PROCEEDINGS OF SCIENCE

# Measurements of $\alpha$ ($\phi_2$)

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We present a summary of the measurements of the CKM angle, $\alpha$ ($\phi_2$), performed by the BaBar and Belle experiments which collect $B\overline{B}$ pairs at the $\Upsilon(4S)$ resonance produced in asymmetric $e^+e^-$ collisions. We discuss the measurements of the the branching fractions and *CP* asymmetries in $B \rightarrow \pi\pi$, $\rho\pi$ and $\rho\rho$ final states that lead to constraints on $\alpha$ ($\phi_2$). Finally, we present the recent measurement of the branching fraction of $B \rightarrow K_{1A}\pi$ decays which can be used to calculate a bound on the shift in the measured $\alpha$ ($\phi_2$) of $B^0 \rightarrow a_1(1260)^{\pm}\pi^{\mp}$ decays caused by second-order loop processes.

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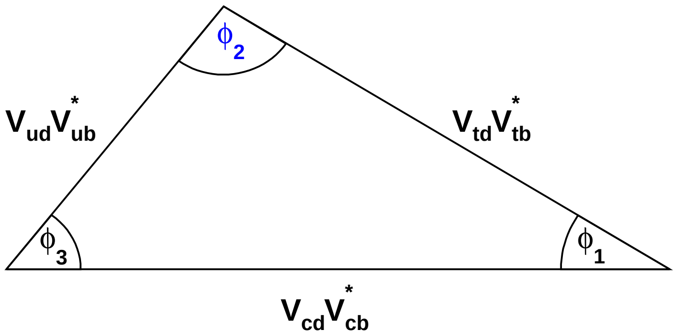


**Figure 1:** The unitarity triangle related to $B$ decays as constructed from CKM matrix elements along with definitions of the internal angles.

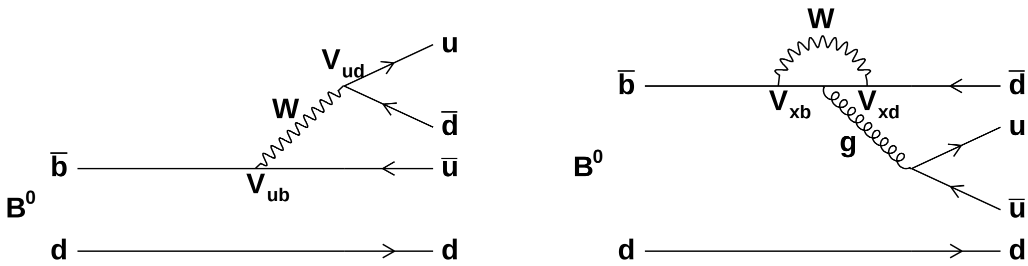


**Figure 2:** The left diagram shows the first-order $b \to u\bar{u}d$ transition while the right diagram shows the second-order $b \to u\bar{u}d$ loop process.

## 1. Introduction

The main goal of the BaBar experiment at SLAC and the Belle experiment at KEK is to constrain the unitarity triangle for $B$ decays shown in Fig. 1. This allows us to test the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for violation of the combined charge-parity ($CP$) symmetry [1, 2], as well as search for new physics effects beyond the Standard Model (SM). These proceedings give a summary of the experimental status of measurements of the CKM phase, $\alpha$, hitherto referred to as $\phi_2$, defined from CKM matrix elements as $\phi_2 \equiv \arg(-V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)$, and shown in Fig. 1.

First-order weak processes (tree) proceeding by $b \to u\bar{u}d$ quark transitions as illustrated in Fig. 2, such as $B^0 \to \pi\pi$, $\rho\pi$, $\rho\rho$ and $a_1(1260)\pi$, are directly sensitive to $\phi_2$. These amplitudes contain the CKM matrix element, $V_{ub}$, which carries the CKM phase, $-\phi_3$. Now, the phenomena of neutral $B$ meson mixing includes the phase, $-2\phi_1$. Considering only the interference between the direct decay of a $B^0$ meson to a $CP$ eigenstate and decays to that same final state where the $B^0$ first mixed to form a $\bar{B}^0$, one obtains a relative phase of $-2\phi_1$ from mixing and $-2\phi_3$ from the difference between $b \to u$ and the conjugate $\bar{b} \to \bar{u}$ process. Thus, assuming a closed triangle, $\phi_1 + \phi_2 + \phi_3 = \pi$, first-order $b \to u\bar{u}d$ transitions are sensitive to $-2\phi_1 - 2\phi_3 = -2\phi_2$.

