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The dark matter is mostly an axion BEC

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Axions differ from ordinary cold dark matter, such as WIMPs or sterile neutrinos, because they form a Bose-Einstein condensate (BEC). As a result, axions accreting onto a galactic halo fall in with net overall rotation. In contrast, ordinary CDM accretes onto galactic halos with an irrotational velocity field. The inner caustics are different in the two cases. It is shown that if the dark matter is axions, the phase space structure of the halos of isolated disk galaxies, such as the Milky Way, is precisely that of the caustic ring model for which observational support exists. The other dark matter candidates predict a far more chaotic phase space structure for galactic halos.

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1. Introduction

One of the outstanding problems in science today is the identity of the dark matter of the universe [1]. The existence of dark matter is implied by a large number of observations, including the dynamics of galaxy clusters, the rotation curves of individual galaxies, the abundances of light elements, gravitational lensing, and the anisotropies of the cosmic microwave background radiation. The energy density fraction of the universe in dark matter is 23%. The dark matter must be non-baryonic, cold and collisionless. *Cold* means that the primordial velocity dispersion of the dark matter particles is sufficiently small, less than about $10^{-8}\,c$ today, so that it may be set equal to zero as far as the formation of large scale structure and galactic halos is concerned. *Collisionless* means that the dark matter particles have, in first approximation, only gravitational interactions. Particles with the required properties are referred to as 'cold dark matter' (CDM). The leading CDM candidates are weakly interacting massive particles (WIMPs) with mass in the 100 GeV range, axions with mass in the 10^{-5} eV range, and sterile neutrinos with mass in the keV range.

Today I argue that the dark matter is axions [2, 3]. The argument has three parts. First, axions behave differently from the other forms of cold dark matter because they form a Bose-Einstein condensate [2]. Second, there is a tool to distinguish axion BEC from the other forms of CDM on the basis of observation, namely the study of the inner caustics of galactic halos. Third, the evidence for caustic rings of dark matter is consistent in every aspect with axion BEC, but not with WIMPs or sterile neutrinos.

Before I start, let me mention that H. Baer and his collaborators have shown that in many supersymmetric extensions of the Standard Model, the dark matter is axions, entirely or in part [4].

2. Axions

Shortly after the Standard Model of elementary particles was established, the axion was postulated [5] to explain why the strong interactions conserve the discrete symmetries P and CP. For our purposes the action density for the axion field $\varphi(x)$ may be taken to be

$$\mathscr{L}_a = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 - \ldots \tag{2.1}$$

where m is the axion mass. The self-coupling strength is

$$\lambda = \frac{m^2}{f^2}\,\frac{m_d^3 + m_u^3}{(m_d + m_u)^3} \simeq 0.35\,\frac{m^2}{f^2} \tag{2.2}$$

in terms of the axion decay constant f and the masses m_u and m_d of the up and down quarks. In Eq. (2.1), the dots represent higher order axion self-interactions and interactions of the axion with other particles. All axion couplings and the axion mass

$$m \simeq 6\cdot 10^{-6}\ \text{eV}\,\frac{10^{12}\ \text{GeV}}{f} \tag{2.3}$$

are inversely proportional to f. f was first thought to be of order the electroweak scale, but its value is in fact arbitrary [6]. However, the combined limits from unsuccessful searches in particle and nuclear physics experiments and from stellar evolution require $f \gtrsim 3\cdot 10^9$ GeV [7].

Furthermore, an upper limit $f \lesssim 10^{12}$ GeV is provided by cosmology because light axions are abundantly produced during the QCD phase transition [8]. In spite of their very small mass, these axions are a form of cold dark matter. Indeed, their average momentum at the QCD epoch is not of order the temperature (GeV) but of order the Hubble expansion rate ($3 \cdot 10^{-9}$ eV) then. In case inflation occurs after the Peccei-Quinn phase transition their average momentum is even smaller because the axion field gets homogenized during inflation. For a detailed discussion see ref. [9]. In addition to this cold axion population, there is a thermal axion population with average momentum of order the temperature.

