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Threshold resummation beyond leading eikonal level

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The modified evolution equation for parton distributions of Dokshitzer, Marchesini and Salam is extended to non-singlet Deep Inelastic Scattering coefficient functions and the physical evolution kernels which govern their scaling violation. Considering the $x \to 1$ limit, it is found that the leading next-to-eikonal logarithmic contributions to the momentum space physical kernels at any loop order can be expressed in term of the one loop cusp anomalous dimension, a result which can presumably be extended to all orders in $(1-x)$. Similar results hold for fragmentation functions in semi-inclusive e^+e^- annihilation. The method does not work for subleading next-to-eikonal logarithms, but, in the special case of the F_1 and F_T structure and fragmentation functions, there are hints of the possible existence of an underlying Gribov-Lipatov like relation.

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1. Threshold resummation in physical evolution kernels

Consider a generic deep inelastic scattering (DIS) *non-singlet* structure function $\mathscr{F}(x,Q^2) = \{2F_1(x,Q^2), F_2(x,Q^2)/x\}$ at large $Q^2 >> \Lambda^2$. We shall be interested in the elastic limit $x \to 1$ where the final state mass $W^2 \sim (1-x)Q^2 << Q^2$. In this limit, large threshold $\ln(1-x)$ logarithms appear. Their resummation is by now standard [1, 2], but usually performed in moment space. However, the result can also be expressed *analytically in momentum space* at the level of so-called "physical evolution kernels" which account for the *physical* scaling violation:

$$\frac{\partial \mathscr{F}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} K(z, a_s(Q^2))\, \mathscr{F}(x/z, Q^2) \equiv K \otimes \mathscr{F} \ , \tag{1.1}$$

where the "physical evolution kernel" $K(x,a_s)$ ($a_s = \alpha_s/4\pi$ is the $\overline{MS}$ coupling) embodies *all* the *perturbative* information about $\mathscr{F}$. For $x \to 1$ threshold resummation yields [3]:

$$K(x, a_s(Q^2)) \sim \left[\frac{\mathscr{J}\left((1-x)Q^2\right)}{1-x}\right]_+ + B_\delta^{DIS}(a_s(Q^2))\, \delta(1-x) \ , \tag{1.2}$$

where $\mathscr{J}(Q^2)$ is a "physical anomalous dimension" (a renormalization scheme invariant quantity), related to the standard "cusp" $A(a_s) = \sum_{i=1}^{\infty} A_i a_s^i$ and final state "jet function" $B(a_s) = \sum_{i=1}^{\infty} B_i a_s^i$ anomalous dimensions by:

$$\mathscr{J}(Q^2) = A\left(a_s(Q^2)\right) + \beta\left(a_s(Q^2)\right) \frac{dB\left(a_s(Q^2)\right)}{da_s} \equiv \sum_{i=1}^{\infty} j_i a_s^i(Q^2) \ . \tag{1.3}$$

The renormalization group invariance of $\mathscr{J}(Q^2)$ yields the standard relation:

$$\begin{aligned} \mathscr{J}\left((1-x)Q^2\right) &= j_1\, a_s + a_s^2[-j_1\beta_0 L_x + j_2] \\ &+ a_s^3[j_1\beta_0^2 L_x^2 - (j_1\beta_1 + 2j_2\beta_0)L_x + j_3] + \mathscr{O}(a_s^4) \ , \end{aligned} \tag{1.4}$$

where $L_x \equiv \ln(1-x)$ and $a_s = a_s(Q^2)$, from which the structure of *all* the eikonal logarithms in $K(x, a_s(Q^2))$, which can be absorbed into the *single* scale $(1-x)Q^2$, can thus be derived.