In the quasi-two-body approach, CKM angles can be determined by measuring the time-dependent asymmetry between $B^0$ and $\bar{B}^0$ decays [3]. For the decay sequence, $\Upsilon(4S) \to B_{CP}B_{\rm Tag} \to f_{CP}f_{\rm Tag}$, where one of the $B$ mesons decays at time, $t_{CP}$, to a $CP$ eigenstate, $f_{CP}$, and the other decays at time, $t_{\rm Tag}$, to a flavour specific final state, $f_{\rm Tag}$, with $q = +1(-1)$ for $B_{\rm Tag} = B^0(\bar{B}^0)$, the decay rate has a time-dependence given by

$$P(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\left[1 + q(A_{CP}\cos\Delta m_d\Delta t + S_{CP}\sin\Delta m_d\Delta t)\right], \tag{1.1}$$

where $\Delta t \equiv t_{CP} - t_{\rm Tag}$ and $\Delta m_d$ is the mass difference between the $B_H$ and $B_L$ mass eigenstates. The parameters, $A_{CP}$ and $S_{CP}$, describe direct and mixing-induced $CP$ violation, respectively. An alternate notation where $C_{CP} = -A_{CP}$ also exists in literature. If a single first-order weak amplitude dominates the decay, then we expect $A_{CP} = 0$ and $S_{CP} = \sin 2\phi_2$.

On the other hand, if second-order processes such as those depicted in Fig. 2 are present, then direct $CP$ violation is possible, $A_{CP} \neq 0$. Additionally, as these loop processes (penguins) are not directly proportional to $V_{ub}$, our measurement of $S_{CP}$ does not directly determine $\phi_2$, rather, $S_{CP} = \sqrt{1-A_{CP}^2}\sin(2\phi_2 - 2\Delta\phi_2)$, where $\Delta\phi_2$ is the shift caused by the second order contributions.

Despite this, it is still possible to determine $\Delta\phi_2$ in $B^0 \to h^+h^-$ with an $SU(2)$ isospin analysis by considering the set of three $B \to hh$ decays where $hh$ is either two pions or two longitudinally polarised $\rho s$ [4]. The main concept behind this is to recognise that the two products in $B^+ \to h^+h^0$ decays must have a total isospin of $I = 1$ or $I = 2$, since $I_3 = 1$. For the penguin terms, only $I = 0$ or $I = 1$ is possible since the gluon carries $I = 0$ and isospin is conserved in the strong interaction. However, as $I = 1$ is forbidden by Bose-Einstein statistics, strong second-order loops are forbidden and hence $B^+ \to h^+h^0$ may only decay weakly at tree-level in the limit of neglecting electroweak penguins.

The $B \to hh$ amplitudes obey the complex relations,

$$A_{+0} = \frac{1}{\sqrt{2}}A_{+-} + A_{00}, \quad \bar{A}_{-0} = \frac{1}{\sqrt{2}}\bar{A}_{+-} + \bar{A}_{00}, \tag{1.2}$$

which can be represented as triangles in Fig. 3. As $B^+ \to h^+h^0$ is a pure first-order mode, these triangles share the same base, $A_{+0} = \bar{A}_{-0}$, and $\Delta\phi_2$ can be determined from the difference between the two triangles. These triangles and $\phi_2$ can be fully determined from the branching fractions, $B(B^0 \to h^+h^-)$, $B(B^0 \to h^0h^0)$, $B(B^+ \to h^+h^0)$ and the $CP$ violation parameters, $A_{CP}(B^0 \to h^+h^-)$, $A_{CP}(B^0 \to h^0h^0)$, $S_{CP}(B^0 \to h^+h^-)$. This method exhibits an 8-fold discrete ambiguity in the determination of $\phi_2$ which arises from the 4 triangle orientations around $A_{+0}$ and the two solutions of $\phi_2$ in the measurement of $S_{CP}$.

At the $B$ factories, two of the key variables in discriminating $B$ signal from the large background are the beam-constrained mass, $M_{\rm bc} = m_{\rm ES} \equiv \sqrt{(E_{\rm beam}^{\rm CMS})^2 - (p_B^{\rm CMS})^2}$, and the energy difference, $\Delta E \equiv E_B - E_{\rm beam}^{\rm CMS}$, which arise as the energy of each $B$ in the $\Upsilon(4S)$ centre-of-mass is known. The full data sets taken at the $\Upsilon(4S)$ resonance for the BaBar and Belle collaborations are 467 million and 772 million $B\bar{B}$ pairs, respectively.

