Combined measurement and QCD analysis of the inclusive $e^\pm p$ scattering cross sections at HERA

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A combination is presented of the inclusive deep inelastic cross sections measured by the H1 and ZEUS Collaborations in neutral and charged current unpolarised $e^\pm p$ scattering at HERA during the period 1994-2000. The data span six orders of magnitude in negative four-momentum-transfer squared, $Q^2$, and in Bjorken $x$. The combination method used takes the correlations of systematic uncertainties into account, resulting in an improved accuracy. The combined data are the sole input in a NLO QCD analysis which determines a new set of parton distribution functions (PDFs), HERAPDF1.0, with small experimental uncertainties. This set includes an estimate of the model and parametrisation uncertainties of the fit result.

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†The results presented here is based on a joint publication by the H1 and ZEUS collaborations[1] which should be referred to for further details.
1. The HERA collider and the H1 and ZEUS detectors

HERA provided the laboratory for the study of deep inelastic electron-proton\textsuperscript{1} scattering. It operated during two running periods – HERA I (1992-2000) and HERA II (2002-2007) – at center of mass energies up to $\sqrt{s} \approx 320$ GeV corresponding to an electron beam energy of 27.5 GeV and a proton beam energy of 920 GeV. During the final stages of operation, the collider was run at reduced proton beam energies of 460 and 575 GeV in order to make a direct measurement of the longitudinal structure function $F_L$.

The H1 and ZEUS colliding-beam experiments observed both neutral current (NC) and charged current (CC) interactions with detectors capable of covering almost $4\pi$ in solid angle. Though the H1 and ZEUS experiments had similar physics objectives, their respective detectors reflected differences in technical solutions both for tracking and calorimetric measurements.

The results in this paper are based on NC and CC unpolarised inclusive cross section measurements\cite{1} using data collected during the HERA I phase of running. The data corresponds to an integrated luminosity per experiment of approximately $100 \text{ pb}^{-1}$ for $e^+ p$ collisions and $15 \text{ pb}^{-1}$ for $e^- p$.

2. Combining the H1 and ZEUS measurements

Published NC and CC measurements from H1 and ZEUS are combined using an averaging procedure described in \cite{2}-\cite{3}. The only assumption adopted in the procedure is that an H1 measurement and a ZEUS measurement are observations of the one true cross section.

The procedure takes into account correlations between measurements due to the various systematic sources of uncertainty. Since H1 and ZEUS have different detectors and so employ different analysis techniques, their respective cross sections show different sensitivities to the systematic sources. Getting the H1 and ZEUS cross sections to fit to one another thus provides a demanding constraint which helps reduce the uncertainty of the combined measurement. The quality of the fit also provides a model-independent check of the consistency of the measurements.

Point-to-point correlations between the measurements of a given data set as well as between data sets of the same experiment are taken into account in the averaging. Apart from the 0.5% uncertainty inherent to both experiments arising from the higher order corrections to the Bethe-Heitler process used for the luminosity calculation, no other systematic sources of uncertainty are assumed to be correlated between H1 and ZEUS in determining the average. Changes in the average due to relaxing this assumption only become significant when treating the photoproduction background estimation and hadronic energy scale as correlated between experiments, and are included as procedural uncertainties of the nominal average; they can be a few percent.

A total of 1402 individual data points are combined to 741 cross section measurements while taking into account 110 sources of correlated systematic uncertainty; the $\chi^2$ per degree of freedom is 636.5/656. The distribution of pulls $p$ which is a measure of the level of agreement between the input data point and the averaged point taking into account uncorrelated errors, is shown in fig.1(a) to (c) for the NC $e^+ p$ process in different kinematic regions; no tension can be seen in the pulls for this and other processes across the kinematic plane. The distribution of the pulls $p_s$

\textsuperscript{1}In this paper “electron” refers both to electrons ($e^-$) and positrons ($e^+$) unless otherwise stated.
for the correlated systematic sources is shown in fig.1(d). The $\chi^2$ per degree of freedom and pull distributions point to a good level of consistency of the H1 and ZEUS data.

The contribution to the uncertainty of several correlated systematic sources are reduced significantly by the combination. For example the uncertainty due to the H1 central calorimeter energy scale is reduced by 55% while that of the ZEUS photoproduction background is reduced by 65%. In regions where one experiment is more precise than the other, there is a reduction of uncertainty as the less precise is fitted to the more precise measurement. This gain in precision is also propagated to regions where the sole input to the averaging is due to the less precise experiment. Fig.2(a) shows the HERA combined NC $e^+ p$ reduced cross section as a function of $Q^2$ for six $x$-bins compared to the input cross sections used in the fit; the corresponding plot for the CC $e^+ p$ reduced cross section is shown in fig.2(b). The total uncertainty of the combined measurement for the NC$e^+ p$ sample is typically smaller than 2% for the range $3 < Q^2 < 500$ GeV$^2$, and reaches 1% for $20 < Q^2 < 100$ GeV$^2$.