However, no analogous result holds at the next-to-eikonal level (except [4] at large-β_0). Indeed, expanding

$$K(x,a_s) = K_0(x)a_s + K_1(x)a_s^2 + K_2(x)a_s^3 + \mathscr{O}(a_s^4) \ , \tag{1.5}$$

the K_i's can be determined as combinations of splitting and coefficient functions. One gets:

$$K_0(x) = P_0(x) = k_{10}\ p_{qq}(x) + \Delta_1\, \delta(1-x) \ , \tag{1.6}$$

with $k_{10} = A_1$ and $p_{qq}(x) = \frac{x}{1-x} + \frac{1}{2}(1-x)$. Moreover for $x \to 1$ one finds [5, 6], barring delta function contributions:

$$\begin{aligned} K_1(x) &= \frac{x}{1-x}(k_{21}L_x + k_{20}) + (h_{21}L_x + h_{20}) + \mathscr{O}((1-x)L_x) \\ K_2(x) &= \frac{x}{1-x}(k_{32}L_x^2 + k_{31}L_x + k_{30}) + (h_{32}L_x^2 + h_{31}L_x + h_{30}) + \mathscr{O}((1-x)L_x^2) \ . \end{aligned} \tag{1.7}$$

Despite the similar logarithmic structure, the *next-to-eikonal* logarithms h_{ij} *cannot* [5] be obtained from a standard renormalization group resummation analogous to the one used (eq.(1.4)) for the *eikonal* logarithms k_{ij}.

2. An alternative approach: the modified physical kernel

Instead, consider [7] a modified physical evolution equation, similar to the one used in [8] (see also [9]) for parton distributions:

$$\frac{\partial \mathscr{F}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} K(z,a_s(Q^2),\lambda)\,\mathscr{F}(x/z,Q^2/z^\lambda)\,, \tag{2.1}$$

where the arbitrary parameter λ shall be set to 1 at the end. Expanding $\mathscr{F}(y,Q^2/z^\lambda)$ around $z=1$, one can relate $K(x,a_s,\lambda)$ to $K(x,a_s)$:

$$K(x,a_s,\lambda) = K(x,a_s) + \lambda[\ln x\, K(x,a_s,\lambda)] \otimes K(x,a_s) + \ldots\,. \tag{2.2}$$

Solving perturbatively, one finds that for $x \to 1$ the corresponding expansion coefficients $K_i(x,\lambda)$ satisfy the analogue of eq.(1.7), with the *same* coefficients k_{ji}'s of the eikonal logarithms, but with the coefficients of the *leading* next-to-eikonal logarithms given by:

$$\begin{aligned} h_{21}(\lambda) &= h_{21} - \lambda k_{10}^2 \\ h_{32}(\lambda) &= h_{32} - \lambda \frac{3}{2} k_{21} k_{10}\,. \end{aligned} \tag{2.3}$$

Setting now $\lambda = 1$, one observes that both $h_{21}(\lambda=1)$ and $h_{32}(\lambda=1)$ *vanish*, which means that $h_{21} = k_{10}^2 = A_1^2 = 16C_F^2$ and $h_{32} = \frac{3}{2}k_{21}k_{10} = -\frac{3}{2}\beta_0 A_1^2 = -24\beta_0 C_F^2$, which agree with the exact results in [5, 6]. It should be stressed that, whereas h_{21} is contributed by the two loop splitting function alone (and thus one simply recovers in this case the result of [8]), h_{32} is instead contributed *only* by the one and two loop coefficient functions, which represents a new result. Similar results are obtained for the coefficients h_{ji} ($j=i+1$) of the *leading* next-to-eikonal logarithms at any loop order, which can all be expressed in term of the one loop cusp anomalous dimension *assuming* the corresponding $h_{ji}(\lambda)$ vanish for $\lambda = 1$. In particular, one predicts $h_{43} = \frac{4}{3}k_{10}k_{32} + \frac{1}{2}k_{21}^2 = \frac{11}{6}\beta_0^2 A_1^2 = \frac{88}{3}\beta_0^2 C_F^2$, which is correct [5, 6], and $h_{54} = \frac{5}{4}k_{10}k_{43} + \frac{5}{6}k_{21}k_{32} = -\frac{25}{12}\beta_0^3 A_1^2 = -\frac{100}{3}\beta_0^3 C_F^2$, which remains to be checked.

