PROCEEDINGS OF SCIENCE

Hadrons from quarks, gluons and first principles

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The aim of this presentation was to review applications of perturbative QCD to global characteristics of multiple hadroproduction in hard processes.

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1. Introduction

Quantum Chromodynamics is a wonder. Take free quarks, supply them with color degrees of freedom, demand invariance with respect to the "repainting" quark fields arbitrarily in each point in space-time — and you get the QCD Lagrangian that describes interacting quarks and gluons. Having done that, you (are supposed to) have the whole hadron world in your hands! Such a beauty and ambition is hard to match.

At the same time, it is worth remembering that QCD is probably the strangest of theories in the history of modern physics. On the one hand, the striking successes of QCD-based phenomenology leave no doubt that QCD is indeed the microscopic theory of hadrons and their interactions. On the other hand, the depth of the conceptual problems that one faces in trying to formulate QCD as a respectable Quantum Field Theory is unprecedented.

QCD nowadays has a split personality. It embodies "hard" and "soft" physics, both being hard subjects, and the softer the harder.

High-energy annihilation $e^+e^- \to$ hadrons, deep inelastic lepton-hadron scattering (DIS), production in hadron-hadron collisions of massive lepton pairs, heavy quarks and their bound states, large transverse momentum jets and photons are classical examples of hard processes. Here a large momentum transfer Q^2, either time-like $Q^2 \gg 1\ \mathrm{GeV}^2$, or space-like $Q^2 \ll -1\ \mathrm{GeV}^2$, is applied to hadrons in order to probe their small-distance quark-gluon structure. Perturbative QCD (PT QCD) controls the relevant cross sections and, to a lesser extent, the structure of final states produced in hard interactions. It should be remembered that, whatever the hardness of the process, it is hadrons, not quarks and gluons, that hit the detectors. For this reason alone, the applicability of the PT QCD approach, even to hard processes, is far from being obvious. One has to rely on plausible arguments (completeness, duality) and look for observables that are less vulnerable towards our ignorance about confinement.

The selection principle for such observables is due to Sterman & Weinberg who have introduced back in 1977 an important notion of Collinear-and-Infrared Safety [1].

An observable is granted the CIS status if it can be calculated in terms of quarks and gluons treated as real particles (partons), without encountering either collinear ($\theta \to 0$) or infrared ($k_0 \to 0$) divergences. The former divergence is a standard feature of (massless) QFT with dimensionless coupling, the latter is typical for massless vector bosons (photons, gluons).

This classification is more than mere zoology. Given CIS quantity, we expect its PT QCD value *predictable* in the quark-gluon framework to be directly comparable with its *measurable* value in the hadronic world. For this reason the CIS observables are the preferred pets of QCD practitioners.

To give an example, we cannot deduce from the first principles parton distributions inside hadrons (PDF, or structure functions). However, the rate of their $\ln Q^2$-dependence (scaling violation) is an example of a CIS measure and stays under PT QCD jurisdiction.

Speaking about the final state structure, we cannot predict, say, the kaon multiplicity or the pion energy spectrum. However, one can decide to be not too picky and concentrate on global characteristics of the final states rather than on the yield of specific hadrons. Being sufficiently inclusive with respect to final hadron species, one can rely on a picture of the energy-momentum flow in hard collisions supplied by PT QCD — the jet pattern.

There are well elaborated procedures for counting jets (CIS jet finding algorithms) and for quantifying the internal structure of jets (CIS jet shape variables). They allow the study of the gross features of the final states while staying away from the physics of hadronization. Along these lines one visualizes asymptotic freedom, checks out gluon spin and color, predicts and verifies scaling violation pattern in hard cross sections, etc. These and similar checks have constituted the basic QCD tests of the first two decades of QCD studies.

2. Multihadron production and QCD

In general, there are three ways to probe the small-distance hadron structure. Firstly, one can excite the vacuum to produce hadrons, like in (but not exclusively) e^+e^- annihilation. Secondly, one can transfer large momentum to a hadron by a sterile probe, like in deep inelastic lepton-hadron scattering (DIS). Finally, there is production of large–$p_\perp$ hadrons in hadron-hadron collisions. (Here sterile probes can be employed in the final state as well, e.g. massive lepton pairs and/or large–$p_\perp$ photons.) Copious production of hadrons is typical for all these processes. On the other hand, at the microscopic level, multiple quark-gluon "production" is to be expected as a result of QCD bremsstrahlung — gluon radiation accompanying abrupt creation/scattering of color partons.

Is there a correspondence between observable hadron and calculable quark-gluon production?

