Neutrino matter with PLANCK

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After reviewing the main mechanisms by which cosmological measurements constrain the sum of neutrino masses, I give the current reached upper limits, emphasizing the level of model-dependence. A large improvement is to be expected with PLANCK’s satellite data, on which I give some news, in particular due to the characterization of the CMB-lensing effect. It will however require a thorough control of many systematics effects upon which progress has been made recently.
1. Neutrino masses in cosmology

1.1 Neutrino physics

During the last decade, a tremendous amount of oscillations experiments brought the clue that neutrinos do have a mass and therefore contribute to the matter budget of the universe [1]. These experiments aim at measuring the full leptonic mixing matrix (similar to the CKM one for fermions) i.e. masses and mixing angles. Unfortunately the oscillation probabilities are related to the difference squared mass of the eigenstates and one is therefore left with a floating absolute normalization. Also, the sign is only experimentally accessible for solar neutrinos (related to $\Delta m^2_{21}$) The combination, through a complicated global fit, gives [1]:

$$m^2_2 - m^2_1 = (7.67 \pm 0.22) \times 10^{-5} \text{ eV}^2$$

$$|m^2_3 - m^2_1| = (2.40 \pm 0.15) \times 10^{-3} \text{ eV}^2$$

(1.1)

where $m_i$ denote the mass eigenstates. The absolute value in these measurements leads to two solutions: the normal and inverted hierarchies, illustrated on Fig. 1.

![Figure 1: Neutrinos mass pattern obtained in the normal (a) / inverted (b) schemes.](image)

The absolute mass scale (level 0) is unknown, but one can get a lower bound by assuming the lighter state to be massless:

$$M_\nu \gtrsim 0.05 (0.10) \text{ eV}$$

(1.2)

for the normal (inverted) hierarchy.

The challenging measurement of neutrino masses in direct $\beta$ decays gives the most direct access to the absolute scale $m_0$. Current results [2] gives an upper bound of $m_0 \lesssim 2 \text{ eV}$ which gives conservatively:

$$M_\nu \lesssim 6 \text{ eV}$$

(1.3)

One waits eagerly for some new results by the 10m spectrometer KATRIN[3] where limits up to $m_0 \simeq 0.2 \text{ eV}$ could be reached...
1.2 How cosmology constrains neutrinos

1.2.1 Cosmic neutrino background

Cosmology provides currently the most precise limits on the sum of neutrino masses ($M_\nu$). This is however an indirect measurement and, as will be discussed, results depend somewhat on the hypotheses and datasets used.

In the following I will only focus on "standard" physics, i.e. 3 massive neutrinos with mixing. I will assume a negligible chemical potential since mixing balances the values among species and Big Bang Nucleosynthesis constrains the electronic one to be close to 0 (for a more general treatment see [4]). I will only give general ideas about neutrino physics in cosmology and forward the interested reader to the very complete report [5].

The history of neutrinos is similar, at the beginning, to that of CMB photons: they decouple at $\simeq 1\,\text{MeV}$, then freeze out with expansion. While thermalized, their mean energy is related to their temperature through:

$$< E_\nu(z) > \simeq 3.15 T_\nu(z) \quad (1.4)$$

where the neutrinos temperature is related, through entropy conservation, to the CMB one by:

$$T_\nu = \left( \frac{4}{11} \right)^{1/3} T_\gamma.$$

Given our knowledge of the CMB temperature today $T_\gamma^0 \simeq 2.7\,\text{K}$ the current neutrino temperature is

$$T_\nu^0 \simeq 1.9\,\text{K} = 1.1 \times 10^{-4} \,\text{eV} \quad (1.5)$$

By comparing their energy Eq.(1.4) to their mass (Fig.1) one sees that at least 2 neutrinos species are not relativistic anymore today. This is true even if the lightest state is massless and for both hierarchies. Neutrinos must therefore be accounted for in the matter budget of the universe. Their non-relativistic transition occurred when $< E > \simeq m_i$ at a redshift of:

$$1 + z_{NR} \simeq 1900 \left( \frac{m_i}{1\,\text{eV}} \right) \quad (1.6)$$

When it was discovered that neutrinos are massive, there has been some excitement about the fact that they could actually be dark matter, but this was tempered by the fact that hot dark matter does not cluster easily to form galaxies. Indeed such matter would lead to a top-down scenario of structure formation (larger structure would form, then crunch into smaller) which is in clear contradiction with observations.