Similar results are obtained for the coefficients f_{ji} ($j=i+1$) of the *leading* next-to-next-to eikonal logarithms, defined by:

$$K_i(x)|_{\rm LL} = L_x^i[p_{qq}(x)\, k_{ji} + h_{ji} + (1-x) f_{ji} + \mathscr{O}((1-x)^2)]\,, \tag{2.4}$$

where the *full* one loop prefactor $p_{qq}(x)$ should be used in the leading term to define the f_{ji}'s. The corresponding $f_{ji}(\lambda)$ coefficients in $K_i(x,\lambda)$ are given by:

$$f_{21}(\lambda) = f_{21} + \lambda \frac{1}{2} k_{10}^2 \tag{2.5}$$

$$f_{32}(\lambda) = f_{32} - \lambda(-\frac{3}{4}k_{10}k_{21} + k_{10}h_{21}) + \lambda^2\frac{1}{2}k_{10}^3$$
$$f_{43}(\lambda) = f_{43} - \lambda\Big(-\frac{2}{3}k_{10}k_{32} + \frac{1}{2}(h_{21} - \frac{1}{2}k_{21})k_{21} + k_{10}h_{32}\Big) + \lambda^2 k_{10}^2 k_{21} \ ,$$

where one notes the presence of contributions *quadratic* in λ. Assuming the $f_{ji}(\lambda)$'s vanish for $\lambda = 1$, the resulting predictions for the f_{ji}'s ($j = i+1$) are again found to agree with the exact results of [6].

3. Fragmentation functions

Similar results hold for physical evolution kernels associated to fragmentation functions in semi-inclusive e^+e^- annihilation (SIA), provided one sets $\lambda = -1$ in the modified evolution equation:

$$\frac{\partial \mathscr{F}_{SIA}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} K_{SIA}(z, a_s(Q^2), \lambda)\, \mathscr{F}_{SIA}(x/z, Q^2/z^\lambda) \ , \tag{3.1}$$

where $\mathscr{F}_{SIA} = \{\mathscr{F}_T, \mathscr{F}_{T+L}\}$ denotes a generic *non-singlet* fragmentation function (I use the notation of [6]). At the *leading* eikonal level, threshold resummation [10] can be summarized in the standard SIA physical evolution kernel by:

$$K_{SIA}(x, a_s(Q^2)) \sim \left[\frac{\mathscr{J}\left((1-x)Q^2\right)}{1-x}\right]_+ + B_\delta^{SIA}(a_s(Q^2))\, \delta(1-x) \ , \tag{3.2}$$

where the "physical anomalous dimension" $\mathscr{J}(Q^2)$ (hence the k_{ji}'s) are the *same* for DIS and SIA, as follows from the results in [11]. Assuming the *leading* threshold logarithms *vanish* beyond the leading eikonal level in the *modified* SIA evolution kernel for $\lambda = -1$, and setting $\lambda = -1$ in eq.(2.3) and (2.5), one derives predictions for h_{ji}^{SIA} and f_{ji}^{SIA} ($j = i+1$) which again agree with the exact results of [6]. In particular, one finds that $h_{ji}^{SIA} = -h_{ji}$.

4. Subleading next-to-eikonal logarithms

The previous approach *does not* work for *subleading* next-to-eikonal logarithms, namely the latter do not vanish in the modified physical evolution kernels for $\lambda = \pm 1$. The following facts are nevertheless worth quoting:

- At *large* β_0, we have a generalization [4] of the leading eikonal *single scale* ansatz (which takes care of *all* subleading logarithms) to *any* eikonal order:

$$K(x,Q^2)\big|_{\text{large } \beta_0} = \left[\frac{x}{1-x}\,\mathscr{J}(W^2)\big|_{\text{large } \beta_0}\right]_+ + (\delta(1-x) term) + \mathscr{J}_0(W^2)\big|_{\text{large } \beta_0} + (1-x)\ \mathscr{J}_1(W^2)\big|_{\text{large } \beta_0} + \dots \tag{4.1}$$

  where $W^2 = (1-x)Q^2$, and the $\mathscr{J}_i$'s (*except* the leading eikonal one) are structure function dependent . A similar result holds for $K_{SIA}(x,Q^2)\big|_{\text{large } \beta_0}$.

- There are remarkable relations between the *momentum space next-to-leading* threshold logarithms of the (DIS) F_1 and the corresponding (SIA) F_T transverse fragmentation function physical evolution kernels at the *next-to-eikonal* level. Namely, using the moment space results of [6], one can derive the following *momentum space* relations:

  1) At two loop for the $\mathcal{O}(L_x^0)$ next-to-eikonal constant term:

$$\begin{aligned} h_{20}^{(F_1)} &= h_{20}^{(F_1)}\Big|_{\text{large }\beta_0} + \Delta h_{20} \\ h_{20}^{(F_T)} &= h_{20}^{(F_T)}\Big|_{\text{large }\beta_0} - \Delta h_{20} \ , \end{aligned} \tag{4.2}$$

  with $h_{20}^{(F_1)}\Big|_{\text{large }\beta_0} = -11\beta_0 C_F$, $h_{20}^{(F_T)}\Big|_{\text{large }\beta_0} = 7\beta_0 C_F$, and $\Delta h_{20} = A_1\Delta_1 = 12C_F^2$.

