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# Logarithmic $O(\alpha_s^3)$ contributions to the DIS Heavy Flavor Wilson Coefficients at $Q^2 \gg m^2$

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The logarithmic contributions to the massive twist-2 operator matrix elements for deep-inelastic scattering are calculated to $O(\alpha_s^3)$for general values of the Mellin variable $N$.

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## 1. Introduction

The heavy flavor contributions to deep-inelastic scattering (DIS) are rather large in the small $x$ region. With the current DIS data a precision of $\sim 1\%$ is reached [1] for $F_2(x,Q^2)$. This requires to describe the heavy flavor corrections to 3-loop order, to perform a consistent next-to-next-to leading order (NNLO) analysis, to measure the strong coupling constant $\alpha_s(M_Z^2)$ and the unpolarized twist-2 parton distribution functions at highest precision possible, cf. [2]. In the region $Q^2/m^2 \geq 10$ one may compute all contributions except the power suppressed terms, $\propto (m^2/Q^2)^k, k \geq 1$ using the factorization theorem given in Ref. [3]. Here, the massive Wilson coefficients factorize into massive operator matrix elements (OMEs), $A_{ij}$, and the massless Wilson coefficients, which are known to $O(\alpha_s^3)$ [4]. The $O(\alpha_s^3)$ Mellin moments of the massive operator matrix elements up to $N = 10...14$, depending on the process, were computed in [5]. The calculation was performed relating the moments of the massive operator matrix elements to massive tadpoles and using `MATAD` [6]. In Ref. [5] also the complete renormalization for a single massive quark has been derived. Different other contributions, needed in the renormalization process, were computed at general values of $N$ in Refs. [7]. For the structure function $F_L(x,Q^2)$ the asymptotic corrections to $O(\alpha_s^3)$ are known for general values of $N$ [8]. They are, however, only applicable at scales $Q^2/m^2 \geq 800$. For transversity the matrix elements were computed for general $N$ at 2-loop order and a series of moments at 3-loop order in [9]. Very recently, the general $N$ results at $O(\alpha_s^3)$ for $F_2(x,Q^2)$ for the color coefficients $\propto n_f$ have been completed [10, 11]. These computations use modern summation technologies encoded in the package `Sigma` [12] and the results can be expressed in terms of nested harmonic sums [13]. From the single pole terms in the massive computations of Refs. [5, 9, 10, 11] the corresponding contributions to the 3–loop anomalous dimensions were derived, either for the respective moments [14], or at general values of $N$ [16].

In this note we report on the logarithmic $O(\alpha_s^3)$ contributions to the massive operator matrix elements, cf. also [17]. They are known for general values of $N$ and depend on the 3-loop anomalous dimensions and massive OMEs up to $O(\alpha_s^2)$.



## 2. The heavy flavor Wilson coefficients in the asymtotic region

The heavy flavor correction to the structure function $F_2(x,Q^2)$ with $n_f$ massless and one heavy flavor reads, [5] :

$$
\begin{aligned}
F_{(2,L)}^{Q\overline{Q}}(x,n_f,Q^2,m^2) = & \sum_{k=1}^{n_f} e_k^2 \Biggl\{ L_{q,(2,L)}^{\mathsf{NS}}\left(x,n_f+1,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) \otimes \left[ f_k(x,\mu^2,n_f) + f_{\overline{k}}(x,\mu^2,n_f)\right] \\
& + \frac{1}{n_f}\left[ L_{q,(2,L)}^{\mathsf{PS}}\left(x,n_f+1,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) \otimes \Sigma(x,\mu^2,n_f) + L_{g,(2,L)}^{\mathsf{S}}\left(x,n_f+1,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) \otimes G(x,\mu^2,n_f)\right]\Biggr\} \\
& + e_Q^2 \left[ H_{q,(2,L)}^{\mathsf{PS}}\left(x,n_f+1,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) \otimes \Sigma(x,\mu^2,n_f) + H_{g,(2,L)}^{\mathsf{S}}\left(x,n_f+1,\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) \otimes G(x,\mu^2,n_f)\right],
\end{aligned}
\tag{2.1}
$$

with boundaries for the Mellin integral $[x(1+4m^2/Q^2),1]$, and $e_i$ the quark charges. Here the different Wilson coefficients are denoted by $L_i, H_i$ in case the photon couples to a light (L) or the heavy (H) quark. For $Q^2 \gg m^2$ they can be expressed in terms of the massive OMEs $A_{ij}$ and the massless Wilson coefficients $C_j$. To $O(a_s^3)$ they read ($a_s = \alpha_s/(4\pi)$)

