and E. Furlan, JHEP **1006** (2010) 101 [1003.4677].

[101] R. Boughezal and F. Petriello, Phys. Rev. D **81** (2010) 114033 [1003.2046].

[102] P. Bolzoni, F. Maltoni, S. O. Moch and M. Zaro, Phys. Rev. Lett. **105** (2010) 011801 [1003.4451].

[103] W. Hollik, T. Plehn, M. Rauch and H. Rzehak, Phys. Rev. Lett. **102** (2009) 091802 [0804.2676].

[104] K. Melnikov and F. Petriello, Phys. Rev. Lett. **96** (2006) 231803 [hep-ph/0603182].

[105] K. Melnikov and F. Petriello, Phys. Rev. D **74** (2006) 114017 [hep-ph/0609070].

[106] S. Catani, L. Cieri, G. Ferrera, D. de Florian and M. Grazzini, Phys. Rev. Lett. **103** (2009) 082001 [0903.2120].

[107] S. Catani, G. Ferrera and M. Grazzini, JHEP **1005** (2010) 006 [1002.3115].

[108] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0711** (2007) 058 [0710.0346].

[109] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0712** (2007) 094 [0711.4711].

[110] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **100** (2008) 172001 [0802.0813].

[111] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0905** (2009) 106 [0903.4658].

[112] S. Weinzierl, Phys. Rev. Lett. **101** (2008) 162001 [0807.3241].

[113] S. Weinzierl, JHEP **0906** (2009) 041 [0904.1077].

[114] S. Weinzierl, JHEP **0907** (2009) 009 [0904.1145].

[115] S. Weinzierl, Phys. Rev. D **80** (2009) 094018 [0909.5056].

[116] S. Catani, L. Trentadue, G. Turnock and B. R. Webber, Nucl. Phys. B **407** (1993) 3.

[117] T. Gehrmann, G. Luisoni and H. Stenzel, Phys. Lett. B **664** (2008) 265 [0803.0695].

[118] T. Becher and M. D. Schwartz, JHEP **0807** (2008) 034 [0803.0342].

[119] R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu and I. W. Stewart, 1006.3080.

[120] Y. T. Chien and M. D. Schwartz, 1005.1644.

[121] Y. L. Dokshitzer, G. Marchesini and B. R. Webber, Nucl. Phys. B **469** (1996) 93 [hep-ph/9512336].

[122] R. A. Davison and B. R. Webber, Eur. Phys. J. C **59** (2009) 13 [0809.3326].

[123] T. Gehrmann, M. Jaquier and G. Luisoni, Eur. Phys. J. C **67** (2010) 57 [0911.2422].

[124] G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, G. Heinrich and H. Stenzel, JHEP **0802** (2008) 040 [0712.0327].

[125] S. Bethke, S. Kluth, C. Pahl and J. Schieck [JADE Collaboration], Eur. Phys. J. C **64** (2009) 351 [0810.1389].

[126] G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, G. Heinrich, G. Luisoni and H. Stenzel, JHEP **0908** (2009) 036 [0906.3436].

[127] G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, G. Heinrich and H. Stenzel, Phys. Rev. Lett. **104** (2010) 072002 [0910.4283].

[128] A. Denner, S. Dittmaier, T. Gehrmann and C. Kurz, Phys. Lett. B **679** (2009) 219 [0906.0372].

[129] A. Denner, S. Dittmaier, T. Gehrmann and C. Kurz, Nucl. Phys. B **836** (2010) 37 [1003.0986].

[130] A. Denner, S. Dittmaier, T. Kasprzik and A. Muck, JHEP **0908** (2009) 075 [0906.1656].

[131] M. Turcato, these proceedings.

[132] K. Krueger, these proceedings.

[133] Q. Li, these proceedings.

[134] M. Casarsa, these proceedings.

[135] T. Gehrmann and E. W. N. Glover, Phys. Lett. B **676** (2009) 146 [0904.2665].

[136] E. W. N. Glover and J. Pires, JHEP **1006** (2010) 096 [1003.2824].

[137] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, JHEP **1001** (2010) 118 [0912.0374].

