PROCEEDINGS OF SCIENCE

TMD parametrizations

M. Anselmino,[ab] **V. Barone,**[cd] **M. Boglione,**[ab] **U. D'Alesio,**[ef] **A. Kotzinian,**[abg] **S. Melis**[*] **,**[cd] **F. Murgia**[f] **and A. Prokudin**[h]

[a] *Dipartimento di Fisica Teorica, Universita di Torino,*
Via P. Giuria 1, I-10125 Torino, Italy
[b] *INFN, Sezione di Torino,*
Via P. Giuria 1, I-10125 Torino, Italy
[c] *Dipartimento di Scienze e Tecnologie Avanzate, Università del Piemonte Orientale,*
Viale T. Michel 11, I-15121 Alessandria, Italy
[d] *INFN, Gruppo Collegato di Alessandria*
Viale T. Michel 11, I-15121 Alessandria, Italy
[e] *Dipartimento di Fisica, Università di Cagliari,*
Cittadella Universitaria, I-09042 Monserrato (CA), Italy
[f] *INFN,Sezione di Cagliari,*
Cittadella Universitaria, I-09042 Monserrato (CA), Italy
[g] *Yerevan Physics Institute,2 Alikhanyan Brothers St.,Yerevan 375036, Armenia*
[h] *Jefferson Laboratory,12000 Jefferson Avenue, Newport News, VA 23606*
E-mail: anselmino@to.infn.it, barone@to.infn.it, boglione@to.infn.it, umberto.dalesio@ca.infn.it, aram.kotzinian@cern.ch, melis@to.infn.it, francesco.murgia@ca.infn.it, prokudin@jlab.org

We review of our recent extractions of the Transverse Momentum Dependent distribution and fragmentation functions (TMDs) performed by analysing the Semi-Inclusive Deep Inelastic Scattering (SIDIS) and $e^+e^- \to h_1 h_2 X$ data on azimuthal asymmetries.

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[*]Speaker.

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1. Sivers function extraction from polarized SIDIS data

Data on the transverse single spin asymmetry $A_{UT}^{\sin(\phi_h-\phi_S)}$ for polarized SIDIS processes collected by the HERMES [1] and COMPASS [2, 3] collaborations allowed us [4] to extract the Sivers distribution functions [5]. The asymmetry $A_{UT}^{\sin(\phi_h-\phi_S)}$ can be written as:

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp \Delta^N f_{q/p^\uparrow}(x,k_\perp)\sin(\varphi-\phi_S)\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2} D_q^h(z,p_\perp)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp f_{q/p}(x,k_\perp)\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2} D_q^h(z,p_\perp)}, \quad (1.1)$$

where ϕ_S and ϕ_h are the azimuthal angles identifying the directions of the proton spin $\boldsymbol{S}$ and of the outgoing hadron h in the $\gamma^* p$ c.m. frame w.r.t. the leptonic plane; φ defines the direction of the incoming (and outgoing) quark transverse momentum, $\boldsymbol{k}_\perp = k_\perp(\cos\varphi, \sin\varphi, 0)$; $f_{q/p}(x,k_\perp)$ is the unpolarized x and $k_\perp$ dependent parton distribution function (PDF); $d\hat\sigma^{\ell q\to\ell q}/dQ^2$ is the unpolarized cross section for the elementary scattering $\ell q \to \ell q$; $D_q^h(z,p_\perp)$ is the fragmentation function (FF) describing the hadronization of the final quark q into the detected hadron h with a light-cone momentum fraction z and a transverse momentum $\boldsymbol{p}_\perp$ with respect to the fragmenting quark; finally, $\Delta^N f_{q/p^\uparrow}(x,k_\perp)$ is the Sivers function, parametrized in terms of the unpolarized distribution function as:

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{M_N} f_{1T}^\perp(x,k_\perp) = 2\,\mathcal{N}_q(x)\, h(k_\perp)\, f_{q/p}(x,k_\perp)\,, \quad (1.2)$$

$$\text{with} \qquad \mathcal{N}_q(x) = N_q x^{\alpha_q}(1-x)^\beta \frac{(\alpha_q+\beta)^{(\alpha_q+\beta)}}{\alpha_q^{\alpha_q}\beta^\beta}\,, \qquad h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2}\,, \quad (1.3)$$

where $N_q \in [-1,1]$, α_q, β and M_1 (GeV/c) are free parameters to be determined by fitting the experimental data. M_N is the nucleon mass and $f_{1T}^\perp(x,k_\perp)$ is the Sivers function in the Amsterdam notation [6]. For the unpolarized PDFs and FFs, we adopt a factorized Gaussian form

$$f_{q/p}(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}\,, \qquad D_q^h(z,p_\perp) = D_q^h(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}\,, \quad (1.4)$$

with $\langle k_\perp^2\rangle = 0.25$ (GeV$/c)^2$ and $\langle p_\perp^2\rangle = 0.20$ (GeV$/c)^2$ as found in Ref. [7]. Collinear PDFs are taken at Leading Order (LO) as in Ref. [8] while for the FFs we used the DSS set [9].