$$
\begin{aligned}
L_{q,(2,L)}^{\mathsf{NS}}(n_f+1) &= a_s^2\left[A_{qq,Q}^{(2),\mathsf{NS}}(n_f+1)\delta_2 + \hat{C}_{q,(2,L)}^{(2),\mathsf{NS}}(n_f)\right] \\
&+ a_s^3\left[A_{qq,Q}^{(3),\mathsf{NS}}(n_f+1)\delta_2 + A_{qq,Q}^{(2),\mathsf{NS}}(n_f+1)C_{q,(2,L)}^{(1),\mathsf{NS}}(n_f+1) + \hat{C}_{q,(2,L)}^{(3),\mathsf{NS}}(n_f)\right] \\
L_{q,(2,L)}^{\mathsf{PS}}(n_f+1) &= a_s^3\Big[\, A_{qq,Q}^{(3),\mathsf{PS}}(n_f+1)\ \delta_2 + A_{gq,Q}^{(2)}(n_f)\ \ n_f\tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + n_f\hat{\tilde{C}}_{q,(2,L)}^{(3),\mathsf{PS}}(n_f)\Big] \\
L_{g,(2,L)}^{\mathsf{S}}(n_f+1) &= a_s^2 A_{gg,Q}^{(1)}(n_f+1)n_f\tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + a_s^3\Big[\, A_{qg,Q}^{(3)}(n_f+1)\ \delta_2 \\
&\quad + A_{gg,Q}^{(1)}(n_f+1)\ \ n_f\tilde{C}_{g,(2,L)}^{(2)}(n_f+1) + A_{gg,Q}^{(2)}(n_f+1)\ \ n_f\tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \\
&\quad + A_{Qg}^{(1)}(n_f+1)\ \ n_f\tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(n_f+1) + n_f\hat{\tilde{C}}_{g,(2,L)}^{(3)}(n_f)\Big]\ , \\
H_{q,(2,L)}^{\mathsf{PS}}(n_f+1) &= a_s^2\Big[\, A_{Qq}^{(2),\mathsf{PS}}(n_f+1)\ \delta_2 + \ \tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(n_f+1)\Big] + a_s^3\Big[\, A_{Qq}^{(3),\mathsf{PS}}(n_f+1)\ \delta_2 \\
&\quad + \tilde{C}_{q,(2,L)}^{(3),\mathsf{PS}}(n_f+1) + A_{gq,Q}^{(2)}(n_f+1)\ \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \\
&\quad + A_{Qq}^{(2),\mathsf{PS}}(n_f+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(n_f+1)\Big]\ , \\
H_{g,(2,L)}^{\mathsf{S}}(n_f+1) &= a_s\Big[\, A_{Qg}^{(1)}(n_f+1)\ \delta_2 + \ \tilde{C}_{g,(2,L)}^{(1)}(n_f+1)\Big] + a_s^2\Big[\, A_{Qg}^{(2)}(n_f+1)\ \delta_2 \\
&\quad + A_{Qg}^{(1)}(n_f+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(n_f+1) + \ A_{gg,Q}^{(1)}(n_f+1)\ \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \\
&\quad + \tilde{C}_{g,(2,L)}^{(2)}(n_f+1)\Big] + \ a_s^3\Big[\, A_{Qg}^{(3)}(n_f+1)\ \delta_2 + \ A_{Qg}^{(2)}(n_f+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(n_f+1) \\
&\quad + A_{gg,Q}^{(2)}(n_f+1)\ \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + \ A_{Qg}^{(1)}(n_f+1)\Big\{C_{q,(2,L)}^{(2),\mathsf{NS}}(n_f+1) \\
&\quad + \tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(n_f+1)\Big\} + \ A_{gg,Q}^{(1)}(n_f+1)\ \tilde{C}_{g,(2,L)}^{(2)}(n_f+1) + \ \tilde{C}_{g,(2,L)}^{(3)}(n_f+1)\Big]\ ,
\end{aligned}
\tag{2.2}
$$

with $\delta_2 = 0(1)$ for $F_L(F_2)$ and $\hat{f}(n_f) = f(n_f+1) - f(n_f)$, $\tilde{f}(n_f) = f(n_f)/n_f$. The massive OMEs depend on the ratio $m^2/\mu^2$, while the scale ratio of the massless Wilson coefficients is $\mu^2/Q^2$. The latter are pure functions of the momentum fraction $z$, or the Mellin variable $N$, if one sets $\mu^2 = Q^2$. The massive OMEs obey then the general structure

$$
A_{ij}^{(3)}\left(\frac{m^2}{Q^2}\right) = a_{ij}^{(3),3}\ln^3\left(\frac{m^2}{Q^2}\right) + a_{ij}^{(3),2}\ln^2\left(\frac{m^2}{Q^2}\right) + a_{ij}^{(3),1}\ln\left(\frac{m^2}{Q^2}\right) + a_{ij}^{(3),0}\ . \tag{2.3}
$$

