The dilaton as a dark matter candidate: An estimate of the dilaton mass from quarkonium spectra.

Mohamed CHABAB

LPHEA, Department of Physics, Cadi Ayyad University
P.O.B. 2390, Marrkesh 40 000, Morocco.
E-mail: mchabab@ucam.ac.ma

Using a string inspired theory and assuming that the dilaton is dark matter candidate, we determine the mass of the dilaton from the heavy quarkonium spectra with Dick potential. This potential has a remarkable property of leading to a confining phase parameterized by the non zero mass for the dilaton and a mass scale $f$. By phenomenological investigation of Dick potential it is shown that the energy levels of charmonium, bottommonium and the $B_c$ fit well the experimental data when the dilaton mass is given a value about $57\text{MeV}$. This estimate lies in the range of values predicted for the dilaton as a cold dark matter, and support the recent experimental evidence for light (1-100 MeV) dark matter as a possible explanation of $511\text{keV}$ gamma-ray emission.

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*Speaker.
†A footnote may follow.
1. Introduction

Elucidating the nature of the dark matter (DM) has become a central question in astrophysics and one of the most fundamental and multidisciplinary quests in science today. However, although the compelling evidence for the existence of the dark matter, there is still an intense debate on its nature and on how dark matter particles interact with Standard model. A large number of candidates have been proposed for (nonbaryonic) DM. Besides the lightest supersymmetric particle, the axion is one of the leading candidate for cold dark matter and very likely made an important contribution to the structure formation of the universe [1]. However, all higher dimmensional unified theories (including superstring, supergravity, Kaluza Klein theories) also predict a universal scalar particle: the dilaton [2]. In supersymmetric gauge theories, the dilaton $\phi$ appears as a companion of the axion: if the dilaton arises in the real part of the lowest component of a chiral field, the imaginary part corresponds to the axion field. In the Kaluza Klein supergravity, the field content of the spacetime is assumed to arise from embedding in the $(4 + d_i)$ dimensional manifold, where the $d_i$ internal dimensions have variable volume from four dimension point of view and is interpreted as the dilaton. Also, the dilaton has various interacting ways: for instance, it couples naturally to super Yang Mills gauge fields in curved space and plays a pivotal role in string theory, to define the string coupling constant as $\exp(\lambda \phi)$, where $\lambda$ is characteristic length typically of the order of the Planck length. The dilaton also couples to the Ricci curvature terms in the Brans-Dicke model of induced gravity as well as to the QCD field strength to generate a confining phase. Therefore, according to recent results on strong weak dualities in gauge theory and string theory, there is strong evidence that the axion and the dilaton should come together and that their properties are intimately connected. For theses features, it is very likely that the dilaton may also be considered as a viable candidate for cold dark matter, once the problem of dilaton stability is solved [3].

In this paper, I will briefly describe the general derivation of the quark-quark interaction potential from a string inspired theory and show that the confining phase, when it exists, depends on the mass of the dilaton and the length scale of the theory. Then, by adopting the dilaton - gauge field coupling function proposed by Dick, we use a semi-relativistic techniques (SLET) to solve heavy quarkonium wave equation and predict the dilaton mass which reproduce the experimental spectra of charmonium, bottomonium and $B_c$ mesons.

2. The string inspired theory

The imprint of dilaton on a 4d effective nonabelian gauge theory is described by a Lagrangian density [5], [7]:

$$\mathcal{L}(\phi, A) = -\frac{1}{4F(\phi)} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + J_\mu^a A_\mu^a$$

(2.1)

where $\phi$ is the dilaton field and $G_{\mu\nu}^a$ is the standard field strength tensor of the theory. $V(\phi)$ denotes the non perturbative dilaton potential and $F(\phi)$ represents the coupling function depending on $\phi$. Several forms of $F(\phi)$ have been proposed in literature. Analysis of the problem of Coulomb gauge theory augmented with dilaton degrees of freedom in (1) performed as follows: First, we consider a point like static Coulomb source defined in the rest frame by the current:
where $C_a$ is the expectation value of $SU(N_c)$ generator. The field equations emerging from the static configuration (2.2) are given by:

$$\left[D_\mu, F^{-1}(\phi)G^{\mu\nu}\right] = J^\nu$$

and

$$\partial_\mu \partial^\mu \phi = -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{4} \frac{\partial F^{-1}(\phi)}{\partial \phi} G^\mu_a G^{\nu\mu}_a$$

After some straightforward algebra, we derive the remarkable formula for interquark potential [6],

$$U(r) = 2\alpha_s \int \frac{F(\phi(r))}{r^2} dr$$

with $\alpha_s = \frac{g^2}{4\pi}$ and $\tilde{\alpha} = \frac{\alpha_s}{8\pi} \left(\frac{N_c-1}{2N_c}\right)$.

