Interpretation of charged Higgs effects in low energy flavour physics

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We discuss two-Higgs-doublet models in view of the present flavour data, in particular present indirect bounds and different techniques of flavour protection.
1. Introduction

The two-Higgs-doublet model (THDM) constitutes one of the simplest extensions of the present Standard Model (SM). Many new-physics scenarios, including supersymmetry, can lead to a low-energy spectrum containing the SM fields plus at least one additional scalar doublet. Also recent developments in string phenomenology indicate that an additional generation of Higgs bosons is generic within this framework [1].

From the two Higgs doublets, three degrees of freedom are eaten and become longitudinal components of the gauge bosons and five degrees are left: the scalar mass eigenstates, $H, h$, the pseudoscalar $A$, and the charged Higgs bosons $H^\pm$. These new degrees of freedom induce new flavour-changing neutral currents in general. In consequence, an analysis of this class of models in view of the huge data sets from the recent flavour experiments is desirable.

The first generation of the $B$ factories at KEK (Belle experiment at the KEKB $e^+e^-$ collider) [2] and at SLAC (BaBar experiment at the PEP-II $e^+e^-$ collider) [3] have collected huge samples of $B$ meson decays and thus established the SM picture of CP violation and other flavour-changing processes in the quark sector. It is remarkable that all present measurements of $B$ meson decays (including the $B$ physics programme at the Tevatron (CDF [4] and D0 [5]) have not observed any unambiguous sign of new physics (NP) yet [6, 7].

![Figure 1: CKM unitarity fit [8].](image)

This means that all flavour-violating processes between quarks are governed by a $3 \times 3$ unitarity matrix referred to as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [11, 12]. The CKM matrix is fully described by four real parameters, three rotation angles and one complex phase. It is this complex phase that represents the only source of CP violation and that allows a unified description of all the CP-violating phenomena in the SM. This can be illustrated by the overconstrained triangle in the complex plane which reflects the unitarity of the CKM matrix, see Fig.1. What is most impressive is the consistency between tree-level and loop-induced processes. This is remarkable
because in the latter ones possible new degrees of freedom might contribute while tree processes are fully dominated by SM physics, see Fig. 2. One also finds consistency when one separates CP-violating and CP-conserving observables, see Fig. 3. As we will see in Sect.3, there is much more data not shown in the unitarity fit which confirms the SM predictions like rare decays.

![Figure 2: Unitarity triangle fixed by tree (left) versus loop (right) processes [8].](image)

![Figure 3: Unitarity triangle fixed by CP-conserving (left) versus CP-violating (right) processes [8].](image)

While this is an impressing success of the CKM theory [11, 12] within the SM which was honored by the Nobel Prize in physics in 2008, there is still room for sizeable new effects from new flavour structures (see i.e. Refs. [9, 10]), as FCNC processes have been tested only up to the 10% level.

The non-existence of large NP effects in the FCNC processes implies the famous flavour problem, namely why FCNC are suppressed. This has to be solved in any viable NP model. Either the mass scale of the new degrees of freedom is very high or the new flavour-violating couplings are small for reasons that remain to be found. For example, assuming generic new flavour-violating couplings, the present data on $K$-$\bar{K}$ mixing implies a very high NP scale of order $10^3$–$10^4$ TeV depending on whether the new contributions enter at loop- or at tree-level. In contrast, theoretical considerations on the Higgs sector, which is responsible for the mass generation of the fundamental particles in the SM, call for NP at order 1 TeV. As a consequence, any NP below the 1 TeV scale must have a non-generic flavour structure. Moreover, the present measurements of $B$ decays, especially of FCNC processes, already significantly restrict the parameter space of NP models.

There has been an intense discussion on how the flavour problem can be solved within the class of models with two or more Higgs doublets [13, 14, 15, 16, 17, 18, 19]. In a first step it is important to find conditions to avoid FCNC at the tree level. However, this might not be sufficient.
It is also important to address the question of the stability of such flavour-protecting conditions under radiative corrections as was pointed out most recently [19].