## 3. The matrix element $A_{Qg}^{(3)}(N)$

In the following we present, as an example, the logarithmic expansion coefficients of Eq. (2.3), $a^{(3),k}, k \geq 1$, for the massive OME $A_{Qg}^{(3)}(N)$ in the $\overline{\mathrm{MS}}$–scheme. They are given by :

$$
\begin{aligned}
a_{Qg}^{(3),3} &= \frac{8(N^2+N+2)T_F}{9N(N+1)(N+2)}\Bigg[T_F n_f\Bigg(C_F\Bigg(\frac{P_1}{(N-1)N^2(N+1)^2(N+2)} - 4S_1\Bigg) \\
&+ C_A\Bigg(4S_1 - \frac{8(N^2+N+1)}{(N-1)N(N+1)(N+2)}\Bigg)\Bigg) - 8T_F^2 + C_A^2\Bigg(-\frac{(11N^4+22N^3-59N^2-70N-48)S_1}{(N-1)N(N+1)(N+2)}
\end{aligned}
$$

$$-12S_1^2+\frac{2(N^2+N+1)(11N^4+22N^3-35N^2-46N-24)}{(N-1)^2N^2(N+1)^2(N+2)^2}\Bigg)+C_AT_F\Bigg(-\frac{56(N^2+N+1)}{(N-1)N(N+1)(N+2)}$$
$$+28S_1\Bigg)+C_F^2\Bigg(-3\frac{(3N^2+3N+2)^2}{4N^2(N+1)^2}+\frac{6S_1(3N^2+3N+2)}{N(N+1)}-12S_1^2\Bigg)+C_FT_F\Bigg(-16S_1$$
$$+\frac{2P_2}{(N-1)N^2(N+1)^2(N+2)}\Bigg)+C_AC_F\Bigg(24S_1^2-\frac{(N^2+N+6)(7N^2+7N+4)S_1}{(N-1)N(N+1)(N+2)}$$
$$-\frac{(3N^2+3N+2)(11N^4+22N^3-59N^2-70N-48)}{4(N-1)N^2(N+1)^2(N+2)}\Bigg)\Bigg]. \quad (3.1)$$

$$a_{Qg}^{(3),2} = 4T_F^2n_f\Bigg(C_F\Bigg(-\frac{4(N^2+N+2)}{3N(N+1)(N+2)}\left(S_2+S_1^2\right)+\frac{8(5N^3+8N^2+19N+6)S_1}{9N^2(N+1)(N+2)}$$
$$-\frac{P_3}{9(N-1)N^4(N+1)^4(N+2)^3}\Bigg)+C_A\Bigg(-\frac{8(5N^4+20N^3+47N^2+58N+20)S_1}{9N(N+1)^2(N+2)^2}$$
$$+\frac{4(N^2+N+2)}{3N(N+1)(N+2)}\left(2S_{-2}+S_2+S_1^2\right)-\frac{2P_4}{9(N-1)N^2(N+1)^3(N+2)^3}\Bigg)\Bigg)+2C_A^2T_F\Bigg($$
$$\frac{8(N^2+N+2)}{N(N+1)(N+2)}\left(2S_{-2,1}-S_1^3-3S_2S_1-S_{-3}-4S_{-2}S_1-S_3\right)-\frac{2P_5S_1^2}{3(N-1)N^2(N+1)^2(N+2)^2}$$
$$+\frac{4P_6S_1}{9(N-1)^2N^3(N+1)^3(N+2)^3}-\frac{4(N^2+N+2)(11N^4+22N^3-59N^2-70N-48)S_{-2}}{3(N-1)N^2(N+1)^2(N+2)^2}$$
$$+\frac{8P_7}{9(N-1)^2N^4(N+1)^4(N+2)^4}-\frac{2(N^2+N+2)(11N^4+22N^3-83N^2-94N-72)S_2}{3(N-1)N^2(N+1)^2(N+2)^2}\Bigg)$$
$$+4C_AT_F^2\Bigg(\frac{4(N^2+N+2)}{N(N+1)(N+2)}\left(S_1^2+S_2+2S_{-2}\right)+\frac{8(5N^4+20N^3-N^2-14N+20)S_1}{9N(N+1)^2(N+2)^2}$$
$$-\frac{2P_8}{9(N-1)N^3(N+1)^3(N+2)^3}\Bigg)+2C_F^2T_F\Bigg(\frac{8(N^2+N+2)}{N(N+1)(N+2)}\left(3S_2S_1-S_1^3+4S_{-2}S_1+2S_3-4S_{-2,1}\right)$$
$$-\frac{16(N^2+N+2)S_{-2}}{N^2(N+1)^2(N+2)}-\frac{6(N^2+N+2)(3N^2+3N+2)S_2}{N^2(N+1)^2(N+2)}+\frac{2(3N^4+14N^3+43N^2+48N+20)S_1^2}{N^2(N+1)^2(N+2)}$$
$$-\frac{4P_9S_1}{N^3(N+1)^3(N+2)}+\frac{P_{10}}{2N^4(N+1)^4(N+2)}+\frac{16(N^2+N+2)S_{-3}}{N(N+1)(N+2)}\Bigg)+4C_FT_F^2\Bigg($$
$$\frac{4(N^2+N+2)}{3N(N+1)(N+2)}\left(S_2-3S_1^2\right)+\frac{8(5N^3+14N^2+37N+18)S_1}{9N^2(N+1)(N+2)}-\frac{P_{11}}{9(N-1)N^4(N+1)^4(N+2)^3}\Bigg)$$
$$+2C_FC_AT_F\Bigg(\frac{4(N^2+N+2)}{N(N+1)(N+2)}\left(4S_1^3-2S_3+4S_{-2,1}-2S_{-3}-3S_{-2}\right)$$
$$+\frac{4P_{12}S_1^2}{3(N-1)N^2(N+1)^2(N+2)^2}-\frac{4P_{13}S_1}{9(N-1)N^3(N+1)^3(N+2)^3}$$
$$+\frac{P_{14}}{18(N-1)N^3(N+1)^3(N+2)^3}+\frac{4(N^2+N+2)(N^4+2N^3+8N^2+7N+18)S_2}{3(N-1)N^2(N+1)^2(N+2)^2}\Bigg). \quad (3.2)$$