## 2. $B \to \pi\pi$

The analysis of $B \to \pi\pi$ performed by the BaBar collaboration is based on their full data set 467 million $B\bar{B}$ pairs [5], while the analysis from the Belle collaboration is based on 535 million

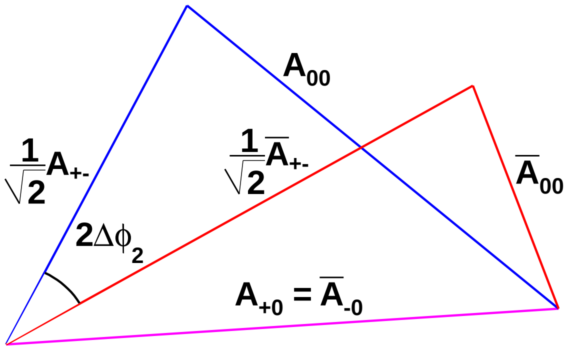


**Figure 3:** Complex isospin triangles from which $\Delta\phi_2$ can be determined.

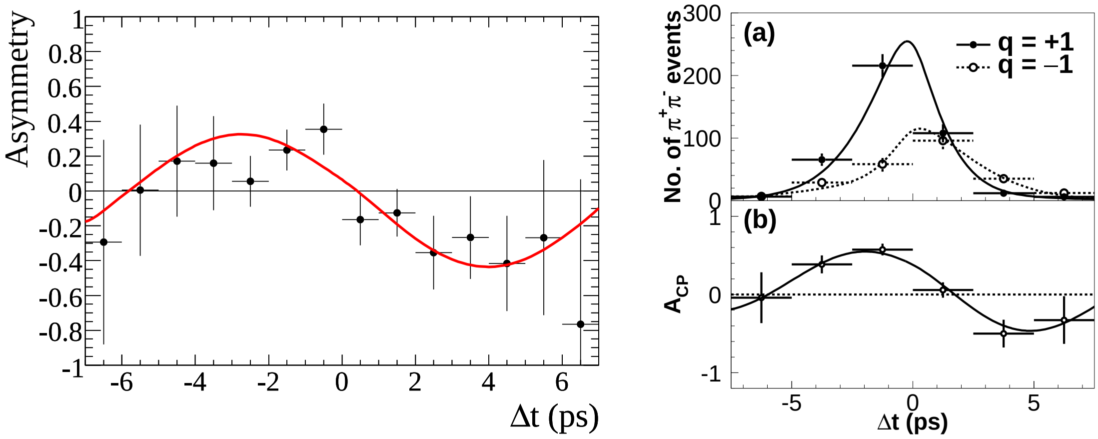


**Figure 4:** The left plot shows the time-dependent asymmetry, $a(\Delta t) \equiv (N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, of $B^0 \to \pi^+\pi^-$ from BaBar. The right plots from Belle show the fit to $\Delta t$ for each flavour tag on top and the resulting asymmetry below. Mixing-induced *CP* violation can be clearly seen in the asymmetry plots and the height difference in the $\Delta t$ projection indicates direct *CP* violation.

$B\bar{B}$ pairs [6]. They obtain the *CP* parameters,

$$
\begin{array}{lcl}
 & \text{BaBar} & \\
\mathcal{A}_{CP} & = & +0.25 \pm 0.08 \pm 0.02 \; (3.0\sigma) \\
\mathcal{S}_{CP} & = & -0.68 \pm 0.10 \pm 0.03 \; (6.3\sigma)
\end{array}
\qquad
\begin{array}{lcl}
 & \text{Belle} & \\
\mathcal{A}_{CP} & = & +0.55 \pm 0.08 \pm 0.05 \; (5.5\sigma) \\
\mathcal{S}_{CP} & = & -0.61 \pm 0.10 \pm 0.04 \; (5.3\sigma),
\end{array}
$$

and the fit projections are shown in Fig. 4. Both experiments have observed *CP* violation in $B \to \pi\pi$ and the difference between the two measurements is $1.9\sigma$. They also find $\mathcal{A}_{CP}$ to be non-zero implying that more than a tree amplitude is present and thus the presence of additional amplitudes should be considered to extract $\phi_2$. A $\chi^2$ of the 6 physical observables is constructed from the 5 constraining amplitudes in Eq. 1.2 and $\phi_2$ which is then minimised in a $\phi_2$ scan. The $\chi^2$ is then converted to a probability for one degree of freedom as shown in Fig. 5, from which BaBar excludes the range $[23^\circ, 67^\circ]$ at the 90% CL and Belle excludes the range $[11^\circ, 79^\circ]$ at the 95% CL.