An indirect evidence that gluons are there, and that they behave, can be obtained from the study of the scaling violation pattern. QCD quarks (and gluons) are not point-like particles, as the orthodox parton model once assumed. Each of them is surrounded by a proper field coat — a coherent virtual cloud consisting of gluons and "sea" $q\bar{q}$ pairs. A hard probe applied to such a dressed parton breaks coherence of the cloud. Constituents of these field fluctuations are then released as particles accompanying the hard interaction. The harder the hit, the larger an intensity of bremsstrahlung and, therefore, the fraction of the energy-momentum of the dressed parton that the bremsstrahlung quanta typically carry away. Thus we should expect, in particular, that the probability that a "bare" core quark carries a large fraction of the energy of its dressed parent will decrease with increase of Q^2. And so it does. The logarithmic scaling violation pattern in DIS structure functions is well established and meticulously follows the QCD prediction based on the parton evolution picture.

In DIS we look for a "bare" quark inside a target dressed one. In e^+e^- hadron annihilation at large energy $s = Q^2$ the chain of events is reversed. Here we produce instead a bare quark with energy $Q/2$, which then "dresses up". In the process of restoring its proper field-coat our parton produces (a controllable amount of) bremsstrahlung radiation which leads to formation of a hadron jet. Having done so, in the end of the day it becomes a constituent of one of the hadrons that hit the detector. Typically, this is the leading hadron. However, the fraction x_E of the initial energy $Q/2$ that is left to the leader depends on the amount of accompanying radiation and, therefore, on Q^2 (the larger, the smaller). In fact, the same rule (and the same formula) applies to the scaling violation pattern in e^+e^- fragmentation functions (time-like parton evolution) as to that in the DIS parton distributions (space-like evolution).

What makes the annihilation channel particularly interesting, is that the present day experiments are so sophisticated that they provide us with a near-to-perfect separation between quark- and gluon-initiated jets (the latter being extracted from heavy-quark-tagged three-jet events).

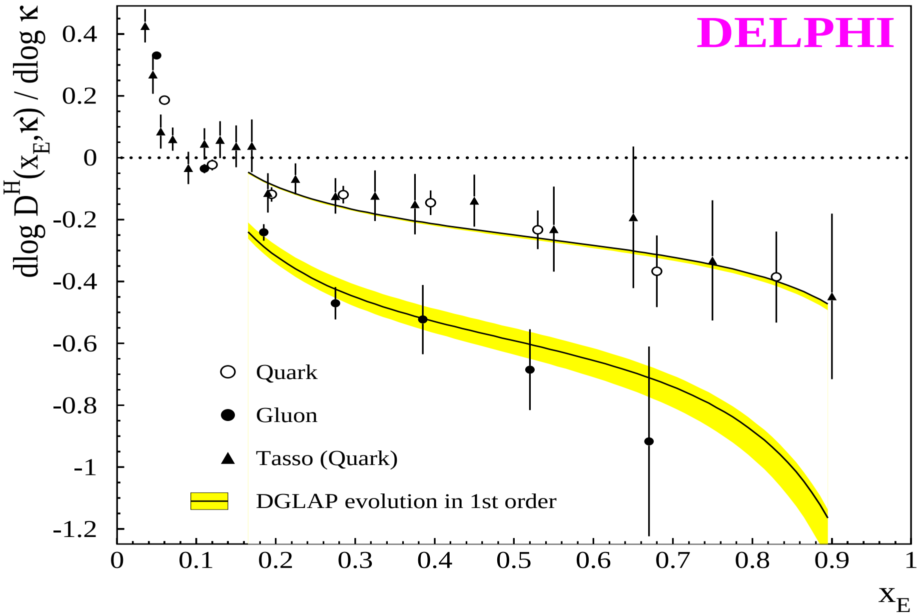

Figure 1: Scaling violation rates in inclusive hadron distributions from gluon and quark jets [2].

In Fig. 1 a comparison is shown of the scaling violation rates in the hadron spectra from gluon and quark jets, as a function of the hardness scale κ that characterizes a given jet [2]. For large values of $x_E \sim 1$ the ratio of the logarithmic derivatives is predicted to be close to that of the gluon and quark "color charges", $C_A/C_F = 9/4$. Experimentally, the ratio is measured to be

$$\frac{C_A}{C_F} = 2.23 \pm 0.09_{\text{stat.}} \pm 0.06_{\text{syst.}}. \tag{2.1}$$

3. Intrajet particle multiplication

3.1 Mean parton and hadron multiplicities

Since accompanying QCD radiation seems to be there, we can make a step forward by asking for a *direct* evidence: what is the fate of those gluons and sea quark pairs produced via multiple initial gluon bremsstrahlung followed by parton multiplication cascades? Let us look at the Q-dependence of the mean hadron multiplicity, the quantity dominated by relatively soft particles with $x_E \ll 1$. This is the kinematical region populated by accompanying QCD radiation.