  2) At three loop for the *single* $\mathcal{O}(L_x)$ next-to-eikonal logarithms:

$$\begin{aligned} h_{31}^{(F_1)} &= h_{31}^{(F_1)}\Big|_{\text{large }\beta_0} + \Delta h_{31} \\ h_{31}^{(F_T)} &= h_{31}^{(F_T)}\Big|_{\text{large }\beta_0} - \Delta h_{31} \ , \end{aligned} \tag{4.3}$$

  with $h_{31}^{(F_1)}\Big|_{\text{large }\beta_0} = -2\beta_0\ h_{20}^{(F_1)}\Big|_{\text{large }\beta_0} = 22C_F\beta_0^2$, $h_{31}^{(F_T)}\Big|_{\text{large }\beta_0} = -2\beta_0\ h_{20}^{(F_T)}\Big|_{\text{large }\beta_0} = -14C_F\beta_0^2$, and:

$$\Delta h_{31} = 2A_1A_2 - 20\beta_0 C_F C_A + 20\beta_0 C_F^2 \ . \tag{4.4}$$

  3) At four loop for the *double* $\mathcal{O}(L_x^2)$ next-to-eikonal logarithms:

$$\begin{aligned} h_{42}^{(F_1)} &= h_{42}^{(F_1)}\Big|_{\text{large }\beta_0} + \Delta h_{42} \\ h_{42}^{(F_T)} &= h_{42}^{(F_T)}\Big|_{\text{large }\beta_0} - \Delta h_{42} \ , \end{aligned} \tag{4.5}$$

  with $h_{42}^{(F_1)}\Big|_{\text{large }\beta_0} = 3\beta_0^2\ h_{20}^{(F_1)}\Big|_{\text{large }\beta_0} = -33C_F\beta_0^3$, $h_{42}^{(F_T)}\Big|_{\text{large }\beta_0} = 3\beta_0^2\ h_{20}^{(F_T)}\Big|_{\text{large }\beta_0} = 21C_F\beta_0^3$, and:

$$\Delta h_{42} = -24\beta_1 C_F^2 + 45\beta_0^2 C_F C_A - 178\beta_0^2 C_F^2 - (47-10\zeta_2)\beta_0 C_F C_A^2 - (60-140\zeta_2)\beta_0 C_F^2 C_A - 16\beta_0 C_F^3 \ . \tag{4.6}$$

  The large-β_0 parts are consistent with eq.(4.1), while the remaining $\pm\Delta h_{ij}$ corrections are suggestive of an underlying (yet to be discovered) Gribov-Lipatov like relation [14].

- *No such relations* exist between the DIS F_2 structure function and the corresponding total angle-integrated F_{T+L} fragmentation function. This fact suggests to focus instead on the *momentum space* physical evolution kernels of the *longitudinal* structure [12, 13] and fragmentation functions. Indeed, some observations in [6] do suggest that the $\mathcal{O}(1/(1-x))$ part of the spacelike and timelike longitudinal evolution kernels might actually be *identical* to any logarithmic accuracy.

5. Conclusions

- Using a kinematically modified [8] physical evolution equation, evidence has been given that the *leading* threshold logarithms at *any* eikonal order in the *momentum space* DIS and SIA *non-singlet* physical evolution kernels can be expressed in term of the *one loop* cusp anomalous dimension A_1, which represents the *first step* towards threshold resummation *beyond* the leading eikonal level. This result also explains the observed *universality* [5, 6] of the *leading* logarithmic contributions to the physical kernels of the various non-singlet structure functions at *any* order [6] in $1-x$.

- The present approach *does not* work for *subleading* next-to-eikonal logarithms. However, there are hints of the possible existence of an underlying (yet to be understood) Gribov-Lipatov like relation in the special case of the F_1 DIS structure function and the corresponding F_T SIA transverse fragmentation function.

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