## 3. $B \to \rho\rho$

$B \to \rho\rho$ decays have an additional complication that the two spin 1 $\rho$ mesons have a relative

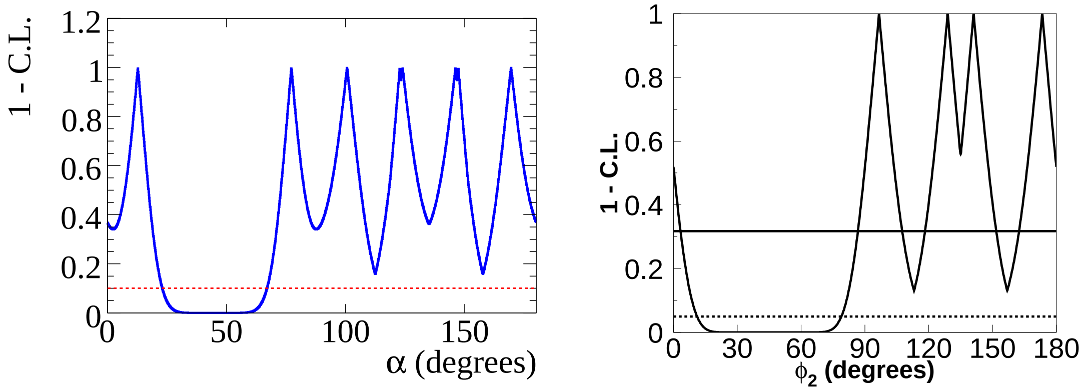


**Figure 5:** The left plot shows the constraint on $\phi_2$ in the $B \to \pi\pi$ system from BaBar where an 8-fold ambiguity can be seen. The right plot shows the constraint from Belle where the apparent 4-fold ambiguity is coincidental.

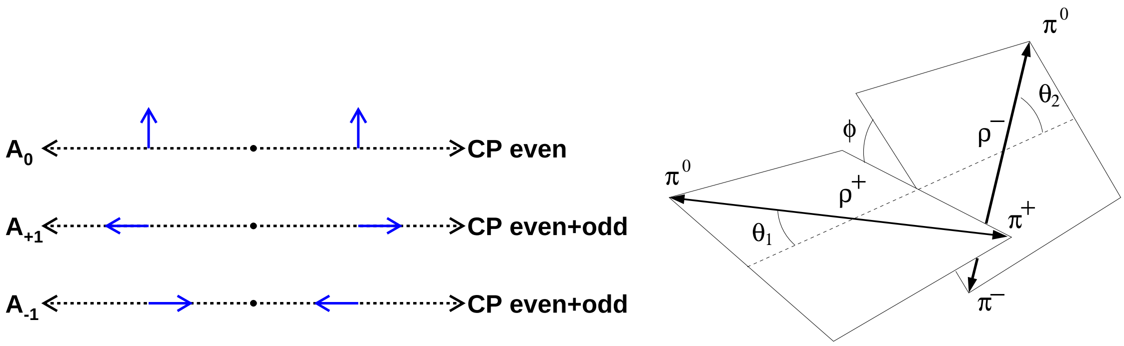


**Figure 6:** The left schematic shows the 3 possible spin projections (blue) onto the momentum direction of each $\rho$ (dashed) in the $B$ rest frame for $B \to \rho\rho$ channels. The right figure defines the planes from which the longitudinal polarisation amplitude can be separated.

orbital angular momentum, $L = 0, 1, 2$. Since the *CP* eigenvalue of $B^0 \to \rho^+\rho^-$ is $(-1)^L$, it is necessary to isolate a definite *CP* component in order to constrain $\phi_2$. Now, the total angular momentum of the $\rho\rho$ system is $J_{\rho\rho} = 0$ and $L$ has no component along the decay axis. Therefore, the final state is a superposition of three possible polarisation amplitudes as shown in Fig. 6: one longitudinal ($A_0 : L = 0, 2$) and two transverse ($A_\pm : L = 0, 1, 2$) amplitudes. By considering the distributions of the $\rho$ helicity angles defined in their respective rest frames as illustrated in Fig. 6, and integrating over the azimuthal angle between the decay planes, the angular decay rate is given by

$$\frac{d^2N}{d\cos\theta_1 d\cos\theta_2} = \frac{9}{16}[4 f_L \cos^2\theta_1 \cos^2\theta_2 + (1 - f_L)\sin^2\theta_1 \sin^2\theta_2], \tag{3.1}$$

where $f_L$ is the fraction of longitudinal polarisation which can be determined in a fit to data. Conveniently, it turns out that the $\rho\rho$ system is dominated by the *CP* even longitudinal amplitude [7, 8], which means the transverse component can be ignored in an isospin analysis.