The non-perturbative QCD effects that give the axion its mass turn on at a temperature of order 1 GeV. The critical time, defined by $m(t_1)t_1 = 1$, is $t_1 \simeq 2 \cdot 10^{-7}$ sec $(f/10^{12}\ \mathrm{GeV})^{\frac{1}{3}}$. Cold axions are the quanta of oscillation of the axion field that result from the turn on of the axion mass. They have number density

$$n(t) \sim \frac{4 \cdot 10^{47}}{\mathrm{cm}^3} \left(\frac{f}{10^{12}\ \mathrm{GeV}}\right)^{\frac{5}{3}} \left(\frac{a(t_1)}{a(t)}\right)^3 \tag{2.4}$$

where $a(t)$ is the cosmological scale factor. Because the axion momenta are of order $\frac{1}{t_1}$ at time t_1 and vary with time as $a(t)^{-1}$, the velocity dispersion of cold axions is

$$\delta v(t) \sim \frac{1}{m t_1} \frac{a(t_1)}{a(t)} \tag{2.5}$$

if each axion remains in whatever state it is in, i.e. if axion interactions are negligible. Let us refer to this case as the limit of decoupled cold axions. If decoupled, the average state occupation number of cold axions is

$$\mathcal{N} \sim n \frac{(2\pi)^3}{\frac{4\pi}{3}(m\delta v)^3} \sim 10^{61} \left(\frac{f}{10^{12}\ \mathrm{GeV}}\right)^{\frac{8}{3}} \quad . \tag{2.6}$$

Clearly, the effective temperature of cold axions is much smaller than the critical temperature

$$T_{\mathrm{c}} = \left(\frac{\pi^2 n}{\zeta(3)}\right)^{\frac{1}{3}} \simeq 300\ \mathrm{GeV} \left(\frac{f}{10^{12}\ \mathrm{GeV}}\right)^{\frac{5}{9}} \frac{a(t_1)}{a(t)} \tag{2.7}$$

for Bose-Einstein condensation. Bose-Einstein (BEC) may be briefly described as follows: if identical bosonic particles are highly condensed in phase space, if their total number is conserved and if they thermalize, most of them go to the lowest energy available state. The condensing particles do so because, by yielding their energy to the remaining non-condensed particles, the total entropy is increased.

Eqs. (2.6) and (2.7) tell us that the first condition is overwhelmingly satisfied. The second condition is also satisfied because all axion number violating processes, such as their decay to two photons, occur on time scales vastly longer than the age of the universe. The only condition for axion BEC that is not manifestly satisfied is thermal equilibrium. Thermal equilibrium of axions may seem unlikely because the axion is very weakly coupled. However, it was found in ref. [2] that dark matter axions do form a BEC, marginally because of their self-interactions, but certainly as a result of their gravitational interactions. No special assumptions are required.

3. Bose-Einstein condensation of cold dark matter axions

Axions are in thermal equilibrium if their relaxation rate Γ is large compared to the Hubble expansion rate $H(t) = \frac{1}{2t}$. At low phase space densities, the relaxation rate is of order the particle interaction rate $\Gamma_s = n\sigma\delta v$ where σ is the scattering cross-section. The cross-section for $\varphi + \varphi \to \varphi + \varphi$ scattering due to axion self interaction is *in vacuum*

$$\sigma_0 = \frac{1}{64\pi}\frac{\lambda^2}{m^2} \simeq 1.5 \cdot 10^{-105}\mathrm{cm}^2 \left(\frac{m}{10^{-5}\ \mathrm{eV}}\right)^6 \quad . \tag{3.1}$$

If one substitutes σ_0 for σ, Γ_s is found much smaller than the Hubble rate, by many orders of magnitude. However, in the cold axion fluid background, the scattering rate is enhanced by the average quantum state occupation number of both final state axions, $\sigma \sim \sigma_0 \mathcal{N}^2$, because energy conservation forces the final state axions to be in highly occupied states if the initial axions are in highly occupied states. In that case, the relaxation rate is multiplied by *one* factor of $\mathcal{N}$ [10]

$$\Gamma \sim n\ \sigma_0\ \delta v\ \mathcal{N} \quad . \tag{3.2}$$

Combining Eqs. (2.4-2.6,3.1), one finds $\Gamma(t_1)/H(t_1) \sim \mathcal{O}(1)$, suggesting that cold axions thermalize at time t_1 through their self interactions, but only barely so.

