Complete next-to-leading-order corrections to $J/\psi$ photoproduction in nonrelativistic QCD

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We calculate the cross section of inclusive direct $J/\psi$ photoproduction at next-to-leading order within the factorization formalism of nonrelativistic quantum chromodynamics, for the first time including the full relativistic corrections due to the intermediate $1^3S_0$, $3^3S_1$, and $3^3P_1$ color-octet states. A comparison of our results to recent H1 data suggests that the color octet mechanism is indeed realized in $J/\psi$ photoproduction, although the predictivity of our results still suffers from uncertainties in the color-octet long-distance matrix elements.
The factorization formalism of nonrelativistic quantum chromodynamics (NRQCD) [1] provides a consistent theoretical framework for the description of heavy-quarkonium production and decay. This implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant $\alpha_s$, from supposedly universal long-distance matrix elements (LDMEs), to be extracted from experiment. The relative importance of the latter can be estimated by means of velocity scaling rules; i.e., the LDMEs are predicted to scale with a definite power of the heavy-quark ($Q$) velocity $v$ in the limit $v \ll 1$. In this way, the theoretical predictions are organized as double expansions in $\alpha_s$ and $v$. A crucial feature of this formalism is that it takes into account the complete structure of the $Q\bar{Q}$ Fock space, which is spanned by the states $n = 2S + 1L^a_J$ with definite spin $S$, orbital angular momentum $L$, total angular momentum $J$, and color multiplicity $a = 1, 8$. In particular, this formalism predicts the existence of color-octet (CO) processes in nature. This means that $Q\bar{Q}$ pairs are produced at short distances in CO states and subsequently evolve into physical, color-singlet (CS) quarkonia by the nonperturbative emission of soft gluons. In the limit $v \to 0$, the traditional CS model (CSM) is recovered in the case of $S$-wave quarkonia.

Fifteen years after the introduction of the NRQCD factorization formalism [1], the existence of CO processes and the universality of the LDMEs are still at issue and far from proven, despite an impressive series of experimental and theoretical endeavors. The greatest success of NRQCD was that it was able to explain the $J/\psi$ hadroproduction yield at the Fermilab Tevatron [2], while the CSM prediction lies orders of magnitude below the data, even if the latter is evaluated at next-to-leading order (NLO) or beyond [3]. Also in the case of $J/\psi$ photoproduction at DESY HERA, the CSM cross section significantly falls short of the data, as demonstrated by a recent NLO analysis [4] using up-to-date input parameters and standard scale choices, leaving room for CO contributions [5]. Similarly, the $J/\psi$ yields measured in electroproduction at HERA and in two-photon collisions at CERN LEP2 were shown [6, 7] to favor the presence of CO processes. As for $J/\psi$ polarization in hadroproduction, neither the leading-order (LO) NRQCD prediction [8], nor the NLO CSM one [3] leads to an adequate description of the Tevatron data. The situation is quite similar for the polarization in photoproduction at HERA [4].

In order to convincingly establish the CO mechanism and the LDME universality, it is an urgent task to complete the NLO description of $J/\psi$ hadro- [3] and photoproduction [4, 9], regarding both $J/\psi$ yield and polarization, by including the full CO contributions at NLO. While the NLO contributions due to the $1^3S_0$ and $3^3S_1$ CO states may be obtained using standard techniques [9], the NLO treatment of $3^1P_1^{[8]}$ states in $2 \to 2$ processes requires a more advanced technology, which has been lacking so far. In fact, the $3^1P_1^{[8]}$ contributions represent the missing links in all those previous NLO analyses [3, 4, 9], and there is no reason at all to expect them to be insignificant. Specifically, their calculation is far more intricate because the application of the $3^1P_1^{[8]}$ projection operators to the short-distance scattering amplitudes produce particularly lengthy expressions involving complicated tensor loop integrals and exhibiting an entangled pattern of infrared singularities. This technical bottleneck, which has prevented essential progress in the global test of NRQCD factorization for the past fifteen years, was overcome for the first time in Ref. [10], which we review here.

In direct photoproduction, a quasi-real photon $\gamma$ that is radiated off the incoming electron $e$
interacts with a parton $i$ stemming from the incoming proton $p$. Invoking the Weizsäcker-Williams approximation and the factorization theorems of the QCD parton model and NRQCD [1], the inclusive $J/\psi$ photoproduction cross section is evaluated from

$$d\sigma(ep \to J/\psi + X) = \sum_{i,n} \int dx dy f_{\gamma/c}(x) f_{i/p}(y) \langle \sigma_i^{J/\psi}[n] \rangle d\sigma(\gamma_i \to c\bar{c}[n] + X),$$

where $f_{\gamma/c}(x)$ is the photon flux function, $f_{i/p}(y)$ are the parton distribution functions (PDFs) of the proton, $\langle \sigma_i^{J/\psi}[n] \rangle$ are the LDMEs, and $d\sigma(\gamma_i \to c\bar{c}[n] + X)$ are the partonic cross sections.

Working in the fixed-flavor-number scheme, $i$ runs over the gluon $g$ and the light quarks $q = u, d, s$ and anti-quarks $\bar{q}$. The Fock states contributing through the order of our calculation include $n = \{q^{(1)}, g^{(8)}, q^{(8)}, g^{(8)}\}$. Example Feynman diagrams for partonic LO subprocesses $\gamma_i \to c\bar{c}[n] + X$ as well as virtual- and real-correction diagrams are shown in Fig. 1.