The BaBar analysis of $B^0 \to \rho^+\rho^-$ is based on 384 million $B\bar{B}$ pairs [7] while the Belle

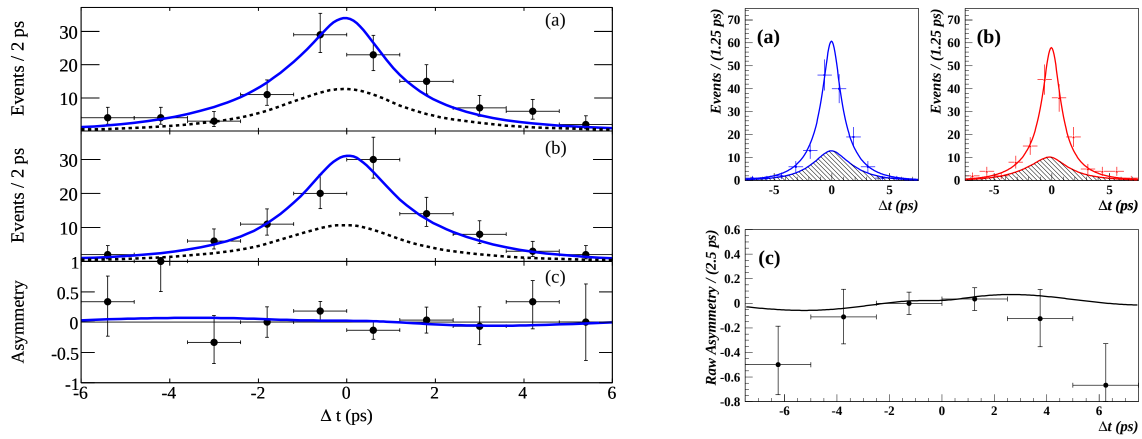


**Figure 7:** The left plot shows the $\Delta t$ distributions of $B^0 \to \rho^+\rho^-$ for (a) $B^0$ and (b) $\bar{B}^0$ tags and (c) the resulting asymmetry for BaBar. The same is shown on the right for Belle.

analysis is based on 535 million $B\bar{B}$ pairs [9]. They obtain the *CP* parameters,

$$\begin{array}{lcl} & \text{BaBar} & \\ A_{CP} & = & 0.01 \pm 0.15 \pm 0.06 \\ S_{CP} & = & -0.17 \pm 0.20^{+0.05}_{-0.06} \end{array} \qquad \begin{array}{lcl} & \text{Belle} & \\ A_{CP} & = & +0.16 \pm 0.21 \pm 0.07 \\ S_{CP} & = & +0.19 \pm 0.30 \pm 0.07, \end{array}$$

shown in Fig. 7 which demonstrate that $A_{CP}$ is consistent with zero, implying no evidence of a penguin contribution.

The BaBar collaboration has recently updated their $B^+ \to \rho^+\rho^0$ analysis with the final data set [10]. They obtain the branching fraction, $\mathcal{B}(B^+ \to \rho^+\rho^0) = (23.7 \pm 1.4 \pm 1.4) \times 10^{-6}$, which allows a precise measurement of the isospin triangle base, and $A_{CP} = 0.054 \pm 0.055 \pm 0.010$, showing no evidence for amplitudes which do not conserve isospin.

Unlike in $B \to \pi\pi$, we can measure $S_{CP}$ from $B^0 \to \rho^0\rho^0$, which can ultimately remove the 4-fold ambiguity of $\Delta\phi_2$, leaving two solutions for $\phi_2$. This is mode is experimentally difficult to isolate due to its relatively low branching fraction in the presence of multiple backgrounds with the same final state. BaBar has observed this mode with a significance of $3.1\sigma$ [11] and obtained the *CP* parameters, $A_{CP} = -0.2 \pm 0.8 \pm 0.3$ and $S_{CP} = +0.3 \pm 0.7 \pm 0.2$, while Belle has obtained an upper limit [12].

A consequence of the small $B^0 \to \rho^0\rho^0$ branching fraction relative to $B^+ \to \rho^+\rho^0$, is that the isospin triangles become flat making the 4 solutions of $\Delta\phi_2$ nearly degenerate. The constraints on $\phi_2$ in $B \to \rho\rho$ are shown in Fig. 8 from which BaBar determines $\phi_2 = (92.4^{+6.0}_{-6.5})^\circ$ and Belle finds $\phi_2 = (91.7 \pm 14.9)^\circ$.