Fig. 2 demonstrates that the hadron multiplicity increases with the hardness of the jet proportional to the multiplicity of secondary gluons and sea quarks. The ratio of the slopes, once again, provides an independent measure of the ratio of the color charges [3], which is consistent with that extracted from scaling violation in fragmentation functions (2.1):

$$\frac{C_A}{C_F} = 2.246 \pm 0.062_{\text{stat.}} \pm 0.008_{\text{syst.}} \pm 0.095_{\text{theo.}}. \tag{3.1}$$

3.2 Inclusive soft particle spectra in jets

Once the total numbers match, we can ask a more delicate question about energy-momentum distribution of final hadrons versus that of the underlying parton ensemble. One should not be too

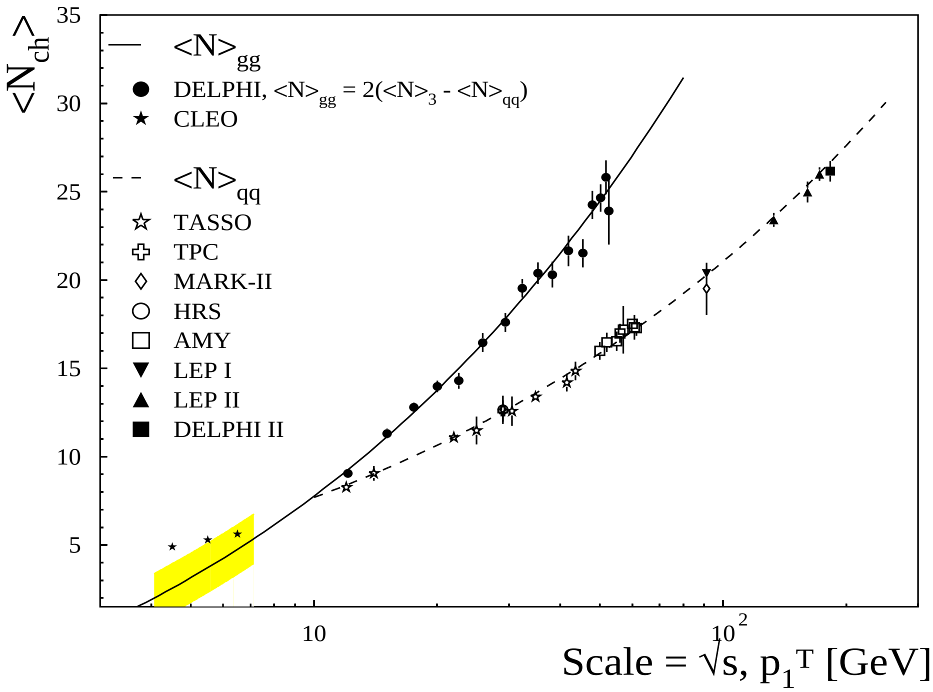

Figure 2: Charged hadron multiplicities in gluon and quark jets (DELPHI 1999).

picky in addressing such a question. It is clear that hadron-hadron correlations, for example, will show resonant structures about which the quark-gluon speaking PT QCD can say little, if anything, at the present state of the art. Inclusive single-particle distributions, however, have a better chance to be closely related. Triggering a single hadron in the detector, and a single parton on paper, one may compare the structure of the two distributions to learn about dynamics of hadronization.

Inclusive energy spectrum of soft bremsstrahlung partons in QCD jets has been derived in 1984 in the so-called MLLA — the Modified Leading Logarithmic Approximation [4]. This approximation takes into account all essential ingredients of parton multiplication in the next-to-leading order. They are: parton splitting functions responsible for the energy balance in parton splitting, the running coupling $\alpha_s(k_\perp^2)$ depending on the relative transverse momentum of the two offspring and exact angular ordering. The last is a consequence of soft gluon coherence and plays an essential rôle in parton dynamics. In particular, gluon coherence suppresses multiple production of very small momentum gluons. It is particles with intermediate energies that multiply most efficiently. As a result, the energy spectrum of relatively soft secondary partons in jets acquires a characteristic hump-backed shape. The position of the maximum in the logarithmic variable $\xi = -\ln x$, the width of the hump and its height increase with Q^2 in a predictable way.

The shape of the inclusive spectrum of all charged hadrons (dominated by $\pi^\pm$) exhibits the same features. This comparison, pioneered by Glen Cowan (ALEPH) and the OPAL collaboration, has later become a standard test of analytic QCD predictions. First scrutinized at LEP, the similarity of parton and hadron energy distributions has been verified at SLC and KEK e^+e^- machines, as well as at HERA and Tevatron where hadron jets originate not from bare quarks dug up from the vacuum by a highly virtual photon/Z^0 but from hard partons kicked out from initial hadron(s).