We fitted the four experimental data sets on $A_{UT}^{\sin(\phi_h-\phi_S)}$, corresponding to pion and kaon production at HERMES [1] and COMPASS [2] with 11 free parameters. We obtain a good description of the data with a $\chi^2/d.o.f. = 1.00$. The best fit values of the parameters can be found in Table I.

2. Transversity and Collins functions from SIDIS and e^+e^- data

A combined fit of SIDIS asymmetries $A_{UT}^{\sin(\phi_h+\phi_S)}$ together with $e^+e^- \to h_1 h_2 X$ data allowed the simultaneous extraction of the transversity distribution and the Collins fragmentation functions [10]. The azimuthal asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}$ is proportional to the convolution of the transversity and the Collins functions:

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto \frac{\sum_q e_q^2\, \Delta_T q(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)}{\sum_q e_q^2\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)}\,, \quad (2.1)$$

$N_u = 0.35^{+0.08}_{-0.08}$	$N_d = -0.90^{+0.43}_{-0.10}$	$N_s = -0.24^{+0.62}_{-0.50}$
$N_{\bar{u}} = 0.04^{+0.22}_{-0.24}$	$N_{\bar{d}} = -0.40^{+0.33}_{-0.44}$	$N_{\bar{s}} = 1^{+0}_{-0.0001}$
$\alpha_u = 0.73^{+0.72}_{-0.58}$	$\alpha_d = 1.08^{+0.82}_{-0.65}$	$\alpha_{sea} = 0.79^{+0.56}_{-0.47}$
$\beta = 3.46^{+4.87}_{-2.90}$	$M_1^2 = 0.34^{+0.30}_{-0.16}$ (GeV/c)^2	

Table 1: Best values of the free parameters for the Sivers function extraction. The errors are determined according to the procedure explained in Appendix A of Ref. [4].

where $\otimes$ stands for a convolution on the transverse momenta; $\Delta_T q(x,k_\perp)$ and $\Delta^N D_{h/q^\uparrow}(z,p_\perp)$ are, respectively, the transversity and the Collins functions. For the analysis of the Belle data, we used the so called "$\cos(\varphi_1+\varphi_2)$ method"in the Collins-Soper frame where the jet thrust axis is used as the $\hat{z}$ direction and the $e^+e^- \to q\,\bar{q}$ scattering defines the $\hat{x}z$ plane (for details see Refs. [11, 12, 13]). We define the quantity A as:

$$A(z_1,z_2,\theta,\varphi_1+\varphi_2) = 1+\frac{1}{4}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(\varphi_1+\varphi_2)\,\frac{\sum_q e_q^2\,\Delta^N D_{h_1/q^\uparrow}(z_1)\,\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\,D_{h_2/\bar{q}}(z_2)}\,, \quad (2.2)$$

where θ is the angle between the lepton beam direction and the thrust axis and

$$\Delta^N D_{h/q^\uparrow}(z) \equiv \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z,p_\perp)\,. \quad (2.3)$$

Unpolarized PDFs and fragmentation functions are taken as in the previous section. For fitting purposes, we define favoured and unfavoured fragmentation functions,

$$D_{\pi^+/u,\bar{d}} = D_{\pi^-/d,\bar{u}} \equiv D_{\rm fav}\,, \qquad D_{\pi^+/d,\bar{u}} = D_{\pi^-/u,\bar{d}} = D_{\pi^\pm/s,\bar{s}} \equiv D_{\rm unf}, \quad (2.4)$$

and similarly for the $\Delta^N D$'s. For the transversity distribution and the Collins function we adopt the following parametrization [12]:

$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\,[f_{q/p}(x)+\Delta q(x)]\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T} \quad (2.5)$$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) \equiv \frac{p_\perp}{zM_h}H_1^\perp(z,p_\perp) = 2\mathcal{N}_q^C(z)\,D_{h/q}(z)h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}\,, \quad (2.6)$$

with

$$\mathcal{N}_q^T(x) = N_q^T\,x^\alpha(1-x)^\beta\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta}\,, \quad (2.7)$$

$$\mathcal{N}_q^C(z) = N_q^C\,z^\gamma(1-z)^\delta\,\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma\delta^\delta}\,, \qquad h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M_h'}\,e^{-p_\perp^2/M_h'^2}\,, \quad (2.8)$$

and $-1 \le N_q^T \le 1$, $-1 \le N_q^C \le 1$. In Eq. (2.6) M_h is the mass of the produced hadron while $H_1^\perp$ denotes the Collins function in the Amsterdam notation. We assume $\langle k_\perp^2\rangle_T = \langle k_\perp^2\rangle$. The helicity distributions $\Delta q(x)$ are taken from Ref. [14]. We take into account, at LO, the QCD evolution of the transversity distribution. For the Collins FF, $\Delta^N D_{h/q^\uparrow}$, as its scale dependence is unknown, we tentatively assume the same Q^2 evolution as for the unpolarized FF, $D_{h/q}$.

