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Scale evolution of kt-distributions

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We review some of the features of the evolutions equations for transverse momentum dependent parton distributions recently proposed by us. We briefly describe the new ingredients entering the equations and their relationship with ordinary evolution equations. We comment on possible choices for the initial conditions and then show results for the evolved distributions obtained by numerical implementation of the equations. By computing the average transverse momentum at different scale we highlight some general properties of the solutions.

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1. Introduction

Transverse momentum dependent, or equivalently, k_t-distributions are currently object of intense research activity due to their wide range of applicability in the description of hadron initiated hard processes [1, 2]. The use of k_t-distributions is indeed phenomenologically appealing since observables constructed upon them show a reasonable agreement with data already in lowest order, which is not the case for predictions based on collinear factorization at the same accuracy, see for instance the discussion in Ref. [3]. Their correct formalization in quantum-chromodynamics appears to be well established only at high energy [4]. Away from this limit the situation is still unclear, although detailed investigations are present in the literature [5, 6, 7]. The relevant issue of factorization within a k_t-dependent approach has also been investigated [8]. The latter property together with a precise knowledge of the scale dependence of k_t-distributions would allow to relate to each other data coming from experiments at different energies and, more important, to test factorization quantitatively. Although a definitive answer is absent in the literature, there have been however some attempts, see for example [9, 10]. In particular we will focus on the equations proposed by us in Ref. [11] which are the space-like version of the ones proposed in Ref. [12]. In a subsequent phemonelogical study [13], performed in the context of semi-inclusive deep inelastic scattering, it was shown that a reasonable description of data could be obtained once unintegrated evolution equations were solved with suitable, but motivated, initial conditions and assuming factorization for the cross-sections of interest. This result stimulated us to apply the same formalism to the description Drell-Yan type processes in hadronic collisions [14]. The p_t-spectrum of the gauge boson has, in fact, a rich transverse structure and involves both perturbative and non-perturbative aspects of the underlying theory. The resummation of the perturbative series in the multiple soft gluon emission limit can be accomodated by using properly modified unintegrated evolution equations [15] so that the structure of the non-perturbative form factor can be investigated.

2. Space-like k_t-dependent evolution equations

Transverse momentum dependent parton distribution function, $\mathscr{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)$, give the probability to find, at a given scale Q^2, a parton i with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ relative to the parent hadron momentum. The evolution equations for k_t-distributions proposed in Refs. [11, 13] read

$$Q^2 \frac{d\mathscr{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} P_{ji}(u) \cdot \\ \cdot \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \delta\big((1-u)Q^2 - l_\perp^2\big) \mathscr{F}_P^j\Big(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{l}_\perp}{u}\Big), \quad (2.1)$$

and resum large logarithms associated to the emission of collinear partons. However, at variance with ordinary evolutions equations [16], the transverse momentum generated at each branching, $\boldsymbol{l}_\perp$, is explicitely taken into accont kinematically and eventually adds to the non-perturbative one due to Fermi motion of the parton in the parent hadron. If we now consider $\mathscr{F}$ to be the bare distribution of a quark in a quark, neglecting flavour indeces, we may insert the source term $\mathscr{F}(x, Q^2, \boldsymbol{k}_\perp) =$

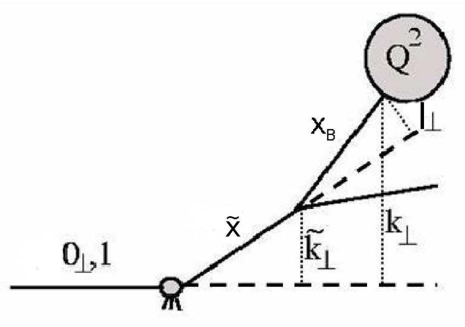

Figure 1: Pictorial representation of the proposed evolution equation. The longitudinal and transverse component of the partons involved in the splitting $\widetilde{k} \to k+l$ are indicated.