This paper is organized as follows. In Section 2 we recall the various conditions on the THDM which avoid FCNC at the tree level. In Section 3 we discuss present bounds on such models from flavour data and compare them with information we get from the direct search for NP in high-energy experiments. In Section 4 we discuss the hypothesis of minimal flavour violation while in the last section we analyze its application to THDM and the stability of the various flavour-protecting conditions.

2. Tree-level FCNC in the THDM

In the most general version of the THDM, the fermionic couplings of the neutral scalars are non-diagonal in flavour leading to FCNC at the tree level. In fact, the most general renormalizable Yukawa interaction reads

\[
-\mathcal{L}_Y^{\text{general}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^\dagger + \bar{Q}_L X_{d2} D_R H_2^\dagger + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.},
\]

with general 3 flavour matrices and \(H_i^\dagger = i\tau_2 H_i^*\). The corresponding mass matrices are given by

\[
M_d = \frac{1}{\sqrt{2}} (v_1 X_{d1} + v_2 X_{d2}) , \quad M_u = \frac{1}{\sqrt{2}} (v_1 X_{u1} + v_2 X_{u2}) .
\]

where \(v_1\) and \(v_2\) are the two vacuum expectation values, \(\langle H_i^\dagger H_i \rangle = v_i\). As already recognized in Refs. [20, 21], the mass matrices and the physical couplings to the physical scalar Higgs bosons cannot be diagonalized simultaneously in general. In fact, after rotating the basis of the Higgs fields, \((H_1, H_2)\) by the angle \(\beta = \arctan(v_2/v_1)\) to the new basis \((S_1, S_2)\), one finds \(\langle S_1^\dagger S_1 \rangle = v = \sqrt{v_1^2 + v_2^2}\) and \(\langle S_2^\dagger S_2 \rangle = 0\). The Lagrangian separates into the mass terms and the interaction terms to the physical Higgs fields:

\[
-\mathcal{L}_Y^{\text{general}} = \bar{Q}_L (\sqrt{2}/v M_d S_1 + Z_d S_2) D_R + \bar{Q}_L (\sqrt{2}/v M_u S_1^\dagger + Z_u S_2^\dagger) U_R + \text{h.c.} .
\]

\[
Z_d = \cos \beta X_{d2} - \sin \beta X_{d1} , \quad Z_u = \cos \beta X_{u2} - \sin \beta X_{u1} .
\]

It is obvious that for general flavour matrices \(X_i\), the matrices \(M_u\) and \(Z_u\) (analogously \(M_d\) and \(Z_d\)) cannot be simultaneously diagonalized. This directly generates tree-level FCNC.

This feature can be cured by the assumption that only one Higgs field can couple to a given quark species, corresponding to the condition \(X_{d1} = X_{d2} = 0\) [20, 21]. Another possibility to induce the same effect is the setting \(X_{u2} = X_{d2} = 0\), which means that all fermions couple only to one of the Higgs fields. The corresponding models are called THDM Type-II and Type-I repectively. These conditions directly imply \(M_{u,d} \sim Z_{u,d}\) which prevents the models from FCNC at the tree-level.

The conditions can be implemented by flavourblind discrete symmetries. For the Type-I model the \(Z_2\) symmetry is just the exchange symmetry \(H_2 \leftrightarrow -H_2\), with all other fields unchanged, while for the Type-II model the \(Z_2\) symmetry transformation is \(H_1 \leftrightarrow -H_1\), \(D_R \leftrightarrow -D_R\). The latter symmetry can be regarded as a subgroup of the well-known Peccei-Quinn \(U(1)\) symmetry [22]. Indeed,
the $U(1)_{\text{PQ}}$ can be defined within this context by attributing $D_R$ and $H_1$ charges $+1$ and $−1$, respectively and no charge to all other fields. Then it is clear that also the flavourblind Peccei-Quinn $U(1)$ symmetry implies the Type-II model.

There is yet another more general flavour-protecting condition on the tree level [13, 16]. Moreover, the caveat of all these tree-level implementations is their instability under quantum corrections which might not assure sufficient flavour protection [19]. Both issues will be discussed in the last section.