A critical aspect of axion BEC phenomenology is whether the BEC continues to thermalize after it has formed. Axion BEC means that (almost) all axions go to one state. However, only if the BEC continually rethermalizes does the axion state track the lowest energy state.

The particle kinetic equations that yield Eq. (3.2) are valid only when the energy dispersion $\frac{1}{2}m(\delta v)^2$ is larger than the thermalization rate [10]. After t_1 this condition is no longer satisfied. One enters then a regime where the relaxation rate due to self interactions is of order [2, 11]

$$\Gamma_\lambda \sim \lambda\ n\ m^{-2} \quad . \tag{3.3}$$

$\Gamma_\lambda(t)/H(t)$ is of order one at time t_1 but decreases as $t\ a(t)^{-3}$ afterwards. Hence, self interactions are insufficient to cause axion BEC to rethermalize after t_1 even if they cause axion BEC at t_1. However gravitational interactions, which are long range, do the job later on. The relaxation rate due to gravitational interactions is of order [2, 11]

$$\Gamma_{\mathrm{g}} \sim G\ n\ m^2\ l^2 \tag{3.4}$$

where $l \sim (m\delta v)^{-1}$ is the correlation length. $\Gamma_{\mathrm{g}}(t)/H(t)$ is of order $4 \cdot 10^{-8}(f/10^{12}\ \mathrm{GeV})^{\frac{2}{3}}$ at time t_1 but grows as $ta^{-1}(t) \propto a(t)$. Thus gravitational interactions cause the axions to thermalize and form a BEC when the photon temperature is of order 100 eV $(f/10^{12}\ \mathrm{GeV})^{\frac{1}{2}}$.

The process of axion Bose-Einstein condensation is constrained by causality. We expect overlapping condensate patches with typical size of order the horizon. As time goes on, say from t to $2t$, the axions in t-size condensate patches rethermalize into $2t$-size patches. The correlation length is then of order the horizon at all times, implying $\delta v \sim \frac{1}{mt}$ instead of Eq. (2.5), and $\Gamma_{\mathrm{g}}/H \propto t^3 a^{-3}(t)$ after the BEC has formed. Therefore gravitational interactions rethermalize the axion BEC on ever shorter time scales compared to the age of the universe. The question now is whether axion BEC has implications for observation.

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4. Dark matter caustics

The study of the inner caustics of galactic halos [12, 13] may provide a useful tool. An isolated galaxy like our own accretes the dark matter particles surrounding it. Cold collisionless particles falling in and out of a gravitational potential well necessarily form an inner caustic, i.e. a surface of high density, which may be thought of as the envelope of the particle trajectories near their closest approach to the center. The density diverges at caustics in the limit where the velocity dispersion of the dark matter particles vanishes. Because the accreted dark matter falls in and out of the galactic gravitational potential well many times, there is a set of inner caustics. In addition, there is a set of outer caustics, one for each outflow as it reaches its maximum radius before falling back in. We will be concerned here with the catastrophe structure and spatial distribution of the inner caustics of isolated disk galaxies.

The catastrophe structure of the inner caustics depends mainly on the angular momentum distribution of the infalling particles [13]. There are two contrasting cases to consider. In the first case, the angular momentum distribution is characterized by 'net overall rotation'; in the second case, by irrotational flow. The archetypical example of net overall rotation is instantaneous rigid rotation on the turnaround sphere. The turnaround sphere is defined as the locus of particles which have zero radial velocity with respect to the galactic center for the first time, their outward Hubble flow having just been arrested by the gravitational pull of the galaxy. The present turnaround radius of the Milky Way is of order 2 Mpc. Net overall rotation implies that the velocity field has a curl, $\vec{\nabla} \times \vec{v} \neq 0$. The corresponding inner caustic is a closed tube whose cross-section is a section of the elliptic umbilic (D_{-4}) catastrophe [12, 13]. It is often referred to as a 'caustic ring', or 'tricusp ring' in reference to its shape. In the case of irrotational flow, $\vec{\nabla} \times \vec{v} = 0$, the inner caustic has a tent-like structure quite distinct from a caustic ring. Both types of inner caustic are described in detail in ref.[13].