In Fig. 3 (DELPHI) the comparison is made of the all-charged hadron spectra at various annihilation energies Q with the so-called “distorted Gaussian” fit [5] which employs the first four moments (the mean, width, skewness and kurtosis) of the MLLA distribution around its maximum.

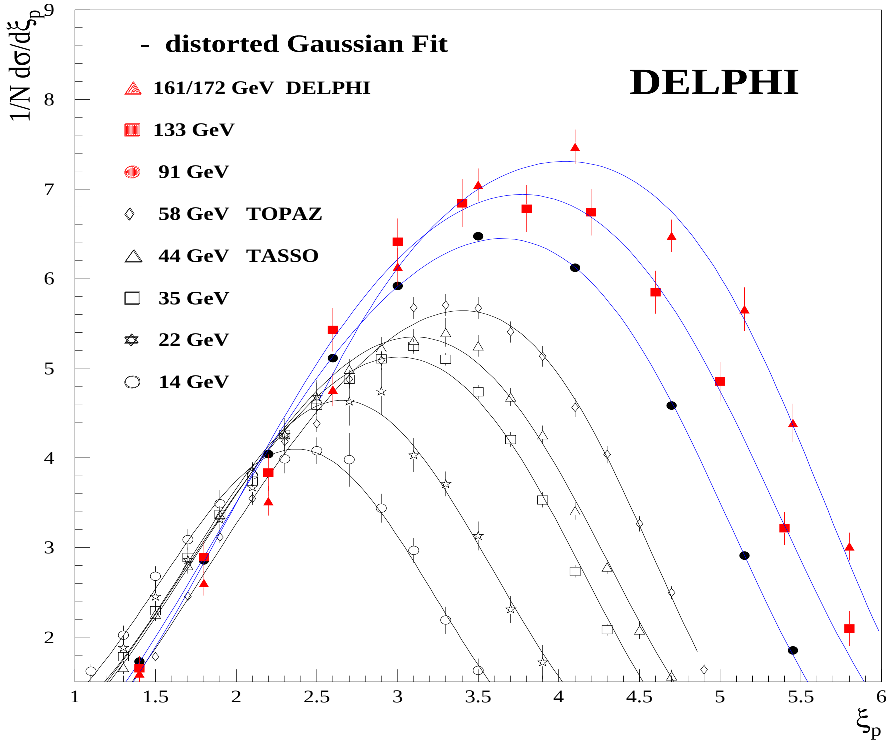

Figure 3: Inclusive energy distribution of charged hadrons in jets produced in e^+e^- annihilation

Shall we say, a (routine, interesting, wonderful) check of yet another QCD prediction? Better not. Such a close similarity offers a deep puzzle, even a worry, rather than a successful test. Indeed, after a little exercise in translating the values of the logarithmic variable $\xi = \ln(E_{\rm jet}/p)$ in Fig. 3 into GeVs you will see that the actual hadron momenta at the maxima are, for example, $p{=}\frac{1}{2}Q\cdot e^{-\xi_{\rm max}} \simeq 0.42$, 0.85 and 1.0 GeV for Q=14, 35 GeV and at LEP-1, Q=91 GeV. Is it not surprising that the PT QCD spectrum is mirrored by that of the pions (which constitute 90% of all charged hadrons produced in jets) with momenta well below 1 GeV?!

For this very reason the observation of the parton-hadron similarity was initially met with a serious and well grounded scepticism: it looked more natural (and was more comfortable) to blame the finite hadron mass effects for falloff of the spectrum at large ξ (small momenta) rather than seriously believe in applicability of the PT QCD consideration down to such disturbingly small momentum scales.

This worry has been answered by the CDF collaboration. Andrey Korytov was the first to hear a theoretical hint [6] and carry out a study of the energy distribution of hadrons produced inside a restricted angular cone Θ around the jet axis. Theoretically, it is not the energy of the jet but the maximal parton transverse momentum inside it, $k_{\perp\max} \simeq E_{\rm jet}\sin\frac{\Theta}{2}$, that determines the hardness scale and thus the yield and the distribution of the accompanying radiation.

This means that by choosing a small opening angle one can study relatively small hardness scales but in a cleaner environment: due to the Lorentz boost effect, eventually all particles that form a short small-Q^2 QCD "hump" are now relativistic and concentrated at the tip of the jet.

For example, selecting hadrons inside a cone $\Theta \simeq 0.14$ around an energetic quark jet with $E_{\rm jet} \simeq 100$ GeV (LEP-2) one should see that very "dubious" $Q = 14$ GeV curve in Fig. 3 but now with the maximum boosted from 450 MeV into a comfortable 6 GeV range.

In the CDF Fig. 4 [7, 8] a close similarity between the hadron yield and the full MLLA parton spectra can no longer be considered accidental and be attributed to non-relativistic kinematical effects.