## 4. $B^0 \to (\rho\pi)^0$

As $B^0 \to (\rho\pi)^0$ is not a *CP* eigenstate, four flavour-charge configurations need to be considered. In principle, one can extend the isospin analysis leading to isospin pentagon relations. However, it is possible to constrain $\phi_2$ explicitly in a time-dependent amplitude analysis that includes variations of the strong phase of interfering $\rho$ resonances over the Dalitz Plot [13]. The relative

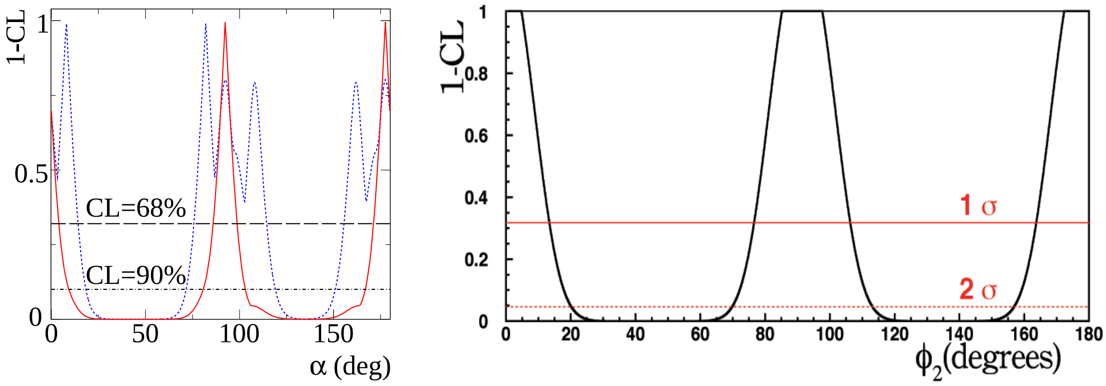


**Figure 8:** The left plot shows the constraint on $\phi_2$ in the $B \to \rho\rho$ system using only BaBar results. The right plot shows the constraint from Belle which uses its latest $B^0 \to \rho^0\rho^0$ result, otherwise world averages. This analysis was performed before the recent update of $B^+ \to \rho^+\rho^0$ from BaBar and a plateau is present as there is no constraint on $A_{CP}(\rho^0\rho^0)$.

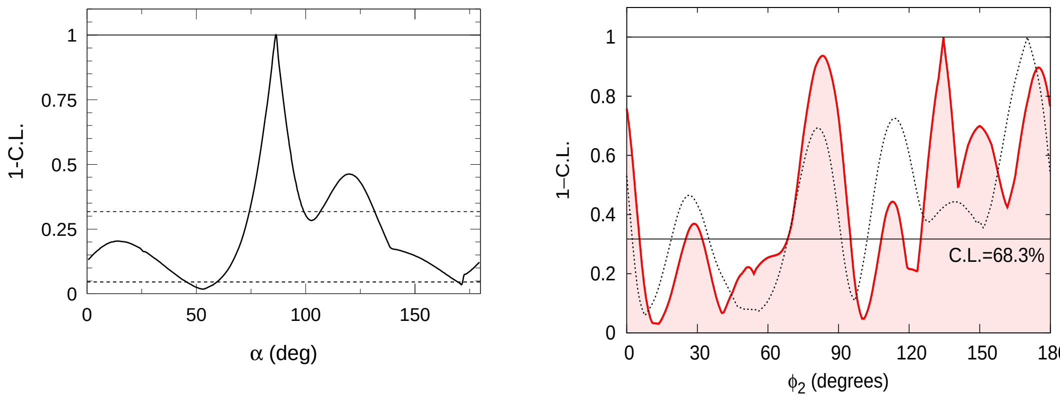


**Figure 9:** The left plot from BaBar shows the constraint on $\phi_2$ in the $B^0 \to (\rho\pi)^0$ system with their time-dependent amplitude analysis. The right plot shows the constraint from Belle where the dashed curve corresponds to the BaBar curve and the red curve contains additional constraints from charged $B^+ \to (\rho\pi)^+$ modes.

moduli and phases of the six possible amplitudes of $B^0(\bar{B}^0) \to \pi^+\pi^-\pi^0$ decays via charged and neutral intermediate $\rho$ resonances are determined. These amplitudes are constructed from isospin relations from which $\phi_2$ can be constrained without ambiguity.

The BaBar and Belle collaborations have performed this analysis with 375 and 449 million $B\bar{B}$ pairs, respectively, and are in good agreement. Their corresponding $\phi_2$ scans are shown in Fig. 9 where BaBar obtains $\phi_2 = (87^{+45}_{-13})^\circ$ while Belle can only constrain $68^\circ < \phi_2 < 95^\circ$ at 68.3% CL for the solution consistent with SM.

