$D^0$-hadronic decays related to the extraction of the CKM angle $\gamma$

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The CKM angle $\gamma$ can be precisely determined by analysing the class of decays $B^\pm \to DK^\pm$, where $D$ represents either $D^0$ or $\bar{D^0}$, both of which decay to a common final state. The parameters associated with the $D$ decay must be well-known in order to precisely measure $\gamma$. Results from several CLEO-c analyses of hadronic $D$ decays (to $K^\pm \pi^\pm$, $K^\pm \pi^\pm \pi^\pm$, $K^\pm \pi^\pm \pi^0$ and $K^0_S \pi^+ \pi^-$) are reported. Preliminary results from the CLEO-c analysis of $D \to K^0_S K^+ K^-$ are also presented. The impact of the CLEO-c measurements on the determination of $\gamma$ at LHCb is discussed.

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1. Introduction

Of the three angles of the $b-d$ Cabibbo-Kobayashi-Maskawa (CKM) [1] triangle ($\alpha/\phi_2$, $\beta/\phi_1$ and $\gamma/\phi_3$), $\gamma$ (defined as $\text{arg}\{-[V_{ud}V_{ub}^*]/[V_{cd}V_{cb}^*]\}$) is currently the least well-determined. The uncertainty on $\gamma$ from direct measurements is $20 - 25^\circ$ [2]. A more precise determination of $\gamma$ is thus one of the central goals of flavour physics.

A very promising method to improve the precision on $\gamma$ is to study interference in decays of the form $B^\pm \rightarrow D K^\pm$, where $D$ is either $D^0$ or $\bar{D}^0$. Feynman diagrams for $B^-$ decays are shown in Figures 1 and 2. For both $B^+$ and $B^-$ decays, the $D^0$ and $\bar{D}^0$ must decay to the same final state in order to produce interference between $b \rightarrow u$ and $b \rightarrow c$ quark transitions. Measuring the relative rates of the $B^+$ and $B^-$ processes enables $\gamma$ to be determined.

The decay of $B^-$ to $\bar{D}^0 K^-$ is suppressed relative to the decay to $D^0 K^-$ by both colour suppression and the ratio of the relevant CKM matrix elements. The ratio between the suppressed and the favoured amplitude is $r_B e^{i(\delta_B - \gamma)} \equiv \mathcal{A}(B^- \rightarrow \bar{D}^0 K^-)/\mathcal{A}(B^- \rightarrow D^0 K^-)$, where $r_B$ is the ratio of the magnitudes of the amplitudes and $\delta_B$ is the relative CP-invariant phase between the two decays.

In analyses of $B^\pm \rightarrow D K^\pm$ decays, the $D$-decay parameters must be known to high precision — in particular the $D$ decay amplitude, which has strong phase $\delta_D$. The $D$-decay parameters will be used as input to $B^\pm \rightarrow D K^\pm$ measurements at LHCb and future $b$-physics detectors. Measurements of these parameters have been conducted at experiments such as BaBar, Belle and CLEO-c, and in the future will also be conducted at BESIII. Of these, CLEO-c and BESIII are particularly important for the present discussion as the quantum correlation present in $\psi(3770) \rightarrow D \bar{D}$ production is of great value in charm studies.

2. CLEO-c and BESIII

CLEO-c was an experiment located at the Cornell Electron Storage Ring (CESR), an electron-positron collider. The detector is described in detail in Ref. [3]. Data were collected at and above the open charm threshold which corresponds to a centre-of-mass energy of 3.77 GeV. A total of $(818 \pm 8) \text{ pb}^{-1}$ of $e^+e^- \rightarrow \psi(3770) \rightarrow D \bar{D}$ data were recorded. The two $D$-mesons are either neutral or charged$^1$ and are quantum-correlated with an overall CP of $-1$. The decay environment at CLEO-c was relatively clean, resulting in low backgrounds and the ability to reconstruct $D$ decay modes with ‘missing’ particles such as a $K^0_L$ or a neutrino.

$^1$Henceforth, only the neutral case will be considered.
The BESIII experiment is located at the Beijing Electron-Positron Collider II (BEPCII). Data have been recorded at several centre of mass energies, including 3.77 GeV. BESIII has a charm physics program that will study quantum-correlated $\psi(3770) \rightarrow D\bar{D}$ decays; the overall luminosity of these decays is expected to be $\gtrsim 4$ fb$^{-1}$. These proceedings will describe several results from the CLEO-c collaboration; future results from BESIII are also anticipated.

At both CLEO-c and BESIII, quantum correlations in the $\psi(3770) \rightarrow D\bar{D}$ system can be exploited to determine the strong phase (etc.) of $D$ decays. The decays of the correlated $D$-mesons are both reconstructed, and the quantum numbers of one $D$ that has decayed to a final state of a particular CP (e.g. $K^+K^-$, which has a CP of +1) are determined. The quantum numbers of the other $D$ that has decayed to the final state of interest can then be inferred. A reconstructed decay of a $D$ to a particular final state is referred to as a tag.