[138] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, **PoS** (RADCOR2009) 052 [1001.2396].

[139] M. Czakon, A. Mitov and S. Moch, Phys. Lett. B **651** (2007) 147 [0705.1975].

[140] M. Czakon, A. Mitov and S. Moch, Nucl. Phys. B **798** (2008) 210 [0707.4139].

[141] M. Czakon, Phys. Lett. B **664** (2008) 307 [0803.1400].

[142] R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maître and C. Studerus, JHEP **0807** (2008) 129 [0806.2301].

[143] R. Bonciani, A. Ferroglia, T. Gehrmann and C. Studerus, JHEP **0908** (2009) 067 [0906.3671].

[144] J. G. Körner, Z. Merebashvili and M. Rogal, Phys. Rev. D **77** (2008) 094011 [0802.0106].

[145] C. Anastasiou and S. M. Aybat, Phys. Rev. D **78** (2008) 114006 [0809.1355].

[146] B. Kniehl, Z. Merebashvili, J. G. Körner and M. Rogal, Phys. Rev. D **78** (2008) 094013 [0809.3980].

[147] S. Dittmaier, P. Uwer and S. Weinzierl, Phys. Rev. Lett. **98** (2007) 262002 [hep-ph/0703120].

[148] S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C **59** (2009) 625 [0810.0452].

[149] K. Melnikov and M. Schulze, 1004.3284.

[150] S. Forte, these proceedings.

[151] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **688** (2004) 101 [hep-ph/0403192].

[152] A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B **691** (2004) 129 [hep-ph/0404111].

[153] E. B. Zijlstra and W. L. van Neerven, Nucl. Phys. B **383** (1992) 525.

[154] I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B **820** (2009) 417 [0904.3563].

[155] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C **63** (2009) 189 [0901.0002].

[156] J. Pumplin, J. Huston, H. L. Lai, P. M. Nadolsky, W. K. Tung and C. P. Yuan, Phys. Rev. D **80** (2009) 014019 [0904.2424].

[157] P. Jimenez-Delgado and E. Reya, Phys. Rev. D **79** (2009) 074023 [0810.4274].

[158] R. D. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. I. Latorre, J. Rojo and M. Ubiali, Nucl. Phys. B **838** (2010) 136 [1002.4407].

[159] S. Alekhin, J. Blümlein, S. Klein and S. Moch, Phys. Rev. D **81** (2010) 014032 [0908.2766].

[160] P. Richardson, these proceedings.

[161] S. Frixione and B. R. Webber, JHEP **0206** (2002) 029 [hep-ph/0204244].

[162] S. Frixione, P. Nason and C. Oleari, JHEP **0711** (2007) 070 [0709.2092].

[163] C. D. White, S. Frixione, E. Laenen and F. Maltoni, JHEP **0911** (2009) 074 [0908.0631].

[164] C. Weydert *et al.*, Eur. Phys. J. C **67** (2010) 617 [0912.3430].

[165] S. Alioli, P. Nason, C. Oleari and E. Re, JHEP **0909** (2009) 111 [0907.4076].

[166] S. Alioli, P. Nason, C. Oleari and E. Re, JHEP **0904** (2009) 002 [0812.0578].

[167] S. Alioli, P. Nason, C. Oleari and E. Re, JHEP **1006** (2010) 043 [1002.2581].

[168] S. Catani, Phys. Lett. B **427** (1998) 161 [hep-ph/9802439].

[169] G. Sterman and M. E. Tejeda-Yeomans, Phys. Lett. B **552** (2003) 48 [hep-ph/0210130].

[170] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **726** (2005) 317 [hep-ph/0506288].

[171] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63** (2001) 114020 [hep-ph/0011336].

[172] T. Becher and M. Neubert, Phys. Rev. Lett. **102** (2009) 162001 [0901.0722].

[173] E. Gardi and L. Magnea, JHEP **0903** (2009) 079 [0901.1091].

[174] T. Becher and M. Neubert, JHEP **0906** (2009) 081 [0903.1126].

[175] L. J. Dixon, E. Gardi and L. Magnea, JHEP **1002** (2010) 081 [0910.3653].

[176] G. Sterman, Nucl. Phys. B **281** (1987) 310.