$$a_{Qg}^{(3),1} = \frac{1}{2}\hat{\gamma}_{qg}^{(2)}(n_f)-\frac{n_f}{2}\hat{\tilde{\gamma}}_{qg}^{(2)}(n_f)+4T_F^2n_f\Bigg(C_F\Bigg(\frac{4(N^2+N+2)}{9N(N+1)(N+2)}\left(4S_3-S_1^3-3S_2S_1\right)+\frac{4(3N+2)S_1^2}{3N^2(N+2)}$$

$$
\begin{aligned}
& + \frac{4(N^4-N^3-20N^2-10N-4)S_1}{3N^2(N+1)^2(N+2)} + \frac{2P_{15}}{3(N-1)N^5(N+1)^5(N+2)^4} + \frac{4P_{16}S_2}{3(N-1)N^3(N+1)^3(N+2)^2}\Biggr) \\
& +C_A\Biggl(\frac{4(N^2+N+2)}{9N(N+1)(N+2)}\Bigl(S_1^3+9S_2S_1+6S_{-3}+12S_{-2}S_1+8S_3-12S_{-2,1}\Bigr) - \frac{4P_{17}S_1}{3N(N+1)^3(N+2)^3} \\
& - \frac{4(N^3+8N^2+11N+2)S_1^2}{3N(N+1)^2(N+2)^2} + \frac{4P_{18}}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{4P_{19}S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& + \frac{16(N^2-N-4)S_{-2}}{3(N+1)^2(N+2)^2}\Biggr)\Biggr) + 2C_A^2T_F\Biggl(\frac{8(N^2+N+2)}{3N(N+1)(N+2)}\Bigl(12S_{-2,1}S_1 - S_1^4 - 9S_2S_1^2 - 8S_3S_1 - 6S_{-3}S_1 \\
& -12S_{-2}S_1^2\Bigr) - \frac{2P_{20}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} + \frac{2P_{21}S_1^2}{3(N-1)N^2(N+1)^3(N+2)^3} \\
& - \frac{2P_{22}S_1}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{2P_{23}S_2S_1}{(N-1)N^2(N+1)^2(N+2)^2} - \frac{2P_{24}}{3(N-1)^2N^5(N+1)^5(N+2)^5} \\
& + \frac{4(N^2+N+2)(11N^4+22N^3-35N^2-46N-24)}{9(N-1)N^2(N+1)^2(N+2)^2}\Bigl(6S_{-2,1}-4S_3-3S_{-3}\Bigr) \\
& - \frac{8(N^2-N-4)(11N^4+22N^3-35N^2-46N-24)S_{-2}}{3(N-1)N(N+1)^3(N+2)^3} - \frac{8P_{25}S_{-2}S_1}{3(N-1)N^2(N+1)(N+2)^2} \\
& + \frac{2(11N^4+22N^3-35N^2-46N-24)P_{19}S_2}{3(N-1)^2N^3(N+1)^3(N+2)^3}\Biggr) + 4C_AT_F^2\Biggl(\frac{8(N^2+N+2)}{9N(N+1)(N+2)}\Bigl(S_1^3+8S_3-12S_{-2,1} \\
& +9S_2S_1+12S_{-2}S_1+6S_{-3}\Bigr) - \frac{8(N^3+8N^2+11N+2)S_1^2}{3N(N+1)^2(N+2)^2} - \frac{8P_{19}S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& + \frac{2P_{26}}{27(N-1)N^4(N+1)^4(N+2)^4} + \frac{32(N^2-N-4)S_{-2}}{3(N+1)^2(N+2)^2} + \frac{4P_{27}S_1}{27N(N+1)^3(N+2)^3}\Biggr) + 2C_F^2T_F\Biggl( \\