$\delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp)$ into eq. (2.1) obtaining, up to $\mathscr{O}(\alpha_s)$, the following result

$$\mathscr{F}(x,Q^2,\boldsymbol{k}_\perp) = \delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp) + \frac{\alpha_s}{2\pi}\frac{1}{\pi}\frac{1}{k_\perp^2}P(x)\,, \tag{2.2}$$

which indeed exposes the singularity associated to the emission of a collinear parton weighted by splitting functions $P(x)$ [16]. In order to clarify the role of the new ingredients appearing in eq. (2.1), it proves useful to consider just one emission, see Fig. (1). The small blob at the bottom left of the figure then symbolizes the iteration of emissions in the parton ladder of whose only the last, $\widetilde{k} \to k+l$, is explicitely shown. Transverse momenta $\widetilde{\boldsymbol{k}}_\perp$ and $\boldsymbol{k}_\perp$ are defined with respect to the incoming hadron direction, while $\boldsymbol{l}_\perp$ is defined with respect to $\widetilde{k}$ direction. The transverse momentum $\widetilde{\boldsymbol{k}}_\perp$ and longitudinal momentum $\widetilde{x}$ of the branching parton $\widetilde{k}$ are written as a function of the relative transverse momentum $\boldsymbol{l}_\perp$ and the fractional momentum u appearing in the splitting. They can be expressed as $\widetilde{\boldsymbol{k}}_\perp = (\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)/u$ and $\widetilde{x} = x/u$, respectively. The former is a result of the transverse boost from $\widetilde{k}$ to hadron direction [17]. Both can be found in the distribution under convolution in the right hand side of eq. (2.1). The additional integration $d^2\boldsymbol{l}_\perp$and the δ-function appearing in eq. (2.1) are associated with the phase space and mass-shell contraint of the emitted partons. The latter can be written as

$$l_\perp^2 = -(1-u)k^2 + u(1-u)\widetilde{k}^2\,, \tag{2.3}$$

where the emission of massless partons has been assumed ($l^2 = 0$). As is well known, logarithmically enanched contributions arise when the interacting parton virtuality increases along the ladder. Therefore in the limit $\widetilde{k}^2 \ll k^2$ we may neglect $\widetilde{k}^2$ with respect to k^2 and set $-k^2 = Q^2$, obtaining $l_\perp^2 = (1-u)Q^2$, which can be found in eq. (2.1). Furthermore, we assume k_t-distributions to fulfil the normalization condition:

$$\int d^2\boldsymbol{k}_\perp \mathscr{F}_P^i(x_B,Q^2,\boldsymbol{k}_\perp) = f_P^i(x_B,Q^2)\,, \tag{2.4}$$

where f_P^i are ordinary parton distributions. It is important to remark that integrating eq. (2.1) over $d^2\boldsymbol{k}_\perp$ and by using eq. (2.4) we recover the ordinary evolution equations for f_P^i [16] .