Also, Trumaine and Gunn [6] exhibited a strong bound that come from the fact that, since neutrinos suffer no inelastic collisions, they cannot be squeezed beyond a certain point. By comparing some dwarfs galaxies core radius to their velocity dispersion, one should assume a very heavy neutrino mass [7] $M_\nu \gtrsim 100 - 300\,\text{eV}$ in clear contradiction with the direct upper limit Eq.(1.3).

On the cosmological side, equating the neutrino energy density to the measured matter one, would require $\Omega_\nu \simeq \frac{M_\nu(\text{eV})}{50\,\text{eV}} \simeq 0.22 \rightarrow M_\nu \simeq 11\,\text{eV}$, which is still too high.

However it is clear that neutrinos contribute to some amount to the matter budget. Anticipating conservative bounds from cosmology $M_\nu \lesssim 1\,\text{eV}$:

$$0.4\% \lesssim f_\nu = \frac{\Omega_\nu}{\Omega_M} \lesssim 8\% \quad (1.7)$$
1.2.2 Effect of neutrinos on CMB power spectra

Direct effect  There are two effects that contribute to the measured CMB power spectra. The first one appears in the case where the non-relativistic transition occurs before the photons decoupling, so that it leaves an imprint in the spectra. Since $z_{\text{dec}} \simeq 1100$, this can only happen (see Eq.(1.6)) if $M_\nu \gtrsim 1.7 \text{eV}$ (more precise calculations [8] show that the limit is rather about $\simeq 1.5 \text{eV}$). In other words, if one just uses CMB power spectra for neutrino mass determination, this is an incompressible limit (that has already been reached by WMAP, see section 1.3).

However, PLANCK will have access to CMB-lensing measurement too (section 3) which will allow to go beyond that limit with a CMB-only consistent dataset...

Indirect effect  There is also an indirect effect on the FRW metric that allows a more precise determination of the neutrino matter when combining CMB measurements with other probes. The scale factor at matter-radiation equality is classically given by $a_{\text{eq}} = \Omega_0^0 r_0 \Omega_0^m$ where the upper-script "0" recalls that we consider densities today. However we know that neutrinos are not relativistic anymore, so the expression must be modified:

$$a_{\text{eq}} = \frac{\Omega_0^0 r_0 (1 - f_\nu)}{(1 - f_\nu) \Omega_0^m} \quad (1.8)$$

$f_\nu$ modifies the moment of matter-radiation equality, in particular the size of the sound horizon at decoupling which results in a slight shift in the temperature power spectrum, as one sees on Fig.2(a). The most visible effect is also an enhancement of the first peak due to the ISW effect just after the recombination.

Degeneracies  It is well known (e.g. [9]) that CMB suffers from what is called the "geometrical degeneracy" which in simple words means that one cannot fully constrain a curvature with just one scale in an expanding universe. Therefore $\Omega_M$ is degenerated for instance with $\Omega_\Lambda$ which will plague the determination of $f_\nu$. Indeed to constrain $f_\nu$ in Eq.(1.8) one needs to pinpoint $\Omega_m$, otherwise variations will be absorbed into this term. This simple argument shows that CMB data must be combined with other probes that fixes some scale, as $H_0$, BAO, and/or large scale structures power spectrum.