## 5. $B^0 \to a_1(1260)^\pm\pi^\mp$

The $B^0 \to a_1(1260)^\pm\pi^\mp$ system is analogous to $B^0 \to (\rho\pi)^0$ however, information on $\phi_2$ is obtained in a quasi-two-body approach since a Dalitz plot analysis is complicated by an additional $\pi^0$ in the final state. The time-dependence of the four flavour-charge configurations where the $a_1$

possesses charge, $c$ ($a_1^+$: $c = +1$, $a_1^-$: $c = -1$), is given by

$$P_{a_1\pi}(\Delta t, q, c) = (1 + cA_{CP})\frac{e^{-|\Delta t|/\tau_{B^0}}}{8\tau_{B^0}}\Big\{1 + q \times \Big[(S_{CP} + c\Delta S)\sin\Delta m_d\Delta t - (C_{CP} + c\Delta C)\cos\Delta m_d\Delta t\Big]\Big\}. \tag{5.1}$$

The parameter $A_{CP}$, measures time and flavour-integrated direct $CP$ violation, $S_{CP}$ measures mixing-induced $CP$ violation and $C_{CP}$ measures flavour-dependent direct $CP$ violation. The quantity $\Delta S$, is related to the strong phase difference between the contributing amplitudes to $B^0 \to a_1(1260)^{\pm}\pi^{\mp}$ decays and $\Delta C$ measures the rate asymmetry between $\Gamma(B^0 \to a_1^+\pi^-) + \Gamma(\overline{B}^0 \to a_1^-\pi^+)$ and $\Gamma(B^0 \to a_1^-\pi^+) + \Gamma(\overline{B}^0 \to a_1^+\pi^-)$. These two parameters are not sensitive to $CP$ violation. From these parameters, an effective $\phi_2^{\rm eff}$ can be determined with a four-fold ambiguity,

$$\phi_2^{\rm eff} = \frac{1}{4}\left[\arcsin\left(\frac{S_{CP} + \Delta S}{\sqrt{1 - (C_{CP} + \Delta C)^2}}\right) + \arcsin\left(\frac{S_{CP} - \Delta S}{\sqrt{1 - (C_{CP} - \Delta C)^2}}\right)\right]. \tag{5.2}$$

The BaBar collaboration has performed this analysis with 384 million $B\overline{B}$ pairs [14] and obtained the parameters

| $CP$ violating | $CP$ conserving |
|---|---|
| $A_{CP} = -0.07 \pm 0.07 \pm 0.02$ | |
| $C_{CP} = -0.10 \pm 0.15 \pm 0.09$ | $\Delta C = +0.26 \pm 0.15 \pm 0.07$ |
| $S_{CP} = +0.37 \pm 0.21 \pm 0.07$ | $\Delta S = -0.14 \pm 0.21 \pm 0.06.$ |

The shift in $\phi_2$ caused by penguin processes can be determined by invoking $SU(3)$ symmetry [15] which involves measuring the branching fractions of the $SU(3)$ related channels, $B \to a_1 K$ and $B \to K_{1A}\pi$. One can then solve this system of inequalities,

$$\cos 2(\phi^{\pm}_{2\,,\,\rm eff} - \phi_2) \geq \frac{1 - 2R^0_{\pm}}{\sqrt{1 - A^{\pm\,2}_{CP}}}, \quad \cos 2(\phi^{\pm}_{2\,,\,\rm eff} - \phi_2) \geq \frac{1 - 2R^+_{\pm}}{\sqrt{1 - A^{\pm\,2}_{CP}}}, \tag{5.3}$$

where

$$R^0_+ \equiv \frac{\overline{\lambda}^2 f^2_{a_1}\overline{\Gamma}(K^+_{1A}\pi^-)}{f^2_{K_{1A}}\overline{\Gamma}(a_1^+\pi^-)}, \quad R^0_- \equiv \frac{\overline{\lambda}^2 f^2_{\pi}\overline{\Gamma}(a_1^- K^+)}{f^2_K\overline{\Gamma}(a_1^-\pi^+)}, \quad R^+_+ \equiv \frac{\overline{\lambda}^2 f^2_{a_1}\overline{\Gamma}(K^0_{1A}\pi^+)}{f^2_{K_{1A}}\overline{\Gamma}(a_1^+\pi^-)}, \quad R^+_- \equiv \frac{\overline{\lambda}^2 f^2_{\pi}\overline{\Gamma}(a_1^+ K^0)}{f^2_K\overline{\Gamma}(a_1^-\pi^+)}, \tag{5.4}$$

where $\lambda^2 = |V_{us}|/|V_{ud}| = |V_{cd}|/|V_{cs}|$, $\overline{\Gamma}$ are averaged decay rates and $f_i$ are decay constants. By inverting these equations, a bound on $|\Delta\phi_2| \equiv |\phi_2^{\rm eff} - \phi_2|$ is calculated from $|\phi_2^{\rm eff} - \phi_2| \leq (|\phi^+_{2\,,\,\rm eff} - \phi_2| + |\phi^-_{2\,,\,\rm eff} - \phi_2|)/2$.