& \frac{8(N^2+N+2)}{3N(N+1)(N+2)}\Bigl(4S_3S_1-S_1^4-3S_2S_1^2\Bigr) + \frac{2(3N^4+42N^3+107N^2+92N+28)S_1^3}{3N^2(N+1)^2(N+2)} \\
& + \frac{2P_{28}S_1^2}{N^3(N+1)^2(N+2)} + \frac{2P_{29}S_1}{N^4(N+1)^4(N+2)} + \frac{2(7N^4+74N^3+79N^2-12N-4)S_2S_1}{N^2(N+1)^2(N+2)} \\
& - \frac{8(N^2+N+2)(3N^2+3N+2)S_3}{3N^2(N+1)^2(N+2)} - \frac{2(3N^2+3N+2)(N^4+17N^3+17N^2-5N-2)S_2}{N^3(N+1)^3(N+2)} \\
& - \frac{P_{30}}{N^5(N+1)^5(N+2)}\Biggr) + 4C_FT_F^2\Biggl(\frac{8(N^2+N+2)}{9N(N+1)(N+2)}\Bigl(4S_3-S_1^3-3S_2S_1\Bigr) \\
& + \frac{P_{31}}{3(N-1)N^5(N+1)^5(N+2)^2} + \frac{8(N^4-N^3-20N^2-10N-4)S_1}{3N^2(N+1)^2(N+2)} + \frac{8(N^4+17N^3+17N^2-5N-2)S_2}{3N^2(N+1)^2(N+2)} \\
& + \frac{8(3N+2)S_1^2}{3N^2(N+2)}\Biggr) + 2C_FC_AT_F\Biggl(\frac{8(N^2+N+2)}{3N(N+1)(N+2)}\Bigl(2S_1^4+12S_2S_1^2+4S_3S_1-12S_{-2,1}S_1+6S_{-3}S_1 \\
& +12S_{-2}S_1^2\Bigr) + \frac{4P_{32}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} - \frac{4P_{33}S_1^2}{3(N-1)N^3(N+1)^3(N+2)^3} \\
& - \frac{4P_{34}S_1}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{4P_{35}S_2S_1}{3(N-1)N^2(N+1)^2(N+2)^2} - \frac{P_{36}}{3(N-1)N^4(N+1)^5(N+2)^4} \\
& - \frac{8P_{37}S_{-2}S_1}{N^2(N+1)^2(N+2)^2} + \frac{4(N^2+N+2)(3N^2+3N+2)}{N^2(N+1)^2(N+2)}\Bigl(2S_{-2,1}-S_{-3}\Bigr) \\
& - \frac{8(N^2-N-4)(3N^2+3N+2)S_{-2}}{N(N+1)^3(N+2)^2} - \frac{2P_{38}S_2}{3(N-1)N^3(N+1)^3(N+2)^2}
\end{aligned}
$$

$$
-\frac{8(N^2+N+2)(29N^4+58N^3-41N^2-70N-48)S_3}{9(N-1)N^2(N+1)^2(N+2)^2}\Bigg) . \tag{3.3}
$$

Here, $S_{\vec{a}} \equiv S_{\vec{a}}(N)$, $P_k$ denote some polynomials in $N$, cf. [17], and $\gamma_{ij}^{(2)}$ are the 3-loop anomalous dimensions. The expansion coefficients given above depend on harmonic sums up weight $\mathsf{w} = 3$. Numerical studies show, that within the kinematic region of HERA the constant terms to (2.3) are as important as the logarithmic contributions. Further details will be given in [17].

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