PROCEEDINGS OF SCIENCE

Phase evolution of Universe to the present inert phase

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It is assumed that current state of the Universe is described by the Inert Doublet Model, containing two scalar doublets, one of which is responsible for EWSB and masses of particles and the second one having no couplings to fermions and being responsible for dark matter. We consider possible evolutions of the Universe to this state during cooling down of the Universe after inflation. We found that in the past Universe could pass through phase states having no DM candidate. In the evolution via such states in addition to a possible EWSB phase transition (2-nd order) the Universe sustained one 1-st order phase transition or two phase transitions of the 2-nd order.

Prospects for Charged Higgs Discovery at Colliders
Uppsala University, Sweden
September 27-30, 2010

*Speaker.

†We are thankful to G. Gil, B. Gorczyca, I. Ivanov, M. Dubinin and R. Nevzorov for useful discussions. Work was partly supported by Polish Ministry of Science and Higher Education Grant N N202 230337. The work of MK and DS was supported in addition by EU Marie Curie Research Training Network HEPTOOLS, under contract MRTN-CT-2006-035505, FLAVIAnet contract No. MRTN-CT-2006-035482. The work of IG and KK was also supported by grants RFBR 08-02-00334-a, 10-02-8579, NSh-3810.2010.2 and Program of Dept. of Phys. Sc. RAS "Experimental and theoretical studies of fundamental interactions related to LHC."

PoS(Charged 2010)031

1. Introduction

About 25% of the Universe is made of Dark Matter (DM). Different candidates for DM particle are now discussed, with masses constrained by the accelerator and astrophysical data.

One of the widely discussed models is the Inert Doublet Model (IDM) [1]. The model contains "standard" scalar (Higgs) doublet ϕ_S, similar to that in the SM, and scalar doublet ϕ_D, which doesn't receive vacuum expectation value (v.e.v.) and doesn't couple to fermions. Four degrees of freedom of the Higgs doublet ϕ_S are as in the SM: three Goldstone modes and one mode that becomes the Higgs boson h_S. All components of the scalar doublet ϕ_D are realized as massive scalar D-particles: charged $D^\pm$ and neutral D_H and D_A. They possess a conserved multiplicative quantum number and therefore the lightest particle among them can be considered as a candidate for DM particle.

Here we assume that IDM with neutral DM particles D_H describes the current state of the Universe. We discuss possible ways of the the phase evolution of Universe during its cooling down after inflation, in continuation of analysis [3]. *Complete version of this paper is given in [2].*

The electroweak symmetry breaking via the Higgs mechanism is described by the Lagrangian $\mathscr{L} = \mathscr{L}_{gf}^{SM} + T - V + \mathscr{L}_Y(\psi_f, \phi_S)$. Here, $\mathscr{L}_{gf}^{SM}$ describes the $SU(2)\times U(1)$ Standard Model interaction of gauge bosons and fermions, which is independent on the realization of the Higgs mechanism, T is the standard kinetic term for two scalar doublets ϕ_S and ϕ_D and V is the potential for these two scalars. The $\mathscr{L}_Y$ describes the Yukawa interaction of fermions ψ_f with only one scalar doublet ϕ_S of the same form as in the SM.

Without loss of generality we write most general potential, which can describe IDM, as

$$\begin{aligned} V = -\frac{1}{2}\left[m_{11}^2(\phi_S^\dagger\phi_S) + m_{22}^2(\phi_D^\dagger\phi_D)\right] + \frac{\lambda_1}{2}(\phi_S^\dagger\phi_S)^2 + \frac{\lambda_2}{2}(\phi_D^\dagger\phi_D)^2 + \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D) + \\ + \lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S) + \frac{\lambda_5}{2}\left[(\phi_S^\dagger\phi_D)^2 + (\phi_D^\dagger\phi_S)^2\right], \qquad \lambda_5 < 0. \end{aligned} \tag{1.1}$$

Here all parameters are real. To make some equations shorter, we use the abbreviations:

$$R = (\lambda_3+\lambda_4+\lambda_5)/\sqrt{\lambda_1\lambda_2}\,, \quad \mu_1 = m_{11}^2/\sqrt{\lambda_1}, \quad \mu_2 = m_{22}^2/\sqrt{\lambda_2}\,. \tag{1.2}$$

This potential is invariant under discrete *S-transformation* and *D-transformation*, defined as

$$\boxed{S:\ \ \phi_S \xrightarrow{S} -\phi_S, \quad \phi_D \xrightarrow{S} \phi_D}\,, \qquad \boxed{D:\ \ \phi_S \xrightarrow{D} \phi_S, \quad \phi_D \xrightarrow{D} -\phi_D}\,, \quad SM \xrightarrow{S,D} SM\,. \tag{1.3}$$

(Here SM denote the SM fermions and gauge bosons). Therefore, both *D-symmetry* and *S-symmetry* are conserved by the potential. The Yukawa term respects D-symmetry but violates S-symmetry.

