PROCEEDINGS OF SCIENCE

On higher-order flavour-singlet splitting functions and coefficient functions at large x

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We discuss the large-x behaviour of the splitting functions $P_{\rm qg}$ and $P_{\rm gq}$ and of flavour-singlet coefficient functions, such as the gluon contributions $C_{2,\rm g}$ and $C_{L,\rm g}$ to the structure functions $F_{2,L}$, in massless perturbative QCD. These quantities are suppressed by one or two powers of $(1-x)$ with respect to the $(1-x)^{-1}$ terms which are the subject of the well-known threshold exponentiation. We show that the double-logarithmic contributions to $P_{\rm qg}$, $P_{\rm gq}$ and C_L at order α_s^4 can be predicted from known third-order results.

XVIII International Workshop on Deep-Inelastic Scattering and Related Subjects
April 19-23, 2010, Convitto della Calza, Firenze, Italy

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PoS(DIS 2010)139

1. Introduction

Inclusive deep-inelastic lepton-nucleon scattering (DIS) via the exchange of a colour-neutral (gauge) boson is a benchmark process of perturbative QCD. Disregarding contributions suppressed by powers of $1/Q^2$, the structure functions in electromagnetic DIS are given by

$$x^{-1} F_a^n(x,Q^2) \; = \; \big[C_{a,i}(\alpha_s(Q^2)) \otimes f_i^n(Q^2) \big](x) \tag{1.1}$$

in terms of the coefficient functions $C_{a,i}$, $a = 2, L$, $i = \mathrm{q}, \mathrm{g}$, and the nucleon parton distributions f_i^n. Here $\otimes$ denotes the standard Mellin convolution, and the summation over i is understood. Without loss of information, we identify the renormalization and factorization scale with the physical scale Q^2 in Eq. (1.1) and throughout this article.

The scale dependence of the parton densities is

$$\frac{d\, f_i(\xi,\mu^2)}{d \ln Q^2} \; = \; \big[P_{ik}(\alpha_s(Q^2)) \otimes f_k(Q^2) \big](\xi)\, . \tag{1.2}$$

The coefficient functions in Eq. (1.1) and the splitting functions P_{ik} can be expanded in powers of the strong coupling constant $a_s \equiv \alpha_s/(4\pi)$,

$$C_{a,i}(x,\alpha_s) \; = \; \textstyle\sum_{l=0} a_s^{l+l_a} c_{a,i}^{(l)}(x) \, , \tag{1.3}$$

$$P_{ik}(x,\alpha_s) \; = \; \textstyle\sum_{l=0} a_s^{l+1} P_{ik}^{(l)}(x) \tag{1.4}$$

with $l_a = 0$ for F_2 (and the Higgs-exchange structure function F_ϕ discussed below), and $l_a = 1$ for the longitudinal structure function F_L. In this notation, the N^{n}LO approximation includes the contributions with $l \leq n$ in both Eqs. (1.3) and (1.4).

The above (spin-averaged) splitting functions are presently known to $n = 2$ [1, 2], i.e., the next-to-next-to-leading order (NNLO $\equiv$ N^{2}LO). The coefficient functions for the most important structure functions (including F_3 for charge-averaged W-exchange) have also been fully computed to order α_s^3 [3–5], while the less important charge-asymmetry W-cases are available only through a couple of low-integer Mellin-N moments [6, 7]. The frontier in the present massless case are now the α_s^4 corrections, for which first results have been obtained at the lowest value of N [8, 9]. See Ref. [10] for the status of the third-order computation of the heavy-quark contributions to DIS.

2. The general large-x behaviour

We are interested in the leading contributions, in terms of powers in $(1-x)$, to Eqs. (1.3) and (1.4). The form of the diagonal splitting functions is stable under higher-order corrections in the $\overline{\mathrm{MS}}$ scheme, [11]

$$P_{ii}^{(l)} \; = \; A_i^{(l)} (1-x)_+^{-1} + B_i^{(l)} \delta(1-x) + \, \ldots \, . \tag{2.1}$$

The off-diagonal quantities, however, receive a double-logarithmic higher-order enhancement,

$$P_{i \neq j}^{(l)} \; = \; \textstyle\sum_{a=0}^{2l} A_{ij,a}^{(l)} \ln^{2l-a}(1-x) + \, \ldots \, , \tag{2.2}$$

where $A_{ij,a}^{(l)} \propto (C_A - C_F)^{l-a}$ for $a < l$ for (at least) $l \leq 2$ [2], i.e., all double logarithms vanish for $C_F = C_A$, which is part of the colour-factor choice leading to an $\mathcal{N} = 1$ supersymmetric theory.