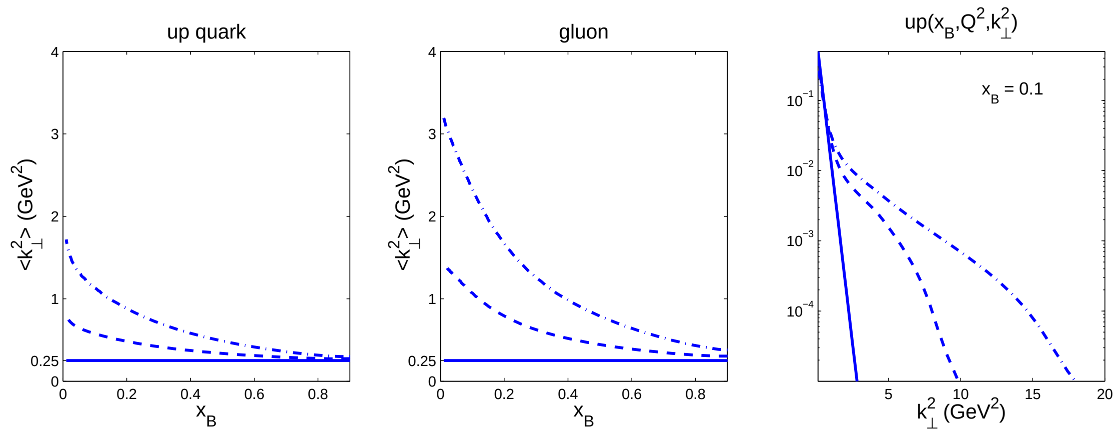

Figure 2: Left and middle panel: average transverse momentum $\langle k_\perp^2 \rangle$ for up-quark and gluon at three different scales: $Q_0^2 = 5$ GeV^2 $(-)$, $Q^2 = 10$ GeV^2 $(--)$ and $Q^2 = 20$ GeV^2 $(-\cdot-)$. Right panel: the transverse spectrum of the up quark distributions at fixed $x_B = 0.1$ at the previously indicated three different scales.

3. Numerical solutions

The unintegrated evolution equations, eq. (2.1), are numerically solved by a finite difference method on a discrete grid in $(x_B, \boldsymbol{k}_\perp)$ space. The initial conditions at the starting scale $Q_0^2 = 5$ GeV^2 are chosen to have the factorized form

$$\mathscr{F}_P^i(x_B, Q_0^2, \boldsymbol{k}_\perp) = f_P^i(x_B, Q_0^2) \frac{1}{\pi < k_{\perp,i}^2 >} e^{\frac{-k_\perp^2}{<k_{\perp,i}^2>}} \qquad i = q, \bar{q}, g \tag{3.1}$$

where $f_P^i(x_B, Q_0^2)$ are ordinary parton distributions [18]. The choise of a energy (x_B) independent guassian transverse factor in eq. (3.1) is supported, for example, by recent semi-inclusive deep inelastic scattering data at low energy [19]. The width $\langle k_{\perp,i}^2 \rangle$ are fixed to a value of 0.25 GeV^2 both for quarks and gluons [13]. Since the initial conditions, eq. (3.1), by construction fulfil eq. (2.4), the latter can be checked for every values of x_B and Q^2 to estimate and, eventually, to improve the numerical accuracy of the evolution. For this check to be meaningful we adopt the same flavour scheme and coupling values used in Ref. [18]. The evolved distributions show indeed some interesting properties. In the left and middle panel of Fig.(2) we present the average transverse momentum, $\langle k_\perp^2 \rangle$, as a function of x_B calculated at three different scale for up quark and gluon, respectively. In both cases, the averaged transverse momentum at the final scale, $Q^2 = 20$ GeV^2, increases with decreasing x_B. The effect is more evident in the gluon case, due to the singular behaviour of $P_{gg}(u)$ at small u. In the opposite limit, $x_B \to 1$, the averaged transverse momentum approaches the value given in the initial condition and indicated by the horizontal line in left and middle panel of Fig.(2). In this limit, phase space only allows the emission of soft partons which generate negligible trasverse momentum. In the right panel of Fig. (2) it is shown the up-quark distribution at fixed $x_B = 0.1$ as a function of $k_\perp^2$. Interestingly, the factorized form of eq. (3.1) is not preserved under evolution. The guassian dependence on $k_\perp^2$ at Q_0^2 is turned into a inverse power law $1/(k_\perp^2)^a$ at the final scale Q^2, the latter dependence in qualitative agreement with matrix elements behaviour at large $k_\perp^2$, eq. (2.2). We also observe that, as Q^2 increases, the distributions start to *populate* the higher $k_\perp^2$ region, as an effect of the leading logarithmic approximation built-in the evolution equations.

Conclusions

In this contribution we have reviewed some of the features of the unintegrated evolution equations proposed by us and discussed some peculiar properties of the evolved k_t-distributions obtained by numerical solution. The use of unintegrated distributions is indeed phenomenologically appealing [13, 14]. Most interestingly, the analysis of jet observables within this framework may give access to valuable informations on the unintegrated gluon distributions which describes hadron structure as probed in high energy collisions. However, the departure from a pure collinear scheme introduces a number of non trivial problems which are at present under investigation by many groups. This implies that, to date, any conclusive statement on the validity of k_t-distributions approach in general, and on the proposed evolution equations in particular, must wait for further theoretical developments.

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