To have a stable vacuum, the potential must be positive at large quasi–classical values of fields $|\phi_i|$ (*positivity constraints*). These conditions limit possible values of λ_i (see e.g. [4]). In terms of parameters (1.2) positivity constraints which are needed in our analysis, can be written as

$$\lambda_1 > 0\,, \quad \lambda_2 > 0, \quad R+1 > 0. \tag{1.4}$$

2. Thermal evolution

Since the Hubble constant is small, a statistical equilibrium at every temperature T takes place. The ground state of such thermal system is given by a minimum of the Gibbs potential

$$V_G = Tr\left(Ve^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right)\,. \tag{2.1}$$

In the first nontrivial approximation and high enough temperature the obtained Gibbs potential has the same form as the basic potential V(1.1) at zero temperature. The coefficients $\lambda's$ of the quartic terms in the potential V_G and V coincide, while the mass terms vary with temperature:

$$\begin{gathered} m_{11}^2(T) = m_{11}^2 - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2\,, \\ c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32} + \frac{g_t^2 + g_b^2}{8}, \quad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32}. \end{gathered} \tag{2.2}$$

Here g and g' are the EW gauge couplings, $g_t \approx 1$ and $g_b \approx 0.03$ are the SM Yukawa couplings for t and b quarks, respectively.

Each of coefficients c_1 and c_2 can be either positive or negative. However, in virtue of positivity conditions (1.4) their sum is positive. According to (4.1), for a realization of the present inert vacuum with neutral DM particle one needs $\lambda_4 + \lambda_5 < 0$. Therefore, at $R > 0$ we have $\lambda_3 > 0$. Since $\lambda_5 < 0$, we have $c_1 > 0$, $c_2 > 0$. At $R < 0$ there are no constraints on signs of c_1, c_2. So,

$$c_1 + c_2 > 0\,, \qquad \boxed{R > 0: \quad c_1 > 0,\ \ c_2 > 0}\,; \quad \boxed{R < 0: \quad \text{arbitrary signs of } c_{1,2}}\,. \tag{2.3}$$

3. Extrema of the potential

We first consider extrema of the potential (1.1) at arbitrary values of parameters. The extrema conditions: $\partial V/\partial \phi_i|_{\phi_i = \langle \phi_i \rangle} = 0$, $\partial V/\partial \phi_i^\dagger\Big|_{\phi_i = \langle \phi_i \rangle} = 0$, $(i = S, D)$ define the extremum values $\langle \phi_S \rangle$ and $\langle \phi_D \rangle$ of the fields ϕ_S and ϕ_D. The extremum with the lowest energy realizes *the vacuum state*.

For each electroweak symmetry violating extremum with $\langle \phi_S \rangle \neq 0$, one can choose the z axis in the weak isospin space so that $\langle \phi_S \rangle$ has only lower component (choosing a "neutral direction" in the weak isospin space) with free form for $\langle \phi_D \rangle$. Therefore, one can write the general solution of extrema conditions as $\langle \phi_S \rangle = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}$, $\langle \phi_D \rangle = \dfrac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix}$ with real, positive v_S.

The solutions of extrema conditions with $u = 0$ are called *neutral extrema*. In this case the extrema conditions can be written as a system of two degenerate cubic equations with four solutions:

$$v_S(-m_{11}^2 + \lambda_1 v_S^2 + \lambda_{345} v_D^2) = 0\,, \qquad v_D(-m_{22}^2 + \lambda_2 v_D^2 + \lambda_{345} v_S^2) = 0\,, \quad v_S^2,\, v_D^2 \geqslant 0\,.$$