3. Parameter bounds on the THDM

Rare $B$ and kaon decays (for reviews see [23, 24, 25]) representing loop-induced or helicity-suppressed processes are highly sensitive probes for new degrees of freedom beyond the SM establishing an alternative way to search for NP. The day the existence of new degrees of freedom is established by the direct search via the Large Hadron Collider (LHC), the present stringent flavour bounds will translate in first-rate information on the NP model at hand.

At present there are two key observables constraining the charged Higgs sector in THDM, the inclusive $\bar{B} \to X_s \gamma$ decay and the leptonic decay $B \to \tau \nu$.

Among the rare decay modes, the inclusive decay $\bar{B} \to X_s \gamma$ is the most important one, because it is theoretically well-understood and at the same time it has been measured extensively at the $B$ factories. While non-perturbative corrections to this decay mode are subleading and recently estimated to be well below 10% [27], perturbative QCD corrections are the most important corrections.
Within a global effort, a perturbative QCD calculation to the next-to-next-to-leading-logarithmic order level (NNLL) has quite recently been performed and has led to the first NNLL prediction of the $\bar{B} \rightarrow X_s \gamma$ branching fraction [29] with a photon cut at $E_\gamma = 1.6\text{GeV}$ (including the error due to nonperturbative corrections):

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{NNLL}} = (3.15 \pm 0.23) \times 10^{-4}.$$  

(3.1)

The combined experimental data leads to (Heavy Flavor Averaging Group (HFAG) [28])

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4},$$

(3.2)

where the first error is combined statistical and systematic, and the second is due to the extrapolation in the photon energy. Thus, the SM prediction and the experimental average are consistent at the 1.2$\sigma$ level, see Fig. 4. This is one important example that the CKM theory is not only confirmed by the data entering into the CKM unitarity fit, but also by many additional flavour mixing phenomena.

![Figure 5: Bound on charged Higgs mass depending on the measured branching ratio and on the experimental error [26].](image)

This specific result in the case of $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ implies very stringent constraints on NP physics models. Such bounds are of course model-dependent, but in general much stronger than the ones derived from other measurements. In any case, the indirect flavour information will be most valuable when the general nature and the mass scale of the NP will be identified in the direct search.

For example one finds a bound on the inverse compactification radius of the minimal universal extra dimension model (mACD) ($1/R > 600\text{GeV}$ at 95% CL) [30]. For the the two-Higgs doublet model (Type-II), one finds an upper bound for the charged Higgs mass, $M_{H^+} > 295\text{GeV}$ at 95% CL [29], see Fig. 5.

In the SM the $b \rightarrow s \gamma$ transition is a loop-induced decay via an exchange of a $W$ boson and a top quark, see left diagram of Fig. 6. In the THDM-II there is an additional contribution due to the charged Higgs (see right diagram of Fig. 6) which always adds to the SM one which implies the stringent lower bound on the charged Higgs mass [31, 32]. However, embedding the THDM-II into a supersymmetric model, one finds the chargino contribution due to quark mixing (see Fig. 7) which
in principle can destructively interfere with the charged Higgs contribution. As a consequence there is no significant bound on the charged Higgs mass within the minimal supersymmetric SM (MSSM), see i.e. Refs. [33, 34, 35, 36].

Moreover, the MSSM allows for generic new sources of flavour violation beyond the CKM structure in the SM. Next to the usual quark mixing also the squark mixing induces flavour mixing due to a possible misalignment of quarks and squarks in flavour space, see i.e. Refs. [37, 38, 39].

The other key observable is the leptonic decay $B \to \tau \nu$. It is induced in the SM at the tree level, see left diagram of Fig. 8. The charged Higgs contribution modifies the SM branching ratio as follows:

$$B_{\text{THDM-II}}(B \to \tau \nu) = B_{\text{SM}}(B \to \tau \nu) \times (1 - \frac{M_B^2}{M_{H^+}^2} \tan^2 \beta)$$

Thus, the measurement of $B(B \to \tau \nu)$ also implies stringent bounds on the charged Higgs mass depending of the value of $\tan \beta$. Complementary, one gets similar information via the measurement of $B(B \to D \tau \nu)$, see right diagram of Fig. 8:

$$B_{\text{THDM-II}}(B \to D \tau \nu) = G_F^2 \tau_B |V_{cb}|^2 f[F_V, F_S, 1 - \frac{M_B^2}{M_{H^+}^2} \tan^2 \beta]$$

The hadronic formfactors $F_V$ and $F_S$ can be studied via the decay $B \to D \ell \nu$.