3. QCD analysis of the combined data – HERAPDF1.0 determination

The combined data presented in the previous section is a consistent data set comprising NC and
CC $e^-p$ and $e^+p$ measurements thus allowing the extraction of the valence quark, sea quark and gluon distributions; it is used as the sole input for a next-to-leading order (NLO) QCD fit to extract the HERAPDF1.0 PDFs. The QCD predictions for the structure functions are obtained by solving the DGLAP evolution equations [4]-[8] at NLO in the $\overline{MS}$ scheme with the renormalization and factorization scales chosen to be $Q^2$. The PDFs are parametrised at the starting scale $Q^2_0 = 1.9$ GeV$^2$ and evolved to all values of $Q^2$ using the DGLAP equations. The heavy quark coefficient functions are calculated in the general-mass variable-flavour-number scheme of [9] with modifications [1]. The mass of the charm and bottom quarks are chosen to be 1.4 and 4.75 GeV respectively. The strong coupling constant is fixed to $\alpha_s(M_Z^2) = 0.1176$. A minimum $Q^2$ cut of $Q^2_{\text{min}} = 3.5$ GeV$^2$ is imposed on the input data limiting it to regions where perturbative QCD should be applicable.

At the starting scale $Q^2_0$ the generic form of the parametrisation of the PDFs is given by:

$$x f(x) = A x^B (1-x)^C (1 + \epsilon \sqrt{x} + Dx + Ex^2) \tag{3.1}$$

where $A$, $B$, $C$, $\epsilon$, $D$ and $E$ are the parameters to be determined from the fit. The PDFs parametrised are the gluon $xg$, valence quarks $xu$, and $xd$, and the $u$-type and $d$-type anti-quark distributions, $x\overline{u}$ and $x\overline{d}$. Here $x\overline{u} = x\overline{u}$ and $x\overline{d} = x\overline{d} + x\overline{s}$ at the starting scale. It is found that the fit having parameters $A$, $B$, $C$ (for all partons parametrised) and $E$ (for the $u_+$ distribution only) is optimum, as introducing $\epsilon$, $D$ and $E$ (for distributions other than $u_+$) as extra parameters brings no improvement to the quality of the fit. With the quark number and momentum sum rules and the additional constraints $B_{u_+} = B_{d_+}$, $B_{u_0} = B_{d_0}$, and $A_{u_0} = A_{d_0}(1 - f_s)$ where $f_s$ is defined by $x\overline{s} = f_s x\overline{T}$ at the starting scale and takes the value 0.31[1]; the optimal parametrisation has 10 free parameters and used to determine the central fit.

The uncertainties on the PDFs are due to experimental, model and parametrisation sources, whose contributions are added in quadrature to determine the total PDF uncertainty. Since the data set is consistent, the conventional $\chi^2$ tolerance of $\Delta\chi^2 = 1$ is enforced when determining the experimental contribution to the PDF uncertainty. Model uncertainties are found by varying the numerical values of the charm quark mass, bottom quark mass, $f_s$ and $Q_{\text{min}}$. Parametrisation uncertainties are obtained by varying $Q^2_0$ allowing for a negative gluon distribution at low $x$ for the case where $Q^2_0$ is set to its lower limit of 1.5 GeV$^2$, relaxing the constraint $B_{u_0} = B_{d_0}$ and considering all possible 11 parameter fits by introducing further $\epsilon$, $D$ and $E$ coefficients to the central fit.

The $\chi^2$ per degree of freedom for the HERAPDF1.0 fit is 574/582. The obtained fit describes well the input cross sections as well as cross sections from fixed target experiments[1]. Fig.3(a) shows the valence, sea and gluon distributions at the starting scale $Q^2 = 1.9$ GeV$^2$ together with the flavour decomposition of the sea. Fractional uncertainty bands are also included highlighting the various contributions. A similar plot for $Q^2 = 10000$ GeV$^2$, the relevant scale for the production of a 100 GeV particle at the LHC say is shown in fig.3(b) where the uncertainty on the gluon is 2% for $x < 0.01$; the sea distributions show similar uncertainties. Such uncertainties on the HERAPDF1.0 PDFs lead to 5% and better uncertainty on predicted $W^\pm$ and $Z$ cross sections at the LHC.

4. Summary

Inclusive NC and CC cross sections from the H1 and ZEUS collaborations have been combined. The combined data set shows significant improvements in precision compared to the indi-
Experimental uncertainties on the PDFs are small. The HERAPDF1.0 PDFs have a total uncertainty of 1% in the best measured region: NC scattering, $20 < Q^2 < 100$ GeV$^2$. A NLO QCD analysis has been performed based exclusively on the combined data resulting in a new set of parton distribution functions – HERAPDF1.0. The resulting uncertainties on the PDFs are small. The HERAPDF1.0 PDFs have a total uncertainty at the level of a few percent at low $x$ due to the precision of the combined data set. A further improvement in precision is expected as the inclusion of HERA II inclusive data (including the 460/575 GeV proton energy measurements) as well as heavy flavour and jet cross sections is forthcoming.

**Figure 3:** The PDFs from HERAPDF1.0: $xu_s$, $xd_s$, $xS = 2x(U + D)$ and $xg$ at the starting scale $Q^2 = 1.9$ GeV$^2$ (a) and $Q^2 = 10000$ GeV$^2$ (b). The break-up of the Sea PDF, $xS$, into the flavours, $xu_{sea}$, $xd_{sea}$, $xs_{sea}$ $xc_{sea}$ and $xb_{sea}$ is illustrated. Fractional uncertainty bands are shown below each PDF. The experimental, model and parametrisation uncertainties are shown separately.

**References**