This system has four solutions (here $\mathscr{E}_a$ are extrema energies):

$$\boldsymbol{EWs}: \ EW symmetric \ \ v_D = 0, \quad v_S = 0, \qquad \mathscr{E}_{EWs} = 0; \tag{3.1}$$

$$\boldsymbol{I_1}: \qquad inert \qquad v_D = 0,\ v^2 \equiv v_S^2 = \frac{m_{11}^2}{\lambda_1}, \quad \mathscr{E}_{I_1} = -\frac{m_{11}^4}{8\lambda_1} \equiv -\frac{\mu_1^2}{8}\,; \tag{3.2}$$

$$\boldsymbol{I_2}: \quad inert-like \quad v_S = 0,\ v^2 \equiv v_D^2 = \frac{m_{22}^2}{\lambda_2}, \quad \mathscr{E}_{I_2} = -\frac{m_{22}^4}{8\lambda_2} \equiv -\frac{\mu_2^2}{8}\,; \tag{3.3}$$

$$\boldsymbol{M}: \qquad mixed \qquad \begin{cases} v_S^2 = \dfrac{\mu_1 - R\mu_2}{\sqrt{\lambda_1}\,(1 - R^2)}\,, \quad v_D^2 = \dfrac{\mu_2 - R\mu_1}{\sqrt{\lambda_2}\,(1 - R^2)}\,, \\ \qquad \mathscr{E}_M = \dfrac{-\mu_1^2 - \mu_2^2 + 2\mu_1\mu_2 R}{8(1 - R^2)}\,. \end{cases} \tag{3.4}$$

$$\text{Note:} \qquad \mathscr{E}_{I_1} - \mathscr{E}_M = \frac{(\mu_1 R - \mu_2)^2}{8(1 - R^2)}\,, \qquad \mathscr{E}_{I_2} - \mathscr{E}_M = \frac{(\mu_2 R - \mu_1)^2}{8(1 - R^2)}\,. \tag{3.5}$$

For $u \neq 0$ the extremum violates not only EW symmetry, but also the $U(1)$ electromagnetic symmetry, leading to the *charge breaking extremum*. This extremum can realize vacuum state only if $\lambda_4+\lambda_5>0$ [5, 3]. This inequality doesn't allow to have neutral DM particle (see (4.1)).

4. Neutral vacuum states

Electroweak symmetric vacuum *EWs* with $\langle\phi_i\rangle=0$ is a minimum realizing vacuum state if $m_{11}^2<0$, $m_{22}^2<0$. Here gauge bosons and fermions are massless, scalar doublets ϕ_S and ϕ_D are massive.

Inert vacuum I_1 preserves D-parity but breaks S-parity. It describes reality in the IDM. The field decomposition near extremum is $\phi_S=\dfrac{1}{\sqrt{2}}\begin{pmatrix}G^+\sqrt{2}\\ v+h_S+iG\end{pmatrix}$, $\phi_D=\dfrac{1}{\sqrt{2}}\begin{pmatrix}D^+\sqrt{2}\\ D_H+iD_A\end{pmatrix}$, where $G^\pm$ and G are Goldstone modes. The Higgs particle h_S interacts with the fermions and gauge bosons just as the Higgs boson in the SM. The scalar D-particles $D_H, D_A, D^\pm$ don't interact with fermions. Because of D-parity conservation *the lightest D-particle is stable, being a good DM candidate*.

The inert *extremum* exists only if $m_{11}^2>0$ (3.2). In accordance with (3.2), (3.3), this extremum can be a *vacuum* only if $\mu_1>\mu_2$. Besides, one should compare I_1 and M extrema. In virtue of (3.5) at $R^2>1$ the energy of extremum M is larger than energy of I_1 extremum – so that the extremum I_1 realizes vacuum. At $R^2<1$ the inert extremum still can be a vacuum in the case, when the mixed extremum does not exist, i. e. if at least one of quantities v_S^2, v_D^2 defined by eq. (3.4) is negative. Note, that due to the positivity constraint $1+R>0$ (1.4) in the case when $R^2>1$ we have $R>1$. For the opposite case, with $R^2<1$, the quantity R can be either positive or negative.

The quadratic part of the potential written in terms of physical fields gives the masses of scalars

$$M_{h_S}^2=m_{11}^2\,,\quad M_{D^\pm}^2=\frac{\lambda_3v^2-m_{22}^2}{2}\,,\quad M_{D_H}^2=M_{D^\pm}^2+\frac{\lambda_4+\lambda_5}{2}v^2\,,\quad M_{D_A}^2=M_{D_H}^2-\lambda_5v^2\,. \qquad (4.1)$$

The requirement that lightest D-particle is a neutral one, results in the condition $\lambda_4+\lambda_5<0$.