It is worthwhile noting that there is some tension between the direct measurement and the indirect fit prediction for $B(B \to \tau \nu)$, see left plot of Fig. 9. The deviation is $2.6 \sigma$. Moreover, as was pointed out by the CKMfitter group [8], there is a specific correlation between $\sin \beta$ and $B(B \to \tau \nu)$ which is also a bit at odds, see right plot of Fig. 9.

A more recent combined analysis of all available bounds within the the THDM-II was presented in Ref. [40] and is shown in the left plot of Fig. 10: For $\tan \beta < 40$ the bound due to the
Figure 8: Tree contributions to $\mathcal{B}(B \to \tau \nu)$ (left) and to $\mathcal{B}(B \to D \tau \nu)$ (right).

Figure 9: Direct measurement versus indirect fit prediction for $\mathcal{B}(B \to \tau \nu)$ (left) and for correlation $(\mathcal{B}(B \to \tau \nu), \sin \beta)$ (right); cross corresponds to the experimental values with 1\,\sigma uncertainties [8].

$\bar{B} \to X_s \gamma$ decay is dominant, while for larger values the tree-level process $B \to \tau \nu$ leads to the strongest bound. The latter is less model-dependent and essentially survives also within the MSSM in contrast to the $\bar{B} \to X_s \gamma$ bound. Further analyses within different types of THDMs can be found in Refs. [41, 42].

Finally, the indirect NP reach via flavour data and the direct NP reach via the ATLAS and CMS experiments should be compared: The expected 95\% CL exclusion limits of the LHC from the processes $gg/gb \to t(b)H^+, H^+ \to \tau \nu/tb$ [45, 46] (see also Fig. 11) are shown in the right, but also in the left plot of Fig. 10; one finds that the present flavour constraints on the THDM-II are comparable and, therefore, nicely complementary to the expected exclusion limits of the LHC. One needs around 10 fb$^{-1}$ at the LHC in order to reach new territory not ruled out by the present flavour data.

4. Minimal flavour violation hypothesis and CP issues

The minimal flavour violation (MFV) hypothesis is a formal solution to the NP flavour problem. It assumes that the flavour and the CP symmetry are broken as in the SM. Thus, it requires that all flavour- and CP-violating interactions be linked to the known structure of Yukawa couplings.

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Figure 10: Combined bound from all flavour observables with combined Higgs search constraint from ATLAS (left) and the original 95% CL exclusion limits from ATLAS [43].

Figure 11: LHC observable tests out the same vertex as the decay $B \to \tau \nu$ [44].

(called $Y_U$ and $Y_D$ in the following). A renormalization-group-invariant definition of MFV based on a symmetry principle is given in [48, 49, 50]; this is mandatory for a consistent effective field theoretical analysis of NP effects.

In fact, a low-energy effective theory with all SM fields including one or two Higgs doublets can be constructed; as the only source of $U(3)^5$ flavour symmetry breaking, the ordinary Yukawa couplings are introduced as background values of fields transforming under the flavour group (‘spurions’) [50]. In the construction of the effective field theory, operators with arbitrary
powers of the dimensionless $Y_U/D$ have in principle to be considered. However, the specific structure of the SM, with its hierarchy of CKM matrix elements and quark masses, drastically reduces the number of numerically relevant operators. For example, it can be shown that in MFV models with one Higgs doublet, all FCNC processes with external $d$-type quarks are governed by the following combination of spurions due to the dominance of the top Yukawa coupling $y_t$:

\[
(Y_U Y_U^\dagger)_{ij} \approx y_t^2 V_{33}^* V_{3j},
\]

where a basis is used in which the $d$-type quark Yukawa is diagonal.