We now describe our theoretical input and the kinematic conditions for our numerical analysis. We set $m_c = m_{J/\psi}/2$, adopt the values of $m_{J/\psi}$, $m_c$, and $\alpha$ from Ref. [11], and use the one-loop (two-loop) formula for $\alpha^{(n_f)}(\mu)$, with $n_f = 3$ active quark flavors, at LO (NLO). As for the proton PDFs, we use set CTEQ6L1 (CTEQ6M) [12] at LO (NLO), which comes with an asymptotic scale parameter of $\Lambda_{\text{QCD}}^4 = 215$ MeV (326 MeV), so that $\Lambda_{\text{QCD}}^3 = 249$ MeV (389 MeV). We evaluate the photon flux function using Eq. (5) of Ref. [13] with the cut-off $Q^2_{\text{max}} = 2 \text{GeV}^2$ [14, 15] on the photon virtuality. Our default choices for the renormalization, factorization, and NRQCD scales are $\mu_r = \mu_f = m_T$ and $\mu_A = m_c$, respectively, where $m_T = \sqrt{p_T^2 + 4m_c^2}$ is the $J/\psi$ transverse mass. We adopt the LDMEs from Ref. [16], which were fitted to Tevatron I data using the CTEQ4 PDFs, because, besides the usual LO set, they also comprise a higher-order-improved set determined by approximately taking into account dominant higher-order effects due to multiple-gluon radiation in inclusive $J/\psi$ hadroproduction, which had been found to be substantial by a Monte Carlo study [17]. We disentangle $\langle \sigma_i^{J/\psi}(S_0) \rangle$ and $\langle \sigma_i^{J/\psi}(P_0) \rangle$, a linear combination of which is fixed by the fit only, as in Ref. [18]. The LO CO LDMEs are similar to the those obtained in Ref. [19] by fitting Tevatron II data using the CTEQ6L1 PDFs [12]. The higher-order-improved CO LDMEs are likely to undershoot the genuine ones, which are presently unknown.

Recently, the H1 Collaboration presented preliminary data on inclusive $J/\psi$ photoproduction taken in collisions of 27.6 GeV electrons or positrons on 920 GeV protons in the HERA II laboratory frame [15]. They nicely agree with their previous measurement at HERA I [14]. These data come as singly differential cross sections in $p_T^2$, $W = \sqrt{(p_T + p_p)^2}$, and $z = (p_{J/\psi} \cdot p_p)/(p_T \cdot p_p)$, in each case with certain acceptance cuts on the other two variables. Here, $p_T$, $p_p$, and $p_{J/\psi}$ are

![Sample diagrams contributing at LO (a and d) and to the virtual (b and e) and real (c and f) NLO corrections.](image_url)
the photon, proton, and $J/\psi$ four-momenta, respectively. In the comparisons below, we impose the same kinematic conditions on our theoretical predictions.

The H1 measurements [14, 15] of the $p_T^2$, $W$, and $z$ distributions of inclusive $J/\psi$ photoproduction are compared with our new NLO predictions in full NRQCD in Fig. 2(a)–(c), respectively. The uncertainty due the LDMEs is indicated by shaded (yellow) bands, whose upper margins (solid lines) refer to the LO set. For comparison, also the default predictions at LO (dashed lines) as well as those of the CSM at NLO (dot-dashed lines) and LO (dotted lines) are shown. Notice that the experimental data are contaminated by the feed-down from heavier charmonia, mainly due to $\psi' \rightarrow J/\psi + X$, which yields an estimated enhancement by about 15% [9]. Furthermore, our predictions do not include resolved photoproduction, which contributes appreciably only at $z < 0.3$ [16], and diffractive production, which is confined to the quasi-elastic domain at $z \approx 1$ and $p_T \approx 0$. These contributions are efficiently suppressed by the cut $0.3 < z < 0.9$ in Figs. 2(a) and (b), so that our comparisons are indeed meaningful. We observe that the NLO corrections enhance the NRQCD cross section, by up to 115%, in the kinematic range considered, except for $z \approx 0.3$, where they are negative. As may be seen from Fig. 2(c), the familiar growth of the LO NRQCD prediction in the upper endpoint region, leading to a breakdown at $z = 1$, is further enhanced at NLO. The solution to this problem clearly lies beyond the fixed-order treatment and may be found in soft collinear effective theory [20]. The experimental data are nicely gathered in the central region of the error bands, except for the two low-$z$ points in Fig. 2(c), which overshoot the NLO NRQCD prediction. However, this apparent disagreement is expected to fade away once the NLO-corrected NRQCD contribution due to resolved photoproduction is included. In fact, the above considerations concerning the large size of the NLO corrections to hadroproduction directly carry over to resolved photoproduction, which proceeds through the same partonic subprocesses. On the other hand, the default CSM predictions significantly undershoot the experimental data, by typically a factor of 4, which has already been observed in Ref. [4]. Except for $p_T^2 \gtrsim 4$ GeV$^2$, the situation is even deteriorated by the inclusion of the NLO corrections.

Despite the caveat concerning our limited knowledge of the CO LDMEs at NLO, we conclude that the H1 data [14, 15] show clear evidence of the existence of CO processes in nature, as predicted by NRQCD, supporting the conclusions previously reached for hadroproduction at the
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Tevatron [2] and two-photon collisions at LEP2 [7]. In order to further substantiate this argument, it is indispensable to complete the NLO analysis of inclusive $J/\psi$ hadroproduction in NRQCD, by treating also the $^{3}P^{0}_{J}$ channels at NLO, so as to permit a genuine NLO fit of the relevant CO LDMEs to Tevatron and CERN LHC data. This goal is greatly facilitated by the technical advancement achieved in the present analysis.

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References