The scalars D_H and D_A have opposite P-parities but since they don't couple to fermions, there is no way to assign to them a definite value of P-parity. However, their relative parity does matter and for example, vertex ZD_HD_A is allowed while vertices ZD_HD_H and ZD_AD_A are forbidden.

The Inert-like vacuum I_2 violates both D-symmetry since $\langle\phi_D\rangle\neq0$ and S-symmetry via Yukawa interaction. It looks as "mirror-symmetric" to the inert vacuum I_1. The interactions among scalars and between scalars and gauge bosons in both cases are identical in form with the change $\phi_S\leftrightarrow\phi_D$ and correspondingly $D_H\to S_H$, $D_A\to S_A$, $D^\pm\to S^\pm$, $h_S\to h_D$. The unique, but important distinction between I_2 and I_1 is given by the Yukawa interaction. The Higgs boson h_D couples to gauge bosons just as the Higgs boson of the SM, however it does not couple to fermions at the tree level. All fermions, by construction interacting only with ϕ_S with vanishing v.e.v. $\langle\phi_S\rangle=0$, are massless. *Here there are no candidates for the dark matter particles.*

The mixed vacuum *M* (with $\langle\phi_S\rangle$, $\langle\phi_D\rangle\neq0$) violates both D- and S-symmetry. It has standard properties of vacuum in CP-conserved 2HDM. *In this vacuum we have massive fermions and no candidates for DM particle*. In accordance with (3.4), (3.5) the mixed extremum is global minimum of potential, i. e. *vacuum*, if and only if $v_S^2>0$, $v_D^2>0$, $R^2<1$. For v.e.v.'s squared given by eqs. (3.4) the latter conditions can be transformed into the relations between mass parameters m_{ii}^2:

$$0<R\mu_1<\mu_2<\mu_1/R\ \ (at\ \ 1>R>0),\quad \mu_2>R\mu_1\,,\quad \mu_1>\mu_2R\ \ (at\ \ 0>R>-1). \qquad (4.2)$$

5. Evolution of phase states of the Universe

Now we consider possible phase history of the Universe, leading to the inert vacuum I_1 today. We depict results in the $(\mu_1(T), \mu_2(T))$ plane[1] – Fig. 1. In accordance with analysis of sect. 4, at $R > 1$ this plane contains one quadrant with EWs phase and two sectors, describing the I_1 and I_2 phases. These sectors are separated by the line $\mu_1 = \mu_2$ (thick black line) – Fig. 1a. At $1 > R > 0$ the phase diagram Fig. 1b is obtained from Fig. 1a by insertion in the upper right quadrant the new sector – the mixed phase M, described in accordance with (4.2) by equation $0 < R\mu_1 < \mu_2 < \mu_1/R$. At $0 > R > -1$ in the phase diagram (Fig. 1c) the the mixed phase M region is realized even beyond an upper right quadrant $\mu_2 > \mu_1/R$, $\mu_2 > \mu_1 R$.

The thermal variations of m_{ii}^2 result in modification of vacuum state. The possible current states of Universe are represented in the Fig. 1 by small black dots $P = (\mu_1, \mu_2)$ with $\mu_1 > 0$. In accordance with (2.2) an evolution leading to a given physical vacuum state P is represented by a

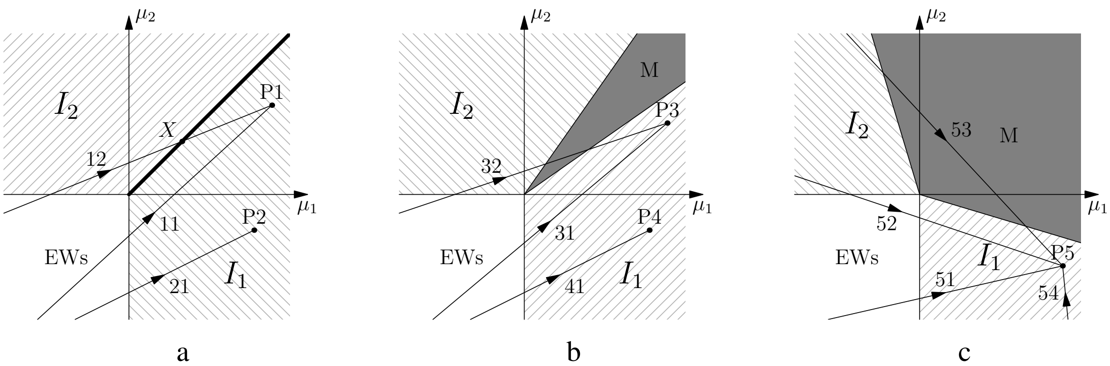

Figure 1: The μ-plane and possible evolutions for $R > 1$ (a), $1 > R > 0$ (b), $0 > R > -1$ (c).

ray, that ends at a point P. Arrows on these rays are directed towards a growth of time (decreasing of temperature). The direction of the ray is determined by parameters $\tilde{c}_1 = c_1/\sqrt{\lambda_1}$, $\tilde{c}_2 = c_2/\sqrt{\lambda_2}$, (2.2). The boundaries between two phases are *the phase transition lines*. These transitions are of the 2-nd order for all rays except the 1-st order transition for the ray 12.