[177] S. Catani and L. Trentadue, Nucl. Phys. B **327** (1989) 323.

[178] A. Idilbi, X. d. Ji and F. Yuan, Nucl. Phys. B **753** (2006) 42 [hep-ph/0605068].

[179] T. Becher, M. Neubert and B. D. Pecjak, JHEP **0701** (2007) 076 [hep-ph/0607228].

[180] V. Ahrens, T. Becher, M. Neubert and L. L. Yang, Eur. Phys. J. C **62** (2009) 333 [0809.4283].

[181] T. Becher and M. D. Schwartz, JHEP **1002** (2010) 040 [0911.0681].

[182] I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, Phys. Rev. D **81** (2010) 094035 [0910.0467].

[183] I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, 1002.2213.

[184] A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, JHEP **0911** (2009) 062 [0908.3676].

[185] M. Beneke, P. Falgari and C. Schwinn, Nucl. Phys. B **828** (2010) 69 [0907.1443].

[186] S. Moch and P. Uwer, Phys. Rev. D **78** (2008) 034003 [0804.1476].

[187] N. Kidonakis and R. Vogt, Phys. Rev. D **78** (2008) 074005 [0805.3844].

[188] M. Czakon, A. Mitov and G. Sterman, Phys. Rev. D **80** (2009) 074017 [0907.1790].

[189] M. Czakon and A. Mitov, Phys. Lett. B **680** (2009) 154 [0812.0353].

[190] V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, 1003.5827.

[191] M. Beneke, M. Czakon, P. Falgari, A. Mitov and C. Schwinn, Phys. Lett. B **690** (2010) 483 [0911.5166].

[192] M. Aliev, H. Lacker, U. Langenfeld, S. Moch, P. Uwer and M. Wiedermann, 1007.1327.

[193] K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192** (1981) 159 .

[194] V. A. Smirnov, Phys. Lett. B **460** (1999) 397 [hep-ph/9905323].

[195] J. B. Tausk, Phys. Lett. B **469** (1999) 225 [hep-ph/9909506].

[196] A. V. Kotikov, Phys. Lett. B **259** (1991) 314.

[197] A. V. Kotikov, Phys. Lett. B **267** (1991) 123.

[198] E. Remiddi, Nuovo Cim. A **110** (1997) 1435 [hep-th/9711188].

[199] T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [hep-ph/9912329].

[200] A. V. Smirnov and M. N. Tentyukov, Comput. Phys. Commun. **180** (2009) 735 [0807.4129].

[201] S. Laporta, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033].

[202] C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046 [hep-ph/0404258].

[203] A. V. Smirnov, JHEP **0810** (2008) 107 [0807.3243].

[204] C. Studerus, Comput. Phys. Commun. **181** (2010) 1293 [0912.2546].

[205] M. Czakon, Comput. Phys. Commun. **175** (2006) 559 [hep-ph/0511200].

[206] G. Heinrich, T. Huber and D. Maître, Phys. Lett. B **662** (2008) 344 [0711.3590].

[207] G. Heinrich, T. Huber, D. A. Kosower and V. A. Smirnov, Phys. Lett. B **678** (2009) 359 [0902.3512].

[208] R. N. Lee, A. V. Smirnov and V. A. Smirnov, JHEP **1004** (2010) 020 [1001.2887].

[209] P. A. Baikov and K. G. Chetyrkin, Nucl. Phys. B **837** (2010) 186 [1004.1153].

[210] A. V. Smirnov and M. Tentyukov, Nucl. Phys. B **837** (2010) 40 [1004.1149].

[211] P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **102** (2009) 212002 [0902.3519].

[212] T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli and C. Studerus, JHEP **1006** (2010) 094 [1004.3653].

[213] C. Anzai, Y. Kiyo and Y. Sumino, Phys. Rev. Lett. **104** (2010) 112003 [0911.4335].

[214] A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **104** (2010) 112002 [0911.4742].

[215] P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, Phys. Rev. Lett. **101** (2008) 012002 [0801.1821].

[216] P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, Phys. Rev. Lett. **104** (2010) 132004 [1001.3606].