We analysed the data released by the HERMES [1] and COMPASS [3] Collaborations for SIDIS, and by the Belle Collaboration [13] for e^+e^- annihilation processes. We fitted data with 9 free parameters obtaining $\chi^2/\text{d.o.f.} = 1.3$ [12]. The best fit parameters can be found in Table II.

$N_u^T = 0.64 \pm 0.34$	$N_d^T = -1.00 \pm 0.02$	$N_{fav}^C = 0.44 \pm 0.07$	$N_{unf}^C = -1.00 \pm 0.06$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$	$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
		$M_h'^2 = 0.91 \pm 0.52$ GeV^2	

Table 2: Best values of the free parameters for the u and d transversity distribution functions and for the favoured and unfavoured Collins fragmentation functions [12].

3. Boer-Mulders function from unpolarized SIDIS data

In Ref. [15] we performed an analysis of the $\cos 2\phi$ asymmetry recently measured by the COMPASS [16] and HERMES [17] collaborations in unpolarized SIDIS. At leading-twist the only $k_\perp$-dependent term contributing to the $\cos 2\phi$ asymmetry contains the Boer-Mulders distribution $h_1^\perp$ coupled to the Collins fragmentation function $H_1^\perp$ of the produced hadron. This contribution to the cross section is given by [6]

$$\left.\frac{\mathrm{d}^5\sigma_{\mathrm{BM}}^{(0)}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_T}\right|_{\cos 2\phi} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\sum_q e_q^2 x(1-y)\int \mathrm{d}^2\boldsymbol{k}_\perp \int \mathrm{d}^2\boldsymbol{p}_\perp\, \delta^2(\boldsymbol{P}_T - z\boldsymbol{k}_\perp - \boldsymbol{p}_\perp) \times \frac{2\,\boldsymbol{h}\cdot\boldsymbol{k}_\perp\,\boldsymbol{h}\cdot\boldsymbol{p}_\perp - \boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{zM_N M_h} h_1^{\perp q}(x,k_\perp^2)\, H_1^{\perp q}(z,p_\perp^2)\cos 2\phi\,, \tag{3.1}$$

where $\boldsymbol{h} \equiv \boldsymbol{P}_T/P_T$. Other contributions to the $\cos 2\phi$ asymmetry come from twist-4 effects. Here we will consider only the twist-4 Cahn contribution. It has the form

$$\left.\frac{\mathrm{d}^5\sigma_{\mathrm{C}}^{(0)}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_T}\right|_{\cos 2\phi} = \frac{8\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\sum_q e_q^2 x(1-y)\int \mathrm{d}^2\boldsymbol{k}_\perp \int \mathrm{d}^2\boldsymbol{p}_\perp\, \delta^2(\boldsymbol{P}_T - z\boldsymbol{k}_\perp - \boldsymbol{p}_\perp) \times \frac{2\,(\boldsymbol{k}_\perp\cdot\boldsymbol{h})^2 - \boldsymbol{k}_\perp^2}{Q^2} f_1^q(x,k_\perp^2)\, D_1^q(z,p_\perp^2)\cos 2\phi\,. \tag{3.2}$$

Notice that the Cahn term is only a part of the complete, still unknown, twist-4 contribution.

The asymmetry determined experimentally is defined as

$$A^{\cos 2\phi} \equiv 2\,\langle\cos 2\phi\rangle = 2\,\frac{\int \mathrm{d}\sigma\,\cos 2\phi}{\int \mathrm{d}\sigma}\,. \tag{3.3}$$

The available data on $\langle\cos 2\phi\rangle$ do not allow a full extraction of the Boer-Mulders function. Thus we simply take $h_1^\perp$ to be proportional to the Sivers function $f_{1T}^\perp$, $h_1^{\perp q}(x,k_\perp^2) = \lambda_q\, f_{1T}^{\perp q}(x,k_\perp^2)$, with a coefficient λ_q to be fitted to the data. The Sivers functions are taken from Ref. [4], see also Sec. 1. Unpolarized PDFs and FFs are taken as in Sec. 1. Fitting the HERMES and COMPASS data we found the following values for the coefficients λ_u and λ_d:

$$\lambda_u = 2.0 \pm 0.1\,, \quad \lambda_d = -1.111 \pm 0.001 \tag{3.4}$$

This implies that $h_1^{\perp u}$ and $h_1^{\perp d}$ are both negative. The χ^2 per degree of freedom of our fit is $\chi^2/d.o.f. = 3.73$. The quality of the description reflects our knowledge of the underlying Cahn mechanism, in particular of the dependence of $\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$ on x and z. More experimental information is needed to unravel this dependence. At present, the quality is good enough to draw some preliminary conclusion on the sign and the approximate size of the Boer-Mulders functions, which appear to be compatible with the theoretical expectations.

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