If a galactic halo has net overall rotation and its time evolution is self-similar, the radii of its caustic rings are predicted in terms of a single parameter, called $j_{\rm max}$. Self-similarity means that the entire phase space structure of the halo is time independent except for a rescaling of all distances by $R(t)$, all velocities by $R(t)/t$ and all densities by $1/t^2$ [14, 15, 16, 17]. For definiteness, $R(t)$ will be taken to be the turnaround radius at time t. If the initial overdensity around which the halo forms has a power law profile

$$\frac{\delta M_i}{M_i} \propto (\frac{1}{M_i})^{\varepsilon} \quad , \tag{4.1}$$

where M_i and δM_i are respectively the mass and excess mass within an initial radius r_i, then $R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$ [14]. In an average sense, ε is related to the slope of the evolved power spectrum of density perturbations on galaxy scales [18]. The observed power spectrum implies that ε is in the range 0.25 to 0.35 [16]. The prediction for the caustic ring radii is (n = 1, 2, 3, ..) [12, 17]

$$a_n \simeq \frac{40\ \text{kpc}}{n} \left(\frac{v_{\rm rot}}{220\ \text{km/s}}\right) \left(\frac{j_{\rm max}}{0.18}\right) \tag{4.2}$$

where $v_{\rm rot}$ is the galactic rotation velocity. Eq.(4.2) is for $\varepsilon = 0.3$. The a_n have a small ε dependence. However, the $a_n \propto 1/n$ approximate behavior holds for all ε in the range 0.25 and 0.35, so that a change in ε is equivalent to a change in $j_{\rm max}$. $(\varepsilon, j_{\rm max})$ = (0.30, 0.180) implies very nearly the same radii as $(\varepsilon, j_{\rm max})$ = (0.25, 0.185) and (0.35, 0.177).

Observational evidence for caustic rings with the radii predicted by Eq. (4.2) was found in the statistical distribution of bumps in a set of 32 extended and well-measured galactic rotation curves [19], the distribution of bumps in the rotation curve of the Milky Way [20], the appearance of a triangular feature in the IRAS map of the Milky Way in the precise direction tangent to the nearest caustic ring [20], and the existence of a ring of stars at the location of the second ($n = 2$) caustic ring in the Milky Way [21]. Each galaxy may have its own value of $j_{\rm max}$. However, the $j_{\rm max}$ distribution over the galaxies involved in the aforementioned evidence is found to be peaked at 0.18. There is evidence also for a caustic ring of dark matter in a galaxy cluster [22].

Recently the rotation curve of our nearest large neighbour, the Andromeda galaxy, was measured with far greater precision and detail than hitherto achieved [23]. The new rotation curve has three prominent bumps, at 10 kpc, 15 kpc and 29 kpc. The positions of these bumps are in the ratios predicted by the caustic ring model and thus provide fresh additional evidence.

5. The caustic ring halo model

The caustic ring model of galactic halos [17] is the phase space structure that follows from self-similarity, axial symmetry, and net overall rotation. Self-similarity requires that the time-dependence of the specific angular momentum distribution on the turnaround sphere be given by [16, 17]

$$\vec{\ell}(\hat{n},t) = \vec{j}(\hat{n})\,\frac{R(t)^2}{t} \tag{5.1}$$

where $\hat{n}$ is the unit vector pointing to a position on the turnaround sphere, and $\vec{j}(\hat{n})$ is a dimensionless time-independent angular momentum distribution. In case of instantaneous rigid rotation, which is the simplest form of net overall rotation,

$$\vec{j}(\hat{n}) = j_{\rm max}\,\hat{n}\times(\hat{z}\times\hat{n}) \tag{5.2}$$

where $\hat{z}$ is the axis of rotation and $j_{\rm max}$ is the parameter that appears in Eq. (4.2). The angular velocity is $\vec{\omega} = \frac{j_{\rm max}}{t}\hat{z}$. Each property of the assumed angular momentum distribution maps onto an observable property of the inner caustics: net overall rotation causes the inner caustics to be rings, the value of $j_{\rm max}$ determines their overall size, and the time dependence given in Eq. (5.1) causes $a_n \propto 1/n$.