The $B \to a_1 K$ branching fraction has been measured by BaBar [16], and they have recently performed a branching fraction measurement of $B \to K_{1A}\pi$ decays with their final data set [17]. Because $K_{1A}$ is a mixture of the $K_1(1270)$ and $K_1(1400)$ states there interference must be considered. As such it was necessary to determine the $K\pi\pi$ model from external WA3 data taken by the ACCMOR collaboration. They obtained the branching fractions,

$$\begin{aligned} B(B^0 \to K_1(1270)^+\pi^- + K_1(1400)^+\pi^-) &= 3.1^{+0.8}_{-0.7} \times 10^{-5}\ (7.5\sigma) \\ B(B^+ \to K_1(1270)^0\pi^+ + K_1(1400)^0\pi^+) &= 2.9^{+2.9}_{-1.7} \times 10^{-5}\ (3.2\sigma), \end{aligned} \tag{5.5}$$



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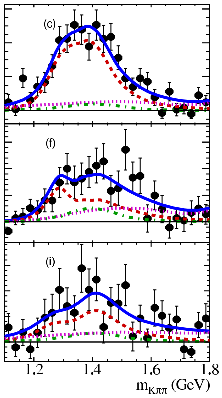


**Figure 10:** The $K\pi\pi$ fit projections from BaBar. The top two plots are from two $B^0$ classes and the bottom is for the $B^+$ channel. The dashed curve represents $K_1(1270)\pi + K_1(1400)\pi$, the dash-dotted, $K^*(1410)\pi$ and the dotted curve is for $K^*(892)\pi\pi$ decays.

where the $K\pi\pi$ fit projections are shown in Fig. 10. Using these results they obtain, $|\Delta\phi_2| < 11^\circ\ (13^\circ)$ at the 68% (90%) CL. Thus, the solution nearest the SM expectation from $B^0 \to a_1(1260)^\pm\pi^\mp$ decays is $\phi_2^{\rm eff} = (79 \pm 7 \pm 11)^\circ$.

## 6. Summary

Many independent measurements of $\phi_2$ have been performed at the $B$ factories including new results from BaBar in $B^+ \to \rho^+\rho^0$ and $B \to K_{1A}\pi$ decays. At this time, $B \to \rho\rho$ is the best environment for constraining $\phi_2$ because of its relatively small penguins. However, only a time-dependent amplitude analysis such as that which can be performed with $B^0 \to (\rho\pi)^0$ will constrain $\phi_2$ without ambiguity. The world average for $\phi_2$ has been determined by the CKMfitter and UTfit groups. They obtain $\phi_2 = (89.0^{+4.4}_{-4.2})^\circ$ [18] and $\phi_2 = (92.0 \pm 3.4)^\circ$ [19], respectively, which was obtained from the $\phi_2$ scans shown in Fig. 11. The $B$ experiments have now accumulated their final data sets and we anticipate the final word on $\phi_2$ from the first generation of $B$ experiments in the near future.

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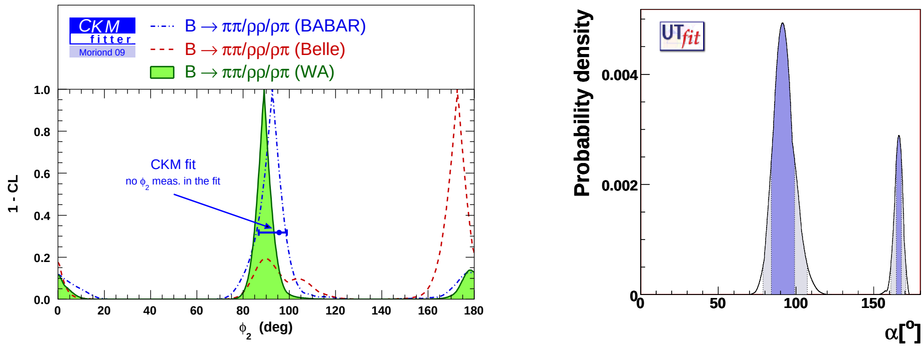


**Figure 11:** The left figure from the CKMfitter group shows the combined $\phi_2$ curve of $B \to \pi\pi$, $\rho\rho$ and $\rho\pi$ for the BaBar (blue) and Belle (red) collaborations. The average is shown in green and the point shows the constraint on $\phi_2$ from measurements of CKM parameters other than $\phi_2$. The right figure shows the combined $\phi_2$ results from the UTfit collaboration.

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