1.2.3 Effect of neutrinos on Large Scale Structures

The main effect of neutrinos on the large scale structure (LSS) power spectrum ($P(k)$) is different and due to their free-streaming. Since neutrinos have a large velocity they cannot cluster below their Jeans-length. Therefore they should not appear in the matter budget below a scale $k_{FS}(z_{NR}) \simeq 10^{-2} \sqrt{\frac{m_\nu}{1 \text{eV}}} \ h \ Mpc^{-1}$ and the suppression should be proportional to the power. Computations of the Boltzmann equations [5] shows that the factor is close to:

$$\frac{p_{f_\nu} - p_{f_\nu=0}}{p_{f_\nu=0}} \simeq -8 f_\nu \quad (1.9)$$

which is a quite strong constraint for a few % $f_\nu$.

Unfortunately LSS measurements are plagued by uncertainties on the bias (luminosity to mass ratio) and non-linear corrections. In particular the high $k$ region (accessible through Lyman-\(\alpha\) analyzes) requires neutrinos-dedicated thermodynamical simulations. The development of a consistent
Figure 2: Effect of massive neutrinos ($M_{\nu}=1\,\text{eV}$) on CMB (a) and Large scale structure $P(k)$ (b) power spectra, using the Boltzmann code “CAMB”. One sees an increase of the first peak due to the ISW effect and a slight shift due to the change in the matter-radiation equality scale factor. For $P(k)$ there is a constant suppression for $k \gtrsim 10^{-2}\,h\,\text{Mpc}^{-1}$ due to the free-streaming of neutrinos.

treatment of massive neutrinos in non-linear structure formation is an active and promising field, but neutrinos bounds given today using this probe are probably over-optimistic.

1.3 Current status on neutrino masses from cosmology

Before presenting the various limits obtained on the neutrino masses, a word of caution is necessary to understand the large spread in the results. As specified before, constraints on neutrinos mass can only be obtained through a combined cosmological fit, in order to resolve the geometrical degeneracy. In order to explain the large disparity of limits published in the literature, the following facts have to be kept in mind:

- global fits are generally performed in the context of a Bayesian analysis which requires the use of priors which can vary according to each scientist sensibility.
- systematic errors related to the different probes, are often treated apart or even sometimes forgotten.
- different cosmological probes are used in global fits depending on the trust in systematics error knowledge, in particular for Supernovae, $Li - \alpha$ forest, LSS data. Recall that in order
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to solve the geometrical degeneracy, only one scale complementary to the CMB one, is necessary and can be chosen as the measurement of the Hubble constant and/or BAO scale. Using other probes will lower the neutrino mass upper limit, but to the extent of introducing some level of "doubt".

• we are living the golden age of cosmology. The "cosmological Standard Model" is still being build up. $\Lambda$CDM is a minimal model that allows to fit well the current data with 6 parameters (half being non-fundamental). It is also referred to simply as a vanilla model. As is well known, a pure cosmological constant is disfavored by theorists due to its strong fine-tuning, which lead to the introduction of the "wCDM " model where the equation of state of the dark energy is constant but not necessarily equal to -1. Until we understand the nature of dark energy there is no strong reasons for this parameter to be independent of redshift. Other uncertainties are related to the nature of primordial fluctuations (tensorial contribution, iso-curvature modes, power spectrum) where some simplifications happen if we assume an initial single-field slow-roll inflation mechanism. Cosmological combinations are sensitive to the model assumed i.e. the number of parameters being fitted. It is therefore crucial to specify explicitly the underlying theoretical model. Results (including neutrino mass limits) may change if the model evolves.

We compile in Table 1 some recent results obtained on the neutrino mass limit emphasizing the hypothesis used. The vanilla model corresponds to a flat universe with 3 neutrinos, negligible tensorial perturbations, a pure power law spectrum of primordial perturbations, negligible iso-curvature contributions and a pure cosmological constant as the source of Dark Energy. These limits are obtained in a Bayesian framework. Many more results can be found in [10] including an interesting comparison to a more "frequentist" approach, leading to similar results.