There are two strict predictions in this general class of models which have to be tested. First, the MFV hypothesis implies the usual CKM relations between $b \to s$, $b \to d$, and $s \to d$ transitions. For example, this relation allows for upper bounds on new-physics effects in $\mathcal{B}(\bar{B} \to X_d \gamma)$, and $\mathcal{B}(\bar{B} \to X_s \nu \bar{\nu})$ using experimental data or bounds from $\mathcal{B}(\bar{B} \to X_d \gamma)$, and $\mathcal{B}(K \to \pi^+ \nu \bar{\nu})$, respectively. This emphasizes the need for high-precision measurements of $b \to s/d$, but also of $s \to d$ transitions such as the rare kaon decay $K \to \pi \nu \bar{\nu}$. A systematic analysis of MFV bounds and relations for $\Delta F = 1$ transitions is given in Ref. [55], for $\Delta F = 2$ in Ref. [56]. The usefulness of MFV-bounds/relations is obvious; any measurement beyond those bounds indicate the existence of new flavour structures.

It is well known that scenarios including two Higgs doublets with large $\tan \beta = O(m_t/m_b)$ allow for the unification of top and bottom Yukawa couplings, as predicted in grand-unified models [52], and for sizeable new effects in helicity-suppressed decay modes [53, 54]. There are more general MFV relations existing in this scenario due to the dominant role of scalar operators. However, since $\tan \beta$ is large, there is a new combination of spurions numerically relevant in the construction of higher-order MFV effective operators, namely

\[
(Y_D Y_D^\dagger)_{ij} \approx y_d^2 \delta_{ij},
\]

which invalidates the general MFV relation between $b \to s/d$ and $s \to d$ transitions.

Within the MFV hypothesis the CKM phase is often assumed to be the only source of CP violation. This implies that any phase measurement is not sensitive to new physics. But flavour and CP violation can be treated separately. In fact, allowing for flavour-blind phases there is a RG-invariant extension of the MFV concept possible, as was first discussed in a phenomenological analysis on CP-violating observables [51]. But in general these phases lead to non-trivial CP effects, which get however strongly constrained by flavour-diagonal observables such as electric dipole moments.

Nevertheless, more recently Batell and Pospelov have given a deeper insight into the concrete EDM constraints on CP phases [58]. They have shown that the large flavourblind CP phases which are compatible with the present EDM constraints almost exclusively contribute to the $B_s$ mixing. In view of the present (slightly anomalous) data on $B_s$ mixing from the Tevatron experiments [4, 5] this is a very interesting new result [59].

\footnote{The MFV hypothesis with flavourblind phases is sometimes called MFV [57].}
5. Minimal flavour-violating THDM and stability issues

In Ref. [13, 16], the authors propose the so-called aligned THDM by fixing all the flavour matrices \( X_i \) in Eq. 2.1 to be proportional to the corresponding Yukawa couplings:

\[
X_{d1} = \text{const}_{d1} Y_D, \quad X_{d2} = \text{const}_{d2} Y_D, \quad X_{u1} = \text{const}_{u1} Y_U, \quad X_{u2} = \text{const}_{u2} Y_U, \tag{5.1}
\]

with real or flavourblind prefactors \( \text{const}_{\cdot} \). Comparing Eqs. 2.2 and 2.4 in the aligned model, one finds that there are no FCNC at the tree level.

But the aligned THDM is just the most general minimal flavour-violating (MFV) renormalizable THDM, but expanded to lowest order in the Yukawa couplings \( Y_i \). Following Ref. [50], the most general MFV ansatz is given by the expansion in the two left-handed spurions \( Y_U Y_U^\dagger \) and \( Y_D Y_D^\dagger \) which were discussed in the last chapter:

\[
X_{d1} = Y_D, \\
X_{d2} = \varepsilon_0 Y_D + \varepsilon_1 Y_D Y_D^\dagger Y_D + \varepsilon_2 Y_D Y_D^\dagger Y_D + \ldots, \\
X_{u1} = \varepsilon_0 Y_U + \varepsilon_1 Y_U Y_U^\dagger Y_U + \varepsilon_2 Y_D Y_D^\dagger Y_U + \ldots, \\
X_{u2} = Y_U. \tag{5.2}
\]