The angular momentum distribution assumed by the caustic ring halo model may seem implausible because it is highly organized in both time and space. Numerical simulations [24] suggest that galactic halo formation is a far more chaotic process. However, since the model is motivated by observation, it is appropriate to ask whether it is consistent with the expected behaviour of some or any of the dark matter candidates. In addressing this question we make the usual assumption, commonly referred to as 'tidal torque theory', that the angular momentum of a galaxy is due to the tidal torque applied to it by nearby protogalaxies early on when density perturbations are still small and protogalaxies close to one another [25, 26]. We divide the question into three parts: 1. is the value of $j_{\rm max}$ consistent with the magnitude of angular momentum expected from tidal torque theory? 2. is it possible for tidal torque theory to produce net overall rotation? 3. does the axis of rotation remain fixed in time, and is Eq. (5.1) expected as an outcome of tidal torque theory?

6. Magnitude of angular momentum

The amount of angular momentum acquired by a galaxy through tidal torquing can be reliably estimated by numerical simulation because it does not depend on any small feature of the initial mass configuration, so that the resolution of present simulations is not an issue in this case. The dimensionless angular momentum parameter

$$\lambda \equiv \frac{L|E|^{\frac{1}{2}}}{GM^{\frac{5}{2}}} \quad , \tag{6.1}$$

where G is Newton's gravitational constant, L is the angular momentum of the galaxy, M its mass and E its net mechanical (kinetic plus gravitational potential) energy, was found to have median value 0.05 [27]. In the caustic ring model the magnitude of angular momentum is given by $j_{\rm max}$. As mentioned, the evidence for caustic rings implies that the $j_{\rm max}$-distribution is peaked at $j_{\rm max} \simeq 0.18$. Is the value of $j_{\rm max}$ implied by the evidence for caustic rings compatible with the value of λ predicted by tidal torque theory?

The relationship between $j_{\rm max}$ and λ may be easily derived. Self-similarity implies that the halo mass $M(t)$ within the turnaround radius $R(t)$ grows as $t^{\frac{2}{3\varepsilon}}$ [14]. Hence the total angular momentum grows according to

$$\frac{d\vec{L}}{dt} = \int d\Omega \frac{dM}{d\Omega dt}\vec{\ell} = \frac{4}{9\varepsilon}\frac{M(t)R(t)^2}{t^2} j_{\rm max}\, \hat{z} \tag{6.2}$$

where we assumed, for the sake of definiteness, that the infall is isotropic and that $\vec{j}(\hat{n})$ is given by Eq. (5.2). Integrating Eq. (6.2), we find

$$\vec{L}(t) = \frac{4}{10+3\varepsilon}\,\frac{M(t)R(t)^2}{t} j_{\rm max}\, \hat{z} \quad . \tag{6.3}$$

Similarly, the total mechanical energy is

$$E(t) = -\int \frac{GM(t)}{R(t)}\frac{dM}{dt}dt = -\frac{3}{5-3\varepsilon}\frac{GM(t)^2}{R(t)} \quad . \tag{6.4}$$

Here we use the fact that each particle on the turnaround sphere has potential energy $-GM(t)/R(t)$ and approximately zero kinetic energy. Combining Eqs. (6.1), (6.3) and (6.4) and using the relation $R(t)^3 = \frac{8}{\pi^2}t^2 GM(t)$ [14], we find

$$\lambda = \sqrt{\frac{6}{5-3\varepsilon}}\,\frac{8}{10+3\varepsilon}\,\frac{1}{\pi}\, j_{\rm max} \quad . \tag{6.5}$$

For $\varepsilon = 0.25$, 0.30 and 0.35, Eq. (6.5) implies $\lambda/j_{\rm max} = 0.281$, 0.283 and 0.284 respectively. Hence there is excellent agreement between $j_{\rm max} \simeq 0.18$ and $\lambda \sim 0.05$.