<table>
<thead>
<tr>
<th>model</th>
<th>dataset</th>
<th>$M_\nu$ 95% CL limit (eV)</th>
<th>ref</th>
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<tr>
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<td>1.3</td>
<td>[11]</td>
</tr>
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<td>[12]</td>
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<td>[11]</td>
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<tr>
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<td>0.89</td>
<td>[13]</td>
</tr>
<tr>
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<td>CMB +H0+BAO</td>
<td>1.47</td>
<td>[13]</td>
</tr>
</tbody>
</table>

Table 1: Compilation of a few robust neutrinos mass limits obtained in cosmology. "CMB" refers to a combination of WMAP7+ACBAR+BICEP+QuAd data, $H_0 = 72.2 \pm 3.6 \text{km s}^{-1}\text{Mpc}^{-1}$ [14], and "BAO" refers to the scale obtained in [15]. More stringent limits can be obtained by adding LSS and/or Supernovae data (see refs, also [10])

Let us highlight a few point:

• CMB-only : we are clearly hitting already the limit described in section 1.2.2. PLANCK data will not allow a much better determination using the same procedure. However, as will be shown, the inclusion of the CMB lensing power spectrum will lower this limit drastically still using a consistent CMB-only dataset.
• $H_0$+BAO : even though the error on $H_0$ is slightly aggressive, these limits are robust. We directly see how relaxing $w$ recovers the geometrical degeneracy and brings back the CMB-only limit.

• An interesting results is pointed out in [13]: when leaving free the number of relativistic species at recombination ($N_{\text{eff}}$) one obtains still very decent limits in $\Lambda$CDM and wCDM models (that can be improved using LSS data).

In this plethora of results, the reader may look for a "reasonable" upper limit on neutrino mass in cosmology. If you believe that $\Lambda$CDM describes the final Standard Cosmological Model (or that Large scale measurements can be used safely) an upper limit of $M_\nu \lesssim 0.6$ eV makes sense. Otherwise $M_\nu \lesssim 1$ eV is safe. In any case, this is the best upper limit obtained in neutrino physics.

2. PLANCK

PLANCK\(^1\) is an ESA space mission designed to measure accurately the CMB temperature anisotropies, including its polarized part. It is composed of a 1.5 m telescope and two instruments lead by consortia: the Low Frequency Instrument (LFI) includes 22 radiometers in the 30-70 GHz range, with spatial resolutions about 10-25 arcmin. The High Frequency Instrument (HFI) is composed of 52 bolometers in the 100-857 GHz range, including 32 polarized, with a 5-9 arcmin spatial resolution. This is the first cryogenic spatial instrument, located at the Lagrange point L2 and operating at only 100 mK. The strong points of PLANCK are:

• a clean environment (space)
• a large number of bands (9) for foreground removal
• a good resolution per detector
• a very low noise and stability at the $\mu$K level

Beside many astrophysics studies, it will allow an exquisite determination of the CMB power spectra in a complete multipole range up to $\ell \simeq 2500$ for polarization and temperature fluctuations. It may even discover primordial tensorial fluctuations, if they are above $\simeq 5\%$ of the scalar ones [16].

PLANCK had a perfect launch with Herschel on may 14, 2009 from an Ariane 5 rocket in Kourou. After $\simeq 2$ months of travel, during which all instruments were checked, it reached the Lagrange point L2, located at around 1.5 million km’s from the Earth. It then undergone a commissioning phase, where all parameters were optimized : it was checked that the instruments behaved as characterized on ground, and the $He^3$–$He^4$ dilution flow which allows to reach ultimately the 100 mK goal and limits the instrument lifetime, was tuned to a low value. Therefore, while originally designed to perform 2 complete surveys of the sky, PLANCK will actually be able to perform 4, until the end of the mission in January 2012.

A first bunch of paper, concerning astrophysics results will be issued in beg. 2011. The cosmological papers will be released as data become public, in 2012. PLANCK policy does not

\(^{1}\)homepage: http://www.rssd.esa.int/planck
allow the presentation of scientific results before, but one can say that the detectors behaves as characterized on ground [17] and that results at least at the level of its scientific program [9] are to be expected.