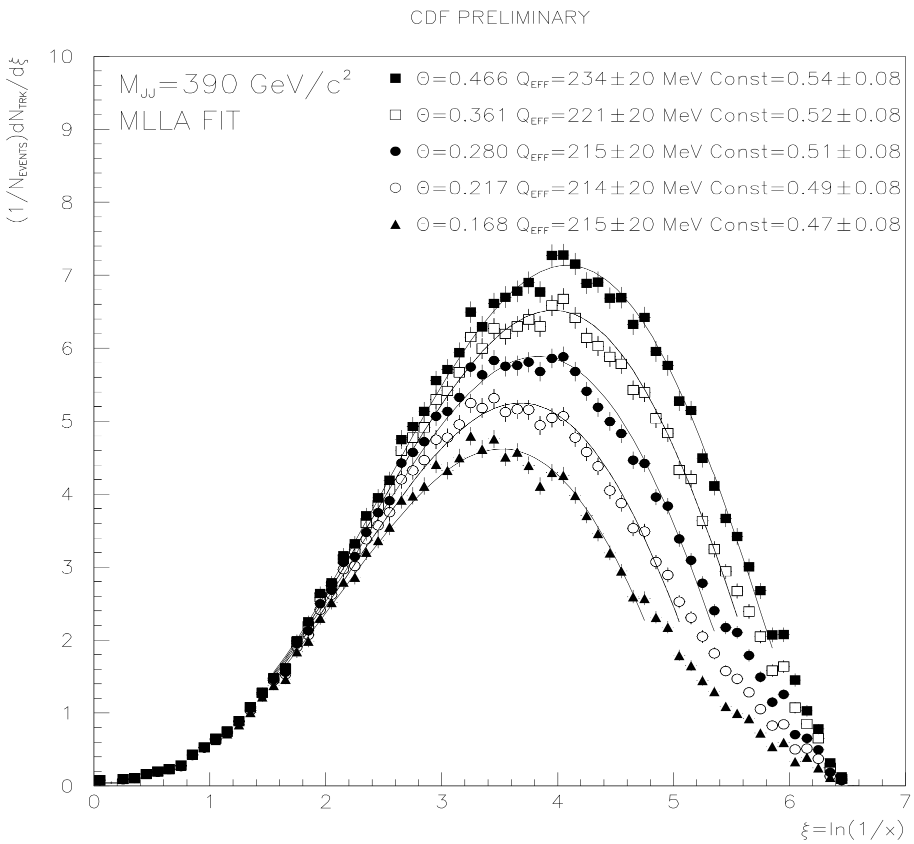

Figure 4: Inclusive energy distribution of charged hadrons in large–$p_\perp$ jets (Goulianos 1997).

3.3 Brave gluon counting

Modulo $\Lambda_{\rm QCD}$, there is only one unknown in this comparison, namely, the overall normalisation of the spectrum of hadrons relative to that of partons (bremsstrahlung gluons).

Strictly speaking, there should/could have been another free parameter, the one which quantifies one's bravery in applying the PT QCD dynamics. It is the minimal transverse momentum cutoff in parton cascades, $k_\perp > Q_0$. The strength of successive $1 \to 2$ parton splittings is proportional to $\alpha_s(k_\perp^2)$ and grows with $k_\perp$ decreasing. The necessity to terminate the process at some low transverse momentum scale where the PT coupling becomes large (and eventually hits the formal "Landau pole" at $k_\perp = \Lambda_{\rm QCD}$) seems imminent. Surprisingly enough, it is not.

Believe it or not, the inclusive parton energy distribution turns out to be a CIS QCD prediction. Its crazy $Q_0 = \Lambda_{\rm QCD}$ limit (the so-called "limiting spectrum") is shown by solid curves in Fig. 4.

Choosing the minimal value for the collinear parton cutoff Q_0 can be looked upon as shifting, as far as possible, responsibility for particle multiplication in jets to the PT dynamics. This brave choice can be said to be dictated by experiment, in a certain sense. Indeed, with increase of Q_0 the parton parton distributions *stiffen* (parton energies are limited from below by the kinematical inequality $xE_{\rm jet} \equiv k \geq k_\perp > Q_0$). The maxima would move to larger x (smaller ξ), departing from the data.