3. Event reconstruction at CLEO-c

At CLEO-c, decays for which all final-state particles are reconstructed are typically selected by determining the beam-constrained mass ($m_{BC}$) of each $D$ candidate. The beam-constrained mass is defined as $m_{BC} \equiv \sqrt{E_{\text{beam}}^2 - p_D^2}$, where $E_{\text{beam}}$ is the beam energy and $p_D$ is the total momentum of the reconstructed final-state daughters of the $D$. The majority of correctly-reconstructed $D^0$ decays are clustered around $m_{BC} \approx m_{D^0}$. A signal region and sidebands are specified on the plot of the $m_{BC}$ of one $D$ against the $m_{BC}$ of the other $D$ (the 'm$_{BC}$ plane'). Figure 3 shows the $m_{BC}$ plane for $D^0 \rightarrow K^\pm \pi^\mp$ tagged with CP$^+$ and CP$^-$ decays [4]. The quantity of flat background in the signal box is estimated by scaling the data yields in the sidebands. The quantities of peaking backgrounds are estimated using high-statistics samples of generic $D\bar{D}$ Monte Carlo simulated data.

The reconstruction of events containing a missing particle proceeds initially with the reconstruction of all other particles in the event. The missing invariant mass, $m_{\text{miss}}$, is then determined using the net four-momentum of the reconstructed particles and information about the detector running conditions. The square of the missing mass for events containing a $K^0_L$ is expected to lie on $m_{K^0_L}^2 \sim 0.25$ GeV$^2$/c$^4$ — an example of this is shown in Figure 4 [4], for which a peak in $m_{\text{miss}}^2$ is clearly visible. The quantity of background present is estimated using Monte Carlo simulated data;

![Figure 3](image_url)
typically, there is a small quantity underneath the $m_{\text{miss}}^2$ peak, but it is predominantly located in the high-mass sidebands.

Figure 4: Square of the missing invariant mass for $D^0 \to K^0_l \pi^0$ decays tagged with $K^\pm \pi^\mp$ (left) and CP$^-$ (right) decays. Black points are data and the solid red region is the estimated background.

4. Hadronic $D$ decays

In this section we consider $D$ decays to a number of different hadronic$^2$ final states. We describe how the parameters of interest (strong phase $\delta$, etc.) can be determined in each case and how these parameters can be used in measurements of $\gamma$ in $B^\pm \to DK^\pm$ decays.

4.1 Self-conjugate multibody $D$ decays: $D \to K^0_S h^+ h^-$

Decays of $B^\pm \to DK^\pm$ in which the $D$ decays to a self-conjugate multibody$^3$ final state can be analysed to determine $\gamma$ using the method proposed by Giri, Grossman, Soffer and Zupan (GGSZ) [5]. Input from an external determination of the $D \to K^0_S h^+ h^-$ decay parameters is extremely useful in order to achieve greater statistical precision. Two such final states produced from $D$ decays have been investigated: $K^0_S \pi^+ \pi^-$ and $K^0_S K^+ K^-$, collectively referred to as $K^0_S h^+ h^-$. In both cases the distribution of events across the Dalitz plane is studied; the formalism for both states is identical.

Define the invariant kinematic variables $x \equiv m^2_{K^0_S h^-}$ and $y \equiv m^2_{K^0_S h^+}$. These variables are coordinates on a Dalitz plane. Define the $D^0 \to K^0_S h^+ h^-$ decay amplitude $f_D(x,y) \equiv A_{xy} e^{i\delta_{xy}}$, where $A_{xy}$ and $\delta_{xy}$ are respectively the magnitude of the decay amplitude and strong phase at $(x,y)$.

In the absence of $D$-mixing and CP violation, $f_D(x,y) = f_D^\ast(y,x)$. The decay amplitude of $B^- \to (K^0_S h^+ h^-)_D K^-$ decays on the $K^0_S h^+ h^-$ Dalitz plane is therefore:

$$\mathcal{A}(B^- \to (K^0_S h^+ h^-)_D K^-)(x,y) \propto f_D(x,y) + r_B e^{i(\delta_B - \gamma)} f_D^\ast(y,x).$$  \hspace{1cm} (4.1)

$^2$Consisting of charged and/or neutral kaons and/or pions.

$^3$‘Multibody’ in this context means three or more particles.
Consequently the $B^-$ decay rate is:
\[
\Gamma(B^- \to (K^0_S h^+ h^-)_D K^-)(x,y) \propto A_{xy}^2 + r_B^2 A_{xy}^2 + 2r_B A_{xy} A_{yx} [\cos(\delta_B - \gamma) \cdot \cos(\Delta \delta_D) - \sin(\delta_B - \gamma) \cdot \sin(\Delta \delta_D)]
\tag{4.2}
\]
where the strong phase difference $\Delta \delta_D \equiv \delta_{xy} - \delta_{yx}$.