Given its spectacular performances, its foreground rejection power and its polarization capabilities, PLANCK will allow a huge improvement on cosmological fits [9]. On the neutrino side, an analysis similar to the WMAP one, will allow roughly speaking a factor of 2 improvement on the upper limit [13]. But there is more...

3. CMB-lensing with PLANCK

In their journey from the last scattering surface to PLANCK detectors, CMB photons encounters slight perturbations from matter gradients, a phenomenon known as gravitational lensing. The effect is a tiny remapping of the photon direction, even though the overall brightness is preserved:

$$T_{\text{obs}}(\hat{n}) = T_{\text{CMB}}(\hat{n} + \vec{d}(\hat{n}))$$

In standard cosmology, the deflection is low: it has an RMS of about 2.7 arcmin. This is insufficient to be resolved by WMAP radiometers, but is at PLANCK’s reach, since HFI CMB channels have a typical spatial resolution of this size.

In the line of sight formalism, the deflection field is related to the gaussian lensing potential:

$$\vec{d}(\hat{n}) = \vec{\nabla} \phi(\hat{n})$$

and one can reconstruct its power spectrum $C_\ell^\phi$ which adds some new interesting information. Indeed, the lensing potential probes the power spectrum of matter [18]

$$C_\ell^\phi \simeq 8\pi^2/\ell^3 \int_0^{r_{\text{LS}}} r dr P_\Psi(\ell/r, \eta_0 - r) \left( \frac{r - r_{\text{LS}}}{r_{\text{LS}}} \right)^2$$

where $r_{\text{LS}}$ denotes the comoving distance to the last scattering surface, and $P_\Psi(k, \eta)$ is related to the conventional matter power spectrum.

Therefore, by reconstructing $C_\ell^\phi$, we obtain a direct determination of the matter power spectrum by just using CMB measurement. This measurement is not affected by the bias, and non-linear corrections are very weak.

The interest of including the lensing effect into cosmological fits is two-fold:

1. the lensing potential slightly changes (at the % level) the measured CMB temperature/polarization power spectra. Taking into account this effect, allows to lift the geometrical degeneracy [19]

2. using $C_\ell^\phi$ obviously adds some information on the matter power spectrum that is complementary to the CMB one.

It is particularly interesting for neutrino mass determination because most of the $C_\ell^\phi$ power comes from low redshifts ($z \lesssim 5$), i.e. a regime in which neutrinos clearly contribute to the matter budget (see Eq.(1.6)). Therefore one has access to the matter content both with (lensing) / without (CMB power) neutrinos contribution, using a single consistent dataset.
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This is illustrated on Figure 3 which shows a realistic level of expected error bars one can achieve in the PLANCK 217 GHz band, starting from the red curve model. The light blue curve shows the best fit model to WMAP CMB-only data, with a neutrino contribution set to its upper limit (see section 1.3) \( M_\nu = 1.3 \text{ eV} \). The lensing power spectrum would clearly constrain the CMB-only measurement.

![Figure 3: Reconstructed deflection power spectrum (points with error bars) on a simulated PLANCK map in the 217GHz band using parameters from the \( \Lambda \)CDM WMAP best fit model (red curve). The light blue one represents the WMAP-only best fit model assuming the measured upper limit \( M_\nu = 1.3 \text{ eV} \). There is a clear discrimination between the two models (from L. Perotto).](image)

Despite its apparent simplicity Eq.(3.1), the lensing reconstruction is difficult and prone to systematic errors, especially at low \( \ell \). On the statistical ground, an estimator based on 4-point correlation functions, has been worked out [20]-[21] and its properties largely studied [22][23][24].