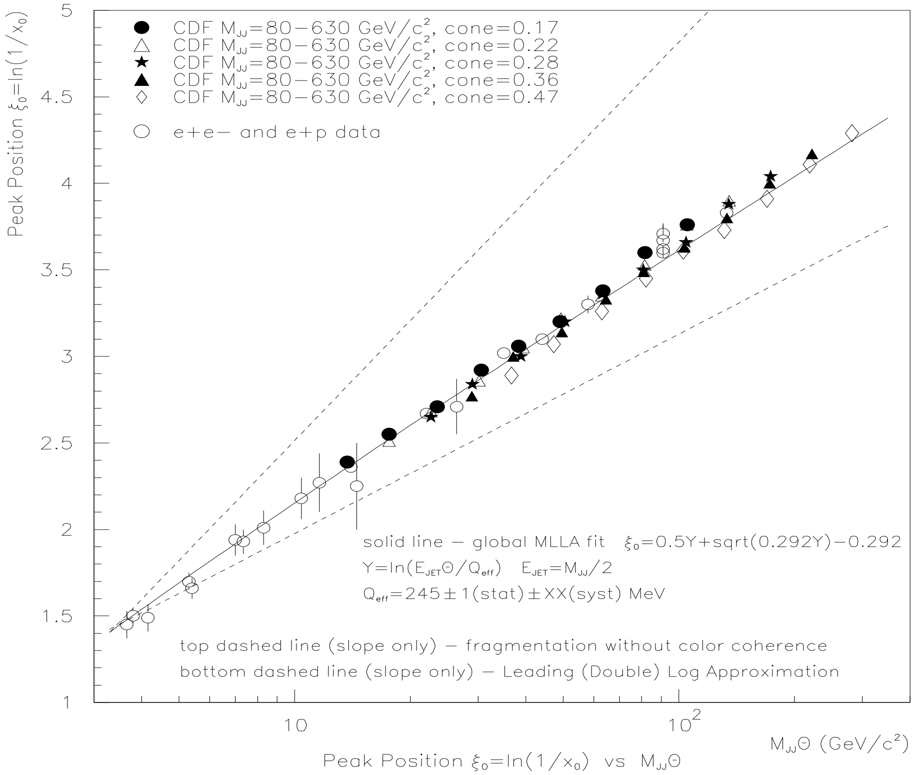

Figure 5: The position of the maximum versus the analytic MLLA prediction [8].

A clean test of "brave gluon counting" is provided by Fig. 5 where the position of the hump, which is insensitive to the overall normalisation, is compared with the parameter-free MLLA QCD prediction [8]. A formal explanation of the tolerance of the *shape* of inclusive parton spectra to the dangerous small-$k_\perp$ domain can be found in the proceedings of the Blois conference [9].

To put a long story short, decreasing Q_0 we start to lose control of the interaction intensity of a parton with a given x and $k_\perp \sim Q_0$ (and thus may err in the overall production rate). However, such partons do not branch any further, do not produce any soft offspring, so that the *shape* of the resulting energy distribution remains undamaged. Color coherence plays here a crucial rôle.

3.4 Local parton-hadron duality

It is important to realize that knowing the spectrum of *partons*, even knowing it to be a CIS quantity in certain sense, does not guarantee on its own the predictability of the *hadron* spectrum.

It is easy to imagine a world in which each quark and gluon with energy k produced at the small-distance stage of the process would have dragged behind its personal "string" giving birth to $\ln k$ hadrons in the final state (the Feynman plateau). The hadron yield then would be given by a convolution of the parton distribution with a logarithmic energy distribution of hadrons from the parton fragmentation.

If it were the case, each parton would have contributed to the yield of non-relativistic hadrons and the hadron spectra would peak at much smaller energies, $\xi_{\max} \simeq \ln Q$, in a spectacular difference with experiment. Physically, it could be possible if the non-perturbative (NP) hadronization physics did not respect the basic rule of the perturbative dynamics, namely, that of color coherence.

There is nothing wrong with the idea of convoluting time-like parton production in jets with the inclusive NP parton→hadron fragmentation function, the procedure which is similar to convoluting space-like parton cascades with the NP initial parton distributions in a target proton to describe DIS structure functions. What nature is telling us, however, is that this NP fragmentation has a finite multiplicity and is *local* in the momentum space. Similar to its PT counterpart, the NP dynamics has a short memory: the NP conversion of partons into hadrons occurs locally in the configuration space.

In spite of a known similarity between the space- and time-like parton evolution pictures ($x \sim 1$), there is an essential difference between *small–x* physics of DIS structure functions and the jet fragmentation. In the case of the space-like evolution, in the limit of small Bjorken–x the problem becomes essentially non-perturbative and PT QCD loses control of the DIS cross sections [10, 11]. On the contrary, studying small Feynman–x particles originating from the time-like evolution of jets offers a gift and a puzzle: all the richness of the confinement dynamics reduces to a mere overall normalisation constant.

The message is, that "brave gluon counting", that is applying the PT language all the way down to very small transverse momentum scales, indeed reproduces the x- and Q-dependence of the observed inclusive energy spectra of charged hadrons (pions) in jets.