The starting point of evolution of Universe to the present day inert phase state can be either electroweak symmetric (EWs) state, at $c_2c_1 > 0$ – rays 11, 12, 21, 31, 32, 41, 51 – or electroweak symmetry violating (*EWv*) state, at $c_2c_1 < 0$ (for $0 > R > -1$ only) – rays 52, 53, 54.

For rays 11, 21, 31, 41, 51 after the second-order EWSB transition at $m_{11}^2(T) = 0$, i. e. at the temperature $T_{EWs,1} = \sqrt{\mu_1/\tilde{c}_1}$ the Universe has entered the present inert phase I_1.

For rays 12, 32 the Universe went through the EWSB second-order phase transition into the inert-like phase I_2 at $m_{22}^2(T) = 0$, i. e. at the temperature $T_{EWs,2} = \sqrt{\mu_2/\tilde{c}_2}$.

For ray 12 the transition from the inert-like phase I_2 into the today's inert phase I_1 at $\mu_2(T) = \mu_1(T)$, i. e. at the temperature $T_{2,1} = \sqrt{(\mu_1 - \mu_2)/(c_1 - \tilde{c}_2)}$, is the first order phase transition with the latent heat $Q_{2,1} = T_{2,1}\left(\partial\mathscr{E}_{I_2}/\partial T - \partial\mathscr{E}_{I_1}/\partial T\right)_{T=T_{2,1}} = (\mu_2\tilde{c}_1 - \mu_1\tilde{c}_2)T_{2,1}^2/4$.

For ray 32 the path from I_2 phase to I_1 is through the mixed phase M. The second-order phase transitions happened at the temperatures $T(I_2 \to M) = \sqrt{(\mu_1 - R\mu_2)/(\tilde{c}_1 - R\tilde{c}_2)}$ and $T(M \to I_1) = \sqrt{(R\mu_1 - \mu_2)/(R\tilde{c}_1 - \tilde{c}_2)}$.

[1] We distinguish present day values of parameters μ_i and their values $\mu_i(T)$ at some temperature T.

For rays 52, 53 a high-temperature state of the Universe is the inert-like vacuum I_2.

For ray 52 during cooling down the Universe goes through electroweak symmetric phase EWs into the present I_1 phase. The second-order phase transitions $I_2 \to EWs$ and $EWs \to I_1$ happened, respectively, at the temperatures $T_{2,EWs} = \sqrt{\mu_2/\tilde{c}_2}$, $T_{EWs,1} = \sqrt{\mu_1/\tilde{c}_1}$.

For ray 53 during cooling down the Universe passes through the mixed phase M into the present I_1 phase. The phase transitions $I_2 \to M$ and $M \to I_1$ are of the second order.

For ray 54 the Universe stays in the inert vacuum I_1 during the whole evolution.

6. Results and discussion

The most important observation we made is as follows: *If current state of the Universe is described by IDM, then during the thermal evolution the Universe can pass through various intermediate phases, different from the inert one. These possible intermediate phases contain no dark matter, which appears only at the relatively late stage of cooling down of the Universe.* To find what scenario of evolution is realized in nature, one should measure all parameters of potential.

Extra phase transitions at lower temperature than EWSB temperature (and especially first order phase transition for the evolution along ray 12) can influence baryogenesis even stronger than transformation of standard second order EWSB transition into the first order one due to term $\phi^3 T$ [6]. Moreover, in contrast to the standard picture, the considered scenarios allow for the phase transition to the current inert phase at relatively low temperature. This gives new starting point for calculation of a today's abundance of the neutral DM components of the Universe and other phenomena.

In this paper we calculated thermal evolution of the Universe in the very high temperature approximation, i. e. for $T^2 \gg |m_{ii}^2|$. The most interesting effects are expected at lower temperatures, where more precise calculations are necessary.

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