In analyzing PLANCK’s data, many systematics have to be studied among which:

- **point sources**: they provide a strong lensing signal. The most luminous ones are to be detected (and masked) by PLANCK, but what about the residual diffuse background? some preliminary studies [25] indicate that the effect is negligible.

- **foregrounds subtraction**: the different frequency maps will be combined, by various component separation algorithms [26], to provide a cleaned CMB map. Will this process preserve lensing? It seems to be the case for at least one method [25].

- **Galaxy masking**: even after component separation, some part of our Galaxy remains and needs to be masked. Formally the likelihood solution of the estimator would require the inversion a \( \approx 50 \times 10^6 \times 50 \times 10^6 \) matrix a challenging task for today’s computers. One solution to this problem, is to inpaint the missing parts by an adequate algorithm that preserves lensing statistics, and recover a full sky map. One algorithm is shown to work in [25]. Another
method can be to work on small patches of the sky, avoiding the gaps. This however requires
to restore periodicity by apodizing the patch, which results into some new systematics...

- **beam asymmetries**: at the level of lensing, the fact that the PSFs are not exactly circular
  affects the low multipoles determination [27].

- **noise inhomogeneity**: given PLANCK scanning strategy, some pixels are visited more often
  that others: this affects the $C_\ell^\phi$ reconstruction. An analytic correction has been proposed for
  low $\ell \lesssim 1000$ modes[28]. Unfortunately PLANCK’s scanning strategy induces higher order
  correlations that bias the reconstruction for larger $\ell$ values too.

- **data treatment**: some 1D signal treatments are applied in order to take into account the
  instruments transfer function, remove spikes due to glitches, etc.. Their effect needs to be
  studied on detailed simulations of the instrument.

- **map-making**: the process of projecting and averaging the (cleaned) timelines on a pixelized
  sky map, using a nearest grid point method, induces some mean remapping within the pixel,
  that may induce a lensing-like effect. It was verified that when combining several channel
  detectors, and using a cutoff $\ell_{\text{max}} = 2000$ in the estimator, this only affects the very low $\ell$
  part of the estimator.

**Conclusion**

Neutrinos contribute to the matter budget of the universe at the $0.4\% < f_\nu = \Omega_\nu / \Omega_M \lesssim 8\%$ level. Cosmology provides an indirect determination of their mass sum, by combining several observables. Upper limits vary according to the authors because of different datasets combinations and explored space of parameters. The “Standard Cosmological Model” is still being build up: we are still in the Dark Ages (Matter and Energy), and don’t know the nature of primordial fluctuations. But we do have a minimal model that is consistent with all cosmological probes and it will be exciting to complement it with PLANCK’s high precision CMB data.

Since we know that standard neutrinos are massive today (at least 2 of them), there is no reason to not include them in cosmological fits. This will degrade the precision on $\Omega_m$ but gives finally a more reliable measurement [29].

Neutrinos masses will be measured within the next decade by cosmology. PLANCK’s data, including its CMB-lensing measurement, will allow to reach a limit $M_\nu \lesssim 0.3\text{eV}$. In case of a measurement, the neutrino hierarchy would remain a mystery [30] for a long time. Theoretical improvements are also to be expected on non-linear structure formation which will allow to use more safely LSS surveys.

Then, the next generation experiments as LSST, Euclid, have the potential to reach the minimal allowed value of $M_\nu \gtrsim 0.05(0.10)\text{eV}$ (direct (inverted) hierarchy) through weak lensing survey. Interestingly, some hints on the hierarchy could then be obtained[30].

There is no such thing as a direct measurement. The expected upper limit of the $\beta$ decay experiment KATRIN could be as low as $m_\nu \lesssim 0.2\text{eV}$, which gives about 0.5 eV on $M_\nu$. This will give an interesting prior for cosmological fits.
Cosmology doesn’t tell us anything about the nature of neutrinos (Dirac or Majorana). It can however complement $2\beta$ decays experiments [31]: for instance a negative result on $2\beta$ measurements and a measurement from cosmology would point out to a Dirac type.

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References


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