Eq. (2.5) shows that existence of confining phases in this effective theory is subject to the following condition,

$$\lim_{r \to \infty} rF^{-1}(\phi(r)) = \text{finite}$$

At this stage, the main objective is to solve the field equations of motion (2.3) and (2.4) and determine analytically $U(r)$. For this, $F(\phi)$ and $V(\phi)$ have to be fixed. In the sequel the dilaton potential is set to $V(\phi) = \frac{1}{2}m^2\phi$, while for $V(\phi)$ we use the form proposed by Dick $\frac{1}{F(\phi)} = \frac{\phi^2}{f^2 + \beta \phi^2}$ [5], where $f$ represents a coupling scale characterizing the strength of the dilaton-gluon coupling and $\beta$ is a parameter in the range $[0,1]$. In this model, the exact analytical solution for the interquark potential is given by (up to a color factor):

$$V(r) = \left[\frac{\beta g^2}{4\pi} - gf \sqrt{\frac{N_c}{2(N_c-1)}} \ln(e^{2Mr} - 1 + \frac{m^2}{\pi^2})\right],$$

with the abbreviation: $k^2 = \frac{\alpha_s f^2}{8\pi} \left(\frac{N_c-1}{N_c}\right)$.

Remarkably the potential $V(r)$ comes with the required behavior: a first term which accommodates the Coulomb interaction at short distances and a second term linearly increasing in the asymptotic regime with a string tension $\sigma \sim gmf$ which depends on the dilatonic degrees of freedom $m, f$.

3. Dilaton mass versus quarkonium spectroscopy

This analysis will be addressed as in [8] where the shifted-$l$ expansion technique is used (SLET) where $l$ is the angular momentum. This method provides a powerful analytic technique for determining the bound states of the semi-relativistic wave equation consisting of two quarks of masses $m_1, m_2$ and total binding meson energy $M$ in any spherically symmetric potential. It is
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rapidly converging, handles highly excited states and includes relativistic corrections in a consistent way.

Dick interquark potential reads,

$$V_D(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{4}{3} g f \sqrt{\frac{N_c}{2(N_c - 1)}} \ln[\exp(2mr) - 1] \tag{3.1}$$

To solve the equation and obtain results from the theory require us to specify several inputs: $m_c$, $m_b$, $m$, $f$ and $\alpha_s$. In our numerical analysis, we set the charm and bottom quark masses to the values $m_c = 1.89$ GeV and $m_b = 5.19$ GeV. For the QCD coupling constant, we take into account the running of $\alpha_s$,

$$\alpha_s(\lambda) = \frac{\alpha_s(m_c)}{1 - (11 - \frac{2}{3}n_f)[\alpha_s(m_c)/2\pi]\ln(m_c/\lambda)} \tag{3.2}$$

where the renormalization scale is fixed to $\lambda = 2\mu$, with $\mu$ is the reduced mass,

$$\mu = \frac{m_1m_2}{m_1 + m_2} \tag{3.3}$$

Thus, combination of the leading order formula (3.2) and the world experimental value $\alpha_s(m_c) = 0.12$ yields,

$$\alpha_s(\text{charmonium}) = 0.31, \quad \alpha_s(\text{bottomonium}) = 0.20, \tag{3.4}$$

while $\alpha_s = 0.22$ for the $b\bar{c}$ quarkonia. On the other hand, the interquark potential parameters $m$ and $f$ are treated as being free in our analysis and are obtained by fitting the spin-averaged $c\bar{c}$, $b\bar{b}$ and $b\bar{c}$ bound states. An excellent fit with the available experimental data can be seen to emerge when the following values are assigned \footnote{if the standard value for the string tension $0.18 \text{ GeV}^2$ is used, the dilaton mass will be shifted to a value about $158 \text{ MeV}$.} [7],

$$m = 57 \text{ MeV} \quad g f \sqrt{\frac{N_c}{2(N_c - 1)}} = 430 \text{ MeV}. \tag{3.5}$$

Therefore, the heavy quarkonium spectra favours a light dilaton with a mass about $57 \text{ MeV}$. Here, I would like to emphasize that the estimate of the dilaton mass discussed in this analysis should be understood as an order estimate, and not as an exact result. Nevertheless, this prediction may shed some light on this (hypothetical) fundamental particle as cold dark matter candidate.

4. General conclusion

Using a string inspired model and assuming that the dilaton is dark matter candidate, we determine the mass of the dilaton from the heavy quarkonium spectra with Dick’s potential. This potential, which results from the analysis of the Coulomb problem of the theory, has a remarkable property of leading to a confining phase which is parameterized by the non zero mass for the dilaton and a mass scale $f$. Through phenomenological investigation of Dick potential we show that the
energy levels of charmonium, bottommonium and the $B_c$ fit well the experimental data when the dilaton mass is given a value about 57 MeV. This estimate lies in the range of values predicted for the dilaton as a cold dark matter [9] [10] and agrees with the light (1-100 MeV) dark matter as a possible explanation 511 keV gamma ray emission [11].

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