

## Threshold resummation with the analytic coupling model in $b$ and $c$ -quark fragmentation

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**G. Ferrera\***

*Dipartimento di Fisica e Astronomia, Università di Firenze and INFN, Sezione di Firenze*

*E-mail: ferrera@fi.infn.it*

We present a model to include non-perturbative corrections to heavy-quark fragmentation. The model is based on next-to-next-to-leading logarithmic perturbative threshold resummation and on an analytic QCD coupling not containing the Landau pole.

We compare the predictions of the model against the experimental data from  $B$  and  $D$  meson fragmentation in  $e^+e^-$  annihilation. Our model gives a good description of the experimental data without introducing any free non-perturbative parameter.

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\*Speaker.

The hadronization of partons into hadrons cannot be calculated from first principles, but it is usually described in terms of phenomenological models, containing few parameters which need to be tuned to experimental data.

We shall propose a different approach to describe heavy quark (bottom and charm) fragmentation in  $e^+e^-$  processes, based on a non-perturbative model [1, 2], including power corrections via an effective strong coupling constant, which does not exhibit the Landau pole. The interesting feature of such a model is that it does not contain any extra free parameter to be fitted to the data, besides the ones entering in the parton-level calculation (i.e.  $\Lambda_{QCD}$  parameter, quark masses). In [3, 4], such a model was also used in the framework of  $B$ -meson decays and it was found good agreement with the experimental data allowing the extraction of  $\alpha_S(m_Z)$  and the Cabibbo-Kobayashi-Maskawa matrix element  $|V_{ub}|$  from such decays.

We consider heavy-quark production in  $e^+e^-$  annihilation

$$e^+e^- \rightarrow P(Q) \rightarrow q(p_q)\bar{q}(p_{\bar{q}})(g(p_g)), \quad (1)$$

in particular  $b$ - and  $c$ -quark production at LEP where  $P$  is a  $Z^0$  boson and  $Q = m_Z$ , as well as charm-quark fragmentation at the  $\Upsilon(4S)$  resonance, i.e.  $Q = m_{\Upsilon(4S)}$  and the  $c\bar{c}$  pair coming from the decay of a virtual photon ( $P = \gamma^*$ ).

The perturbative fragmentation approach [5, 6], up to power corrections, factorizes the energy distribution of a heavy quark as the convolution of a coefficient function, associated with the emission off a massless parton, and a perturbative fragmentation function, expressing the transition of the light parton into a heavy quark. The factorized heavy-quark spectrum reads:

$$\frac{1}{\sigma} \frac{d\sigma}{dx}(x, Q, m_q) = C(x, Q, \mu_F) \otimes D(x, \mu_F, m_q). \quad (2)$$

where  $Q$  is the hard scale of the process,  $x$  is the heavy-quark energy fraction  $x = \frac{2E_q}{Q}$  and  $\mu_F \sim Q$  is the factorization scale.

The perturbative fragmentation function fulfills the DGLAP evolution equations: its value at a any scale  $\mu_F$  can be obtained once an initial condition at  $\mu_{0F} \sim m_q$  is given. In [1, 2] we have used the coefficient function and the initial condition at next-to-leading order (NLO) and solved the DGLAP equations with a NLO kernel, i.e. resumming up to next-to-leading (NLL) mass logarithms  $(\alpha_S^n \ln^{n-1}(\mu_F^2/\mu_{0F}^2))$ <sup>1</sup>.

Both coefficient function and initial condition contain terms,  $\sim 1/(1-x)_+$  and  $\sim [\ln(1-x)/(1-x)]_+$ , enhanced when  $x$  approaches 1, which corresponds to soft- or collinear-gluon radiation. One needs to resum such contributions to all orders to improve the perturbative prediction (threshold resummation). We have resummed these contribution to all order in the next-to-next-to-leading logarithmic (NNLL) approximation, following the general method of [7, 8].

We now briefly discuss the phenomenological model which includes non-perturbative power corrections through an effective QCD coupling [10, 3, 1]. We start to construct a general analytic QCD coupling from the standard one, by means of an analyticity requirement: the analytic coupling is defined to have the same discontinuity of the standard coupling in the time-like region and no

<sup>1</sup>One could go beyond such a level of accuracy and include NNLO corrections to the coefficient function, initial condition and to the non-singlet splitting functions .

other singularity [9], at one loop it reads:

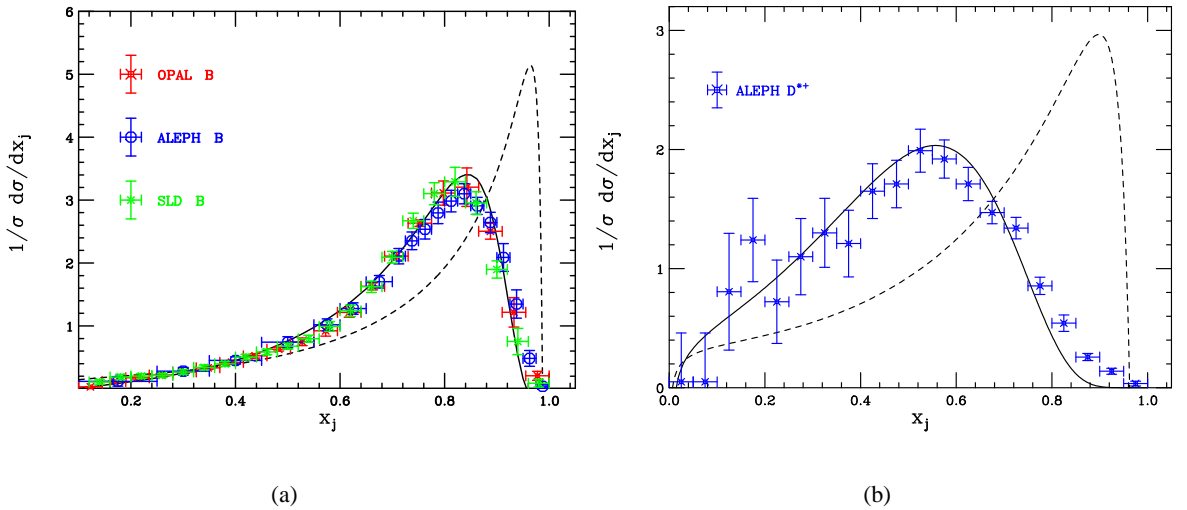
$$\bar{\alpha}_s(Q^2) = \frac{1}{\beta_0} \left[ \frac{1}{\log(Q^2/\Lambda^2)} - \frac{\Lambda^2}{Q^2 - \Lambda^2} \right], \quad (3)$$

where  $\Lambda = \Lambda_{QCD}$  and  $\beta_0$  is the first coefficient of the QCD beta function  $\beta(\alpha_s)$ . The coupling above has no Landau pole, which has been subtracted by a power correction, while has the same discontinuity of the standard one for  $Q^2 < 0$ , related to gluon branching. The last term on the r.h.s. of Eq. (3) produces a series of power corrections once it is expanded for  $Q^2 \gg \Lambda^2$ . Since heavy quark fragmentation is a time-like process, we have also included the absorptive parts of the gluon polarization function into the effective coupling [1]: that amounts to a resummation of constant terms to all orders. At one-loop for example one obtains for the analytic time-like coupling:

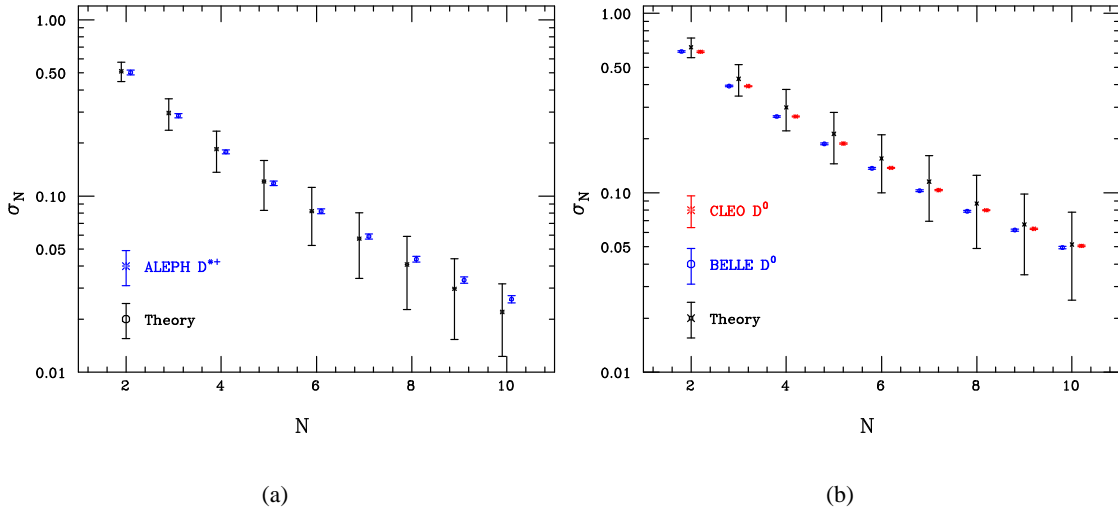
$$\bar{\alpha}_s(k^2) = \frac{1}{\beta_0} \left[ \frac{1}{2} - \frac{1}{\pi} \arctan \left( \frac{\log(k^2/\Lambda^2)}{\pi} \right) \right]. \quad (4)$$

The prescription at the base of our model is to replace the standard QCD coupling with the analytic time-like coupling in the next-to-next-to-leading order resummed formulas. Let us stress that even if our model does not contain any free parameter to be fitted to the data we have constructed the model among different possibilities (e.g. different possible prescription for the low energy behaviour of the QCD coupling) with the goal of describing at best the experimental data. Anyway it is highly not trivial that such a simple model, can describe in a good way rather different processes, involving different hard scales, such as  $B$ -decays and bottom/charm fragmentation.

In Figs. 1 and 2 we compare the prediction of the model with experimental data from ALEPH [11, 12], OPAL [13], SLD [14], CLEO [15] and BELLE [16].



**Figure 1:** Results on bottom (left) and charm (right) hadron production (solid line) predicted with the analytic coupling model compared with the parton level calculation (dashes) and with experimental data.



**Figure 2:** Moments of charmed-hadron cross section with the analytic coupling model (“Theory”) compared with the moments of  $D^{*+}$  production (left) and  $D^0$  production. Theoretical errors are estimated by varying the parameters entering in the perturbative calculation ( $\mu_F$ ,  $\mu_{0F}$ ,  $m_q$  and  $\alpha_S(m_Z)$ ).

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