The agreement between $j_{\rm max}$ and λ gives further credence to the caustic ring model. Indeed if the evidence for caustic rings were incorrectly interpreted, there would be no reason for it to produce a value of $j_{\rm max}$ consistent with λ. Note that the agreement is excellent only in Concordance Cosmology. In a flat matter dominated universe, the value of $j_{\rm max}$ implied by the evidence for caustic rings is 0.27 [12, 17].

7. Net overall rotation

Next we ask whether net overall rotation is an expected outcome of tidal torquing. The answer is clearly no if the dark matter is collisionless. Indeed, the velocity field of collisionless dark matter satisfies

$$\frac{d\vec{v}}{dt}(\vec{r},t) = \frac{\partial \vec{v}}{\partial t}(\vec{r},t) + (\vec{v}(\vec{r},t)\cdot\vec{\nabla})\vec{v}(\vec{r},t) = -\vec{\nabla}\phi(\vec{r},t) \tag{7.1}$$

where $\phi(\vec{r},t)$ is the gravitational potential. The initial velocity field is irrotational because the expansion of the universe caused all rotational modes to decay away [28]. Furthermore, it is easy to show [13] that if $\vec{\nabla}\times\vec{v} = 0$ initially, then Eq. (7.1) implies $\vec{\nabla}\times\vec{v} = 0$ at all later times. Since net overall rotation requires $\vec{\nabla}\times\vec{v} \neq 0$, it is inconsistent with collisionless dark matter, such as WIMPs or sterile neutrinos. If WIMPs or sterile neutrinos are the dark matter, the evidence for caustic rings, including the agreement between $j_{\max}$ and λ obtained above, is purely fortuitous.

Axions [5, 6, 8, 7] differ from WIMPs and sterile neutrinos. Axions are not collisionless, in the sense of Eq. (7.1), because they form a rethermalizing Bose-Einstein condensate. This process is quantum mechanical in an essential way and not described by Eq. (7.1). By *rethermalizing* we mean that thermalization rate remains larger than the Hubble rate so that the axion state tracks the lowest energy available state. The compressional (scalar) modes of the axion field are unstable and grow as for ordinary CDM, except on length scales too small to be of observational interest [2]. Unlike ordinary CDM, however, the rotational (vector) modes of the axion field exchange angular momentum by gravitational interaction. Most axions condense into the state of lowest energy consistent with the total angular momentum, say $\vec{L} = L\hat{z}$, acquired by tidal torquing at a given time. To find this state we may use the WKB approximation because the angular momentum quantum numbers are very large, of order 10^{20} for a typical galaxy. The WKB approximation maps the axion wavefunction onto a flow of classical particles with the same energy and momentum densities. It is easy to show that for given total angular momentum the lowest energy is achieved when the angular motion is rigid rotation. So we find Eq. (5.2) to be a prediction of tidal torque theory if the dark matter is axions.

Thermalization by gravitational interactions is only effective between modes of very low relative momentum because only in this case is the correlation length l, that appears in Eq. (3.4), large. After the axions fall into the gravitational potential well of the galaxy, they form multiple streams and caustics like ordinary CDM [29]. The momenta of particles in different streams are too different from each other for thermalization by gravitational interactions to occur across streams. The wavefunction of the axions inside the turnaround sphere is mapped by the WKB approximation onto the flow of classical particles with the same initial conditions on that sphere. The phase space structure thus formed has caustic rings since the axions reach the turnaround sphere with net overall rotation. The axion wavefunction vanishes on an array of lines. These lines, numbering of order 10^{20}, may be thought of as the vortices characteristic of a BEC with angular momentum. However, the transverse size of the axion vortices is of order the inverse momentum associated with the radial motion in the halo, $(mv_r)^{-1} \sim 20$ meters for a typical value (10^{-5} eV) of the axion mass. In a BEC without radial motion the size of vortices is of order the healing length [30], which is much larger than $(mv_r)^{-1}$.