In order to achieve greater statistical precision, it is desirable to use an external source of $D$-decay data to determine $\Delta \delta_D$. One method is to use an additional flavour-tagged $D$-decay sample to determine the $D$ decay parameters. Both the BaBar and Belle collaborations use this method; a high-statistics sample of $D^+ \to D^0 \pi$ is used to determine a model of the $D^0 \to K^0_S h^+ h^-$ decay.

There is an uncertainty associated with this procedure, which is difficult to assess reliably, arising from how well the model represents the underlying physics of the $D$ decay. In particular, it is possible to use this method to measure the magnitude of the $D$ decay amplitude, but not the phase.

At BaBar, both $K^0_S \pi^+ \pi^-$ and $K^0_S K^+ K^-$ have been analysed [6, 7]. The $K^0_S K^+ K^-$ model is parameterised using the isobar formalism; the $K^0_S \pi^+ \pi^-$ model is parameterised with the K-matrix formalism. The value of $\gamma$ obtained from measurements of $B^\pm \to (K^0_S h^+ h^-)_D K^\pm$ decays is:
\[
\gamma = 68.0^{+14.0}_{-11.6} \text{(stat)} \pm 4.0 \text{(syst)} \pm 3.0 \text{(model)}.
\tag{4.3}
\]

At Belle, the $K^0_S \pi^+ \pi^-$ decay has been modelled using the isobar formalism [8]. The value of $\gamma$ obtained using this model in the analysis of $B^\pm \to (K^0_S \pi^+ \pi^-)_D K^\pm$ decays is:
\[
\gamma = 78.4^{+10.8}_{-11.6}\text{(stat)} \pm 3.6\text{(syst)} \pm 8.9\text{(model)}.
\tag{4.4}
\]

An alternative approach to modelling the decays using a flavour-tagged $D^0$ sample is to divide the $K^0_S h^+ h^-$ Dalitz plane into bins and count the number of events reconstructed in each bin. In a sample of $B^\pm \to (K^0_S h^+ h^-)_D K^\pm$ decays, the number of $B^\pm$ events in the $i$th bin of the $K^0_S h^+ h^-$ Dalitz plane is:
\[
N_i(B^\pm \to (K^0_S h^+ h^-)_D K^\pm) \propto T_i + r_B^2 T_{i-1} + 2r_B \sqrt{T_iT_{i-1}} [\cos(\delta_B \pm \gamma)c_i + \sin(\delta_B \pm \gamma)s_i]
\tag{4.5}
\]
where
\[
T_i \equiv \int_{i} |f_{pp}(x,y)|^2 \, dx \, dy,
\tag{4.6}
\]
\[
c_i \equiv \frac{1}{\sqrt{T_iT_{i-1}}} \int_{i} |f_{pp}(x,y)||f_{pp}(y,x)| \cos(\Delta \delta_D(x,y)) \, dx \, dy
\tag{4.7}
\]
and
\[
s_i \equiv \frac{1}{\sqrt{T_iT_{i-1}}} \int_{i} |f_{pp}(x,y)||f_{pp}(y,x)| \sin(\Delta \delta_D(x,y)) \, dx \, dy.
\tag{4.8}
\]

The parameters $c_i$ and $s_i$ are the weighted-average cosine and sine of the strong phase difference in the $i$th bin. CLEO-c and BESIII are uniquely positioned to make model-independent measurements of these quantities using quantum-correlated $D^0\bar{D}^0$ decays.

At CLEO-c, both $K^0_S \pi^+ \pi^-$ [9] and $K^0_S K^+ K^-$ have been studied; all results for $K^0_S K^+ K^-$ are preliminary. To first order, the CP of $K^0_S h^+ h^-$ is the opposite CP of that to $K^0_S h^+ h^-$. Such decays can therefore be used to provide additional statistical precision. The parameters associated with $K^0_S h^+ h^-$ are primed (e.g. $c'_i$).

\[\text{PoS (FPCP 2010) 008}\]
To determine $c_i(\gamma)$, the yield of $K_{SL}^0 h^+ h^-$ tagged with CP eigenstates is used. Constraints on $s_i(\gamma)$, and additional constraints on $c_i(\gamma)$, are provided by the yields of $K_{SL}^0 h^+ h^-$ tagged with $K_{SL}^0 h^+ h^-$. A maximum likelihood fit is used to extract $c_i(\gamma)$ and $s_i(\gamma)$.5

The kinematic distributions of CP-tagged $K_{SL}^0 K^+ K^-$ events are shown in Figures 5–8. The effect of quantum correlations can clearly be seen; resonances appear and disappear depending on the CP state of the $