4. Interjet particle flows

Another class of multihadron production phenomena speaking in favour of the "brave gluon counting" is the so-called interjet physics. It deals with particle flows in the angular regions between jets in various multi-jet configurations. These particles do not belong to any particular jet, and their production, at the PT QCD level, is governed by coherent soft gluon radiation off the multi-jet system as a whole. Due to QCD coherence, these particle flows are insensitive to internal structure of underlying jets. The only thing that matters is the color topology of the primary system of hard partons and their kinematics.

The ratios of particle flows in different inter-jet valleys are given by parameter-free PT QCD predictions and reveal the so-called "string" or "drag" effects. For a given kinematical jet configuration such ratios depend only on the number of colors (N_c).

For example, the ratio of the multiplicity flow between a quark (antiquark) and a gluon to that in the $q\bar{q}$ valley in symmetric ("Mercedes") three-jet $q\bar{q}g$ e^+e^- annihilation events is predicted to be

$$\frac{dN_{qg}^{(q\bar{q}g)}}{dN_{qq}^{(q\bar{q}g)}} \simeq \frac{5N_c^2-1}{2N_c^2-4} = \frac{22}{7}. \tag{4.1}$$

Comparison of the denominator with the density of radiation in the $q\bar{q}$ valley in $q\bar{q}\gamma$ events with a gluon jet replaced by an energetic photon results in

$$\frac{dN_{qq}^{(q\bar{q}\gamma)}}{dN_{qq}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2-1)}{N_c^2-2} = \frac{16}{7}. \tag{4.2}$$

Emitting an energetic gluon off the initial quark pair depletes accompanying radiation in the backward direction: color is *dragged* out of the $q\bar{q}$ valley. This destructive interference effect is so strong that the resulting multiplicity flow falls below that in the least favourable direction transversal to the three-jet event plane:

$$\frac{dN_{\perp}^{(q\bar{q}\gamma)}}{dN_{qq}^{(q\bar{q}g)}} \simeq \frac{N_C+2C_F}{2(4C_F-N_c)} = \frac{17}{14}. \tag{4.3}$$

At the level of the PT accompanying gluon radiation (QCD radiophysics) such predictions are quite simple and straightforward to derive. The fact that these and many similar numbers have been seen experimentally offers a serious puzzle. The problem is, the naive perturbative wisdom is being impressed upon 100–300 MeV pions which dominate hadron flows between jets in the present-day experiments such as, for example, the OPAL study shown in Fig. 6.

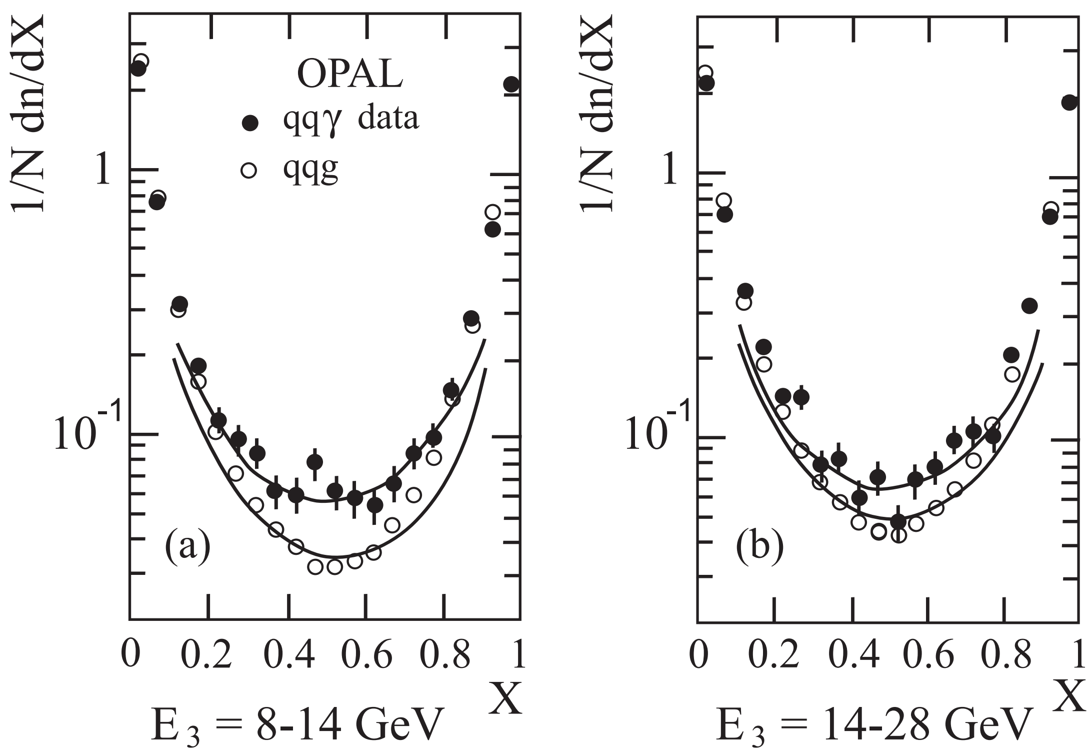

Figure 6: Particle flows in the $q\bar{q}$ valley in $q\bar{q}\gamma$ and $q\bar{q}g$ events [12] versus an analytic parameter-free prediction based on the soft gluon radiation pattern [13].

The fact that even such soft junk follows the PT QCD rules is truly amazing.

5. Conclusion

Meticulous studies of basic hard processes — e^+e^- annihilation, DIS, large-$k_\perp$ hadron–hadron interactions — taught us a serious lesson: the bulk production of final hadrons from jets that develop in the vacuum closely follows the pattern of the underlying PT gluon radiation (and subsequent quark–gluon cascades). Hard energetic partons form a sort of color antenna that determines the structure of the emerging gluon field and, in the end of the day, the flow of final hadrons. This picture applies to *global characteristics* of multihadron production such as the inclusive energy spectrum of charged hadrons inside jets and the pattern of soft particle multiplicity flow between jets. The fact that even a legitimate finite smearing due to hadronization effects does not look mandatory here, makes one think of a deep duality between the hadron and quark-gluon languages.

Put together, the ideas behind the brave gluon counting are known as the hypothesis of Local Parton-Hadron Duality [4]. Experimental evidence in favour of LPHD is mounting, and so is the list of challenging questions to be answered by the future quantitative theory of color confinement.

Heavy ions have an important role to play in elucidating the nature of the parton–hadron conversion. Though strange it might seem, in high energy heavy ion collisions the perturbative QCD approach gains in legitimacy. On the one hand, HI environment as a multi-body problem offers a good source of headache for the theory, and always will. On the other hand, in a QCD medium even soft — minimally biased — processes tend to turn hard(ish). Discussing this phenomenon some will refer to the growing "saturation scale" that characterizes the gluon field inside the energetic projectile, others — to Brownian transverse momentum broadening due to multiple scattering inside the target. By hook or by crook, typical transverse momenta should increase with the size of the medium: $k_\perp^2 \propto A^{1/3}$, thus enlarging the domain of applicability of the perturbative quark–gluon language.

The fact that the momentum distribution — the Poynting vector — of soft hadrons follows that of the underlying color field should certainly manifest itself in the bulk properties of the hadron matter produced in HI collisions too. A better understanding of the pattern of coherent gluon radiation in a multiple collision environment, geometry and color topology of the corresponding field, should result, in particular, in establishing yet another duality: between the microscopic QCD and hydrodynamical descriptions of collective flow effects.

References

[1] G. Sterman and S. Weinberg, Phys. Rev. Lett. **39** (1977) 1436.

[2] K. Hamacher et al., DELPHI Collab., Int. Europhysics Conference on High-Energy Physics (EPS-HEP 99), Tampere, Finland, July 1999.

[3] P. Abreu, et al., DELPHI Collab., Phys. Lett. **449 B** (1999) 383; Eur. Phys. J. **C 13** (2000) 573.

[4] Yu.L. Dokshitzer and S.I. Troyan, "*Asymptotic Freedom and Local Parton-Hadron Duality in QCD Jet Physics*". 19th Leningrad Winter School (in Russian), vol. 1, p. 144, Leningrad 1984;
Ya.I. Azimov et al., Zeit. Phys. **C 27** (1985) 65;
Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, *Basics of Perturbative QCD*, ed. J. Tran Thanh Van, Gif-sur-Yvette, Editions Frontiéres, 1991.

[5] C.P. Fong and B.R. Webber, Nucl. Phys. **B 355** (1991) 54.

[6] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, Rev. Mod. Phys. **60** (1988) 373.

[7] D. Goulianos, CDF Collab., 32nd Recontres de Moriond, Les Arcs, France, March 22, 1997.

[8] A.N. Safonov, CDF Collab., Int. Euroconference in Quantum Chromodynamics, Montpellier, France, July 7, 1999;
D. Acosta et al. (CDF Collab.), Phys. Rev. **D 68** (2003) 012003.

[9] Yu.L. Dokshitzer, *Perturbative QCD and Power Corrections*, Invited talk at the International Conference "Frontiers of Matter", Blois, France, 1999 [arXiv:hep-ph/9911299]

[10] A.H. Mueller, Phys. Lett. **B 396** (1997)251.

[11] G. Camici and M. Ciafaloni, Phys. Lett. **B 395** (1997) 118.

[12] R. Akers et al., OPAL Collab., Zeit. Phys. **C 68** (1995) 531.

[13] M.A. Buican, V.A. Khoze and W. Ochs, Eur. Phys. J. **C 28** (2003) 313.