One might ask whether there is a way in which net overall rotation may be obtained other than by BEC of the dark matter particles. I could not find any. General relativistic effects may produce a curl in the velocity field but are only of order $(v/c)^2 \sim 10^{-6}$ which is far too small for the purposes described here. One may propose that the dark matter particles be collisionfull in the sense of having a sizable cross-section for elastic scattering off each other. The particles then share angular momentum by particle collisions after they have fallen into the galactic gravitational potential well. However, the collisions fuzz up the phase space structure that we are trying to account for. The angular momentum is only fully shared among the halo particles after the flows and caustics of the model are fully destroyed. Axions appear singled out in their ability to produce the net overall rotation implied by the evidence for caustic rings of dark matter.

If the dark matter is WIMPs or sterile neutrinos, the velocity field of dark matter is curl-free. As already mentioned, the inner caustics of galactic halos have then a tent-like structure which is quite distinct from caustic rings [13]. Also, the angular momentum of the dark matter accreting onto a halo is not shared among the infalling particles. The total angular momentum vector $\vec{L}$ of the halo is the same as for axion dark matter, since it is determined by the outcome of tidal torque theory, but unlike the axion case is the sum of many contributions randomly oriented with respect to one another. The tent-like inner caustics have therefore random orientations, whereas the caustic rings of the axion case lie all in the galactic plane.

8. Self-similarity

The third question provides a test of the conclusions reached so far. If galaxies acquire their angular momentum by tidal torquing and if the dark matter particles are axions in a rethermalizing Bose-Einstein condensate, then the time dependence of the specific angular momentum distribution on the turnaround sphere is predicted. Is it consistent with Eq. (5.1)? In particular, is the axis of rotation constant in time?

Consider a comoving sphere of radius $S(t) = Sa(t)$ centered on the protogalaxy. $a(t)$ is the cosmological scale factor. S is taken to be of order but smaller than half the distance to the nearest protogalaxy of comparable size, say one third of that distance. The total torque applied to the volume V of the sphere is

$$\vec{\tau}(t) = \int_{V(t)} d^3r \; \delta\rho(\vec{r},t)\; \vec{r} \times (-\vec{\nabla}\phi(\vec{r},t)) \tag{8.1}$$

where $\delta\rho(\vec{r},t) = \rho(\vec{r},t) - \rho_0(t)$ is the density perturbation. $\rho_0(t)$ is the unperturbed density. In the linear regime of evolution of density perturbations, the gravitational potential does not depend on time when expressed in terms of comoving coordinates, i.e. $\phi(\vec{r} = a(t)\vec{x},t) = \phi(\vec{x})$. Moreover $\delta(\vec{r},t) \equiv \frac{\delta\rho(\vec{r},t)}{\rho_0(t)}$ has the form $\delta(\vec{r} = a(t)\vec{x},t) = a(t)\delta(\vec{x})$. Hence

$$\vec{\tau}(t) = \rho_0(t)a(t)^4 \int_V d^3x \; \delta(\vec{x})\; \vec{x} \times (-\vec{\nabla}_x\phi(\vec{x})) \quad . \tag{8.2}$$

Eq. (8.2) shows that the direction of the torque is time independent. Hence the rotation axis is time independent, as in the caustic ring model. Furthermore, since $\rho_0(t) \propto a(t)^{-3}$, $\tau(t) \propto a(t) \propto t^{\frac{2}{3}}$ and hence $\ell(t) \propto L(t) \propto t^{\frac{5}{3}}$. Since $R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$, tidal torque theory predicts the time dependence of

Eq. (5.1) provided $\varepsilon = 0.33$. This value of ε is in the range, $0.25 < \varepsilon < 0.35$, predicted by the evolved spectrum of density perturbatuions and supported by the evidence for caustic rings. So the time dependence of the angular momentum distribution on the turnaround sphere is also consistent with the caustic ring model.

9. Conclusion

If the dark matter is axions, the phase space structure of galactic halos predicted by tidal torque theory is precisely, and in all respects, that of the caustic ring model proposed earlier on the basis of observations. The other dark matter candidates predict a different phase space structure for galactic halos. Although the QCD axion is best motivated, a broader class of axion-like particles behaves in the manner described here.

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