The simple form of \( X_{d1} \) and \( X_{u2} \) can be assumed without loss of generality by redefining the two spurions \( Y_U \) and \( Y_D \). But if the higher-order terms in \( X_{d2} \) and \( X_{u1} \) are not included on the tree-level, as in ansatz 5.1, they are automatically generated by radiative corrections. This is assured by the RG invariance of the MFV hypothesis which is implemented by the flavour \( SU(3)^3 \) symmetry. Thus, the functional form in Eq. 5.2 is preserved, only the coefficients \( \varepsilon_i \) and \( \varepsilon_j \) change and are related via the RG equations. In view of this, it is also clear that setting all coefficients to zero leads to heavy fine-tuning. Thus, there is no Yukawa alignment in general within the MFV framework.

In Ref. [19], the stability of the various tree-level implementations by flavourful and flavour-blind symmetries regarding flavour protection is discussed. In the MFV case, the FCNC induced by higher-order terms in the spurions are under control. Even when the coefficients in Eq. 5.2 are of order \( O(1) \) the expansion in the spurions is rapidly convergent due to small CKM matrix elements and small quark masses as was already shown in Ref. [50].

This is not in general true in the case of the implementation via exact flavourblind symmetries if there are additional degrees of freedom at higher scales. Integrating out the latter, one can easily construct higher-dimensional operators which are \( Z_2 \) invariant, but which destroy the flavour protection:

\[
L_Y^{(4)} = \frac{c_1}{\Lambda^2} \tilde{Q}_L X_{d1}^{(6)} U_R H_2 |H_1|^2 + \frac{c_2}{\Lambda^2} \tilde{Q}_L X_{d2}^{(6)} U_R H_2 |H_2|^2 \\
+ \frac{c_3}{\Lambda^2} \tilde{Q}_L X_{u1}^{(6)} D_R H_1 |H_1|^2 + \frac{c_4}{\Lambda^2} \tilde{Q}_L X_{u2}^{(6)} D_R H_1 |H_2|^2, \tag{5.3}
\]

These operators are \( Z_2 \) exact in the sense of the Type-II model \((H_1 \leftrightarrow H_1 \text{ and } D_R \leftrightarrow -D_R)\), but after electroweak symmetry breaking they induce new FCNC. With \( c_i = O(1) \) and the new physics scale \( \Lambda = O(1\text{TeV}) \) one finds too large FCNC inconsistent with present flavour data [19]. Further protection via the MFV hypothesis is needed. This problem already occurs in the case of one Higgs doublet [60, 61, 62].
A similar argument for the implementation of the tree-level condition using the Peccei-Quinn $U(1)$ is valid. However, in contrast to the $Z_2$ symmetry, the Peccei-Quinn symmetry must be explicitly broken in other sectors of the theory to avoid a massless pseudoscalar Higgs field. The spontaneous breaking via the vev of $H_2$ would imply a Goldstone boson. In general, the explicit breaking terms induce too large FCNC [19].

6. Future opportunities

There are great experimental opportunities in flavor physics in the near future. LHCb [63] has finally started taking data and promises to overwhelm many $B$ factory results. In addition, two Super-$B$ factories, Belle II at KEK [64, 65] and SuperB in Italy [66, 67, 68], have been approved and partially funded to accumulate two orders of magnitude larger data samples. The Super-$B$ factories are Super-Flavour factories: Besides precise $B$ measurements they allow for precise analyses of CP violation in charm and of lepton flavour-violating modes like $\tau \rightarrow \mu \gamma$ (for more details see Ref. [69]).

Regarding the measurement of clean $B$ modes, the Super-$B$ factories will push the experimental precision to its limit. For example, the present experimental error of $\mathcal{B}(B \rightarrow \tau \nu)$ discussed in Sect. 3 will be reduced from 20% down to 4%. Thus, the NP reach of this observable will significantly improve; exclusion regions within the THDM-II and the MSSM are shown in Fig. 12.

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