The leading large-x parts of 'diagonal' coefficient functions, e.g., $C_{2,\mathrm{q}}$ and $C_{\phi,\mathrm{g}}$, are given by

$$c^{(l)}_{\mathrm{diag}} \;=\; \sum_{a=0}^{2l-1} D^{(l)}_{i,a} \left[\frac{\ln^{2l-1-a}(1-x)}{1-x} \right]_{+}^{-1} + \;\ldots\;. \tag{2.3}$$

These terms are resummed by the soft-gluon exponentiation [12–15]. For DIS structure functions (and some other semi-leptonic processes) this resummation is known at the next-to-next-to-next-to-leading logarithmic accuracy, i.e., the highest six logs are completely known to all orders [16].

No resummation has been derived so far for the off-diagonal (flavour-singlet) coefficient functions such as $C_{2,\mathrm{g}}$ and $C_{\phi,\mathrm{q}}$ which are of the form

$$c^{(l)}_{\mathrm{off-d}} \;=\; \textstyle\sum_{a=0}^{2l-1} O^{(l)}_{i,a} \ln^{2l-1-a}(1-x) + \;\ldots\;. \tag{2.4}$$

The coefficient functions for F_L are suppressed by one power in $(1-x)$ with respect to those of F_2,

$$c^{(l)}_{L,i} \;=\; \textstyle\sum_{a=0}^{2l} L^{(l)}_{L,i} (1-x)^{\delta_{ig}} \ln^{2l-a}(1-x) + \;\ldots\;, \tag{2.5}$$

recall our notation with $l_L = 1$ in Eq. (1.3). The double-log contributions to $C_{L,\mathrm{q}}$ (and the $C_F = 0$ part of $C_{L,\mathrm{g}}$) have been resummed in Ref. [17], i.e., the respective highest three logarithms ($a = 0$, 1 and 2 in Eq. (2.5)) are known to all orders.

Our aim is to derive corresponding predictions for all quantities in Eqs. (2.2), (2.4) and (2.5). This contribution is a brief status report of this programme, which has not been finished so far.

3. Physical evolution kernel for (F_2, F_ϕ)

The results of Ref. [17] and their extension to the non-leading corrections for $C_{2,\mathrm{q}}$ and other quantities at all orders in $(1-x)$ [18], see Ref. [19] for a brief summary, have been obtained by studying the non-singlet physical evolution kernels for the respective observables. It is thus natural to study also flavour-singlet physical kernels.

The most natural complement to the standard quantity F_2 with $c^{(0)}_{2,i} = \delta_{iq}\,\delta(1-x)$ is a structure function for a probe which directly interacts only with gluons, such as a scalar ϕ with a $\phi\, G^{\mu\nu} G_{\mu\nu}$ coupling to the gluon field [20]. In the Standard Model this interaction is realized for the Higgs boson in the limit of a heavy top-quark [21,22]. The coefficient functions $C_{\phi,i}$ have been determined recently in Refs. [23] and [24] to the second and third order in α_s, respectively.

We thus consider the 2-vector singlet structure function and 2×2 coefficient-function matrix

$$F \;=\; \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix} \;,\;\; C \;=\; \begin{pmatrix} C_{2,\mathrm{q}} & C_{2,\mathrm{g}} \\ C_{\phi,\mathrm{q}} & C_{\phi,\mathrm{g}} \end{pmatrix} \;. \tag{3.1}$$

With P denoting the matrix of the splitting functions (2.3) and (2.4), the evolution kernel for F reads

$$\begin{aligned} \frac{dF}{d\ln Q^2} \;&=\; \frac{dC}{d\ln Q^2}\, f + CP f \\ &=\; \Big(\beta(a_\mathrm{s})\, \frac{dC}{da_\mathrm{s}} C^{-1} + CPC^{-1}\Big) F \;=\; KF \quad \text{with} \quad K \;=\; \begin{pmatrix} K_{22} & K_{2\phi} \\ K_{\phi 2} & K_{\phi\phi} \end{pmatrix} \;. \end{aligned} \tag{3.2}$$

$\beta(a_\mathrm{s}) = -\beta_0 a_\mathrm{s}^2 + \ldots$ with $\beta_0 = 11 C_A/3 - 2\, n_f/3$ is the standard beta function of QCD. All products of x-dependent quantities have to be read as convolutions (or products of their Mellin transforms).

After expanding in α_s, the first term in the second line of Eq. (3.2) receives double-logarithmic contributions from the non-singlet and singlet coefficient functions (2.3) and (2.4). The second term, absent in the non-singlet cases of Refs. [17, 18], includes also the double-log terms of Eq. (2.2).

The crucial observation, proven by available three-loop calculations to order α_s^4 for the non-singlet parts (thanks to Eq. (2.1)) and to order α_s^3 for the singlet contribution, is that the physical kernel K is only single-log enhanced [24], i.e.,

$$K_{ab}^{(l)} = \sum_{\eta=0}^{l} A_{ab,\eta}^{(l)} (1-x)^{-\delta_{ab}} \ln^{l-\eta}(1-x) + \ldots \qquad (3.3)$$

where the expansion coefficients $K_{ab}^{(l)}$ are defined as in Eq. (1.4) for the splitting functions above.

We conjecture that also the flavour singlet part remains single-log enhanced at the fourth order. This implies a cancellation between the double-logarithmic contributions to the, so far unknown, off-diagonal $l = 3$ splitting functions (2.2) and the known [4, 24] coefficient functions to order α_s^3 from which the former can be deduced. The results are

$$\begin{aligned} P_{\rm qg}^{(3)}/n_f \;=\; & \ln^6(1-x)\cdot 0 \,+\, \ln^5(1-x)\Big[\frac{22}{27}C_{AF}^3 - \frac{14}{27}C_{AF}^2 C_F + \frac{4}{27}C_{AF}^2 n_f\Big] \\ & + \ln^4(1-x)\Big[\Big(\frac{293}{27} - \frac{80}{9}\zeta_2\Big)C_{AF}^3 - \frac{116}{81}C_{AF}^2 n_f + \Big(\frac{4477}{16} - 8\zeta_2\Big)C_{AF}^2 C_F - \frac{13}{81}C_{AF}C_F^2 \\ & + \frac{17}{81}C_{AF}C_F n_f - \frac{4}{81}C_{AF}n_f^2\Big] \,+\, \mathscr{O}\left(\ln^3(1-x)\right) \;, \end{aligned} \qquad (3.4)$$

$$\begin{aligned} P_{\rm gq}^{(3)}/C_F \;=\; & \ln^6(1-x)\cdot 0 \,+\, \ln^5(1-x)\Big[\frac{70}{27}C_{AF}^3 - \frac{14}{27}C_{AF}^2 C_F - \frac{4}{27}C_{AF}^2 n_f\Big] \\ & + \ln^4(1-x)\Big[\Big(\frac{3280}{81} + \frac{16}{9}\zeta_2\Big)C_{AF}^3 - \frac{256}{27}C_{AF}^2 n_f + \Big(\frac{637}{18} - 8\zeta_2\Big)C_{AF}^2 C_F - \frac{49}{81}C_{AF}C_F^2 \\ & + \frac{17}{81}C_{AF}C_F n_f + \frac{32}{81}C_{AF}n_f^2\Big] \,+\, \mathscr{O}\left(\ln^3(1-x)\right) \end{aligned} \qquad (3.5)$$

with $C_{AF} \equiv C_A - C_F$. The vanishing of the leading $\ln^6(1-x)$ contributions is due to a cancellation of contributions. This cancellation turns out to be structural feature [25], i.e., the leading coefficients vanish at all even orders in α_s. Eqs. (3.4) and (3.5) show the colour-factor pattern already noted for $l \le 2$ below Eq. (2.2). The feature is not an obvious consequence of our derivation and can thus be viewed as a non-trivial check of the above conjecture. The extension of the above results to all powers of $(1-x)$ can be found in Ref. [24].

4. Physical evolution kernel for (F_2, F_L)

The system of standard DIS structure functions

$$F \;=\; \begin{pmatrix} F_2 \\ \widehat{F}_L \end{pmatrix} \;,\quad \widehat{F}_L \;=\; F_L/\big(a_s\, c_{L,\rm q}^{(0)}\big) \;, \qquad (4.1)$$

studied before in Refs. [26, 27], can be analyzed in complete analogy to the previous section. Our normalization of $\widehat{F}_L$ (of course Eq. (4.1) involves a simple division only in Mellin-N space) leads to

$$C \;=\; \begin{pmatrix} \delta(1-x) & 0 \\ \delta(1-x) & \hat{c}_{L,\rm g}^{(0)} \end{pmatrix} \,+\, \sum_{l=1} a_s^l \begin{pmatrix} c_{2,\rm q}^{(l)} & c_{2,\rm g}^{(l)} \\ \hat{c}_{L,\rm q}^{(l)} & \hat{c}_{L,\rm g}^{(l)} \end{pmatrix} . \qquad (4.2)$$

The resulting elements physical evolution kernel are again single-log enhanced at large x and read

$$K \;=\; \begin{pmatrix} K_{22} & K_{2\mathrm{L}} \\ K_{\mathrm{L}2} & K_{\mathrm{LL}} \end{pmatrix} \;, \qquad K_{ab}^{(l)} = \sum_{\eta=0}^{l} \widehat{A}_{ab,\eta}^{(l)} (1-x)^{-1} \ln^{l-\eta}(1-x) \,+\, \ldots \tag{4.3}$$

at, at least, $l \leq 3$ for the upper row, with $\widehat{A}_{2\mathrm{L},0}^{(l)} = 0$, and at $l \leq 2$ for the lower row.

Conjecturing that this behaviour holds at $l = 3$ also for K_{L2} and K_{LL}, the three-loop results of Refs. [1–4] together with Eq. (3.4) yield

$$\begin{aligned} c_{L,\mathrm{q}}^{(3)}/C_F \;=\;& \ln^6(1-x)\, \frac{16}{3}\, C_F^3 \,+\, \ln^5(1-x) \Big[(72-64\,\zeta_2)\, C_F^3 + \frac{80}{9}\, C_F^2 n_f - \Big(\frac{728}{9} - 32\,\zeta_2 \Big)\, C_F^2 C_A \Big] \\ & + \ln^4(1-x) \cdot \big[\text{known coefficients} \big] \,+\, \mathscr{O}\left(\ln^3(1-x) \right) \;, \end{aligned} \tag{4.4}$$

$$\begin{aligned} c_{L,\mathrm{g}}^{(3)}/n_f \;=\;& (1-x)\ln^6(1-x)\, \frac{32}{3}\, C_A^3 \,+\, (1-x)\ln^5(1-x) \Big[-\frac{2080}{9}\, C_A^3 + \frac{64}{9}\, C_A^2 n_f + \frac{104}{3}\, C_A^2 C_F \\ & + \frac{40}{3}\, C_F^3 \Big] \,+\, (1-x)\ln^4(1-x) \Big[\Big(\frac{70760}{27} - 352\,\zeta_2 \Big) C_A^3 - \Big(\frac{25306}{27} - \frac{320}{3}\,\zeta_2 \Big) C_A^2 C_F \\ & - \frac{4192}{27}\, C_A^2 n_f + \Big(\frac{1600}{27} + 32\,\zeta_2 \Big) C_A C_F^2 + \frac{556}{27}\, C_A C_F n_f + \Big(38 - \frac{320}{3}\,\zeta_2 \Big) C_F^3 \\ & + \frac{32}{27}\, C_A n_f^2 + \frac{308}{27}\, C_F^2 n_f \Big] \,+\, \mathscr{O}\left((1-x)\ln^3(1-x) \right) \;, \end{aligned} \tag{4.5}$$

where the coefficient of $\ln^4(1-x)$ in Eq. (4.4) has been suppressed for brevity. The complete form of this equation has been given in Ref. [17], where it was derived in another manner which did not involve the off-diagonal splitting functions. Consequently the consistency of the two derivations provides another confirmation of the correctness of the above result for $P_{\mathrm{qg}}^{(3)}$. The non-C_F parts of Eq. (4.5) – here, as App. C of Ref. [24], given for W-exchange, i.e., without the fl_{11}^{g} contribution for the photon case [4] – have also been derived, but not explicitly written down, in Ref. [17].

5. Summary and Outlook

We have summarized the status of our large-x predictions of higher-order off-diagonal splitting functions and DIS coefficient functions. The coefficients of the highest three powers of $\ln(1-x)$ have been derived for the four-loop contributions to the splitting functions P_{qg} and P_{gq} from the three-loop coefficient functions and the single-logarithmic enhancement of the physical evolution kernel for the system (F_2, F_ϕ) of flavour-singlet structure functions at order α_s^4 [24]. In the present contribution we have employed these results to derive also the leading three large-x logarithms for the fourth-order gluon coefficient function $C_{L,\mathrm{g}}$ for the longitudinal structure function from the analogous kernel for (F_2, F_L).

These results will become phenomenologically relevant, via effective x-space parametrizations analogous to, e.g., those of Ref. [28], once the next major step towards a full fourth-order calculation of deep-inelastic scattering, the extension of Ref. [29] to order α_s^4, has been taken.

The determination of flavour-singlet quantities from the physical kernels is neither rigorous, nor – unlike in flavour non-singlet cases [17, 18] – can it be extended to all orders in α_s. First all-order leading-logarithmic results have been presented in Ref. [25] of a rigorous and powerful approach, the prediction of the coefficients of the highest double logarithms from the D-dimensional structure of the unfactorized structure functions together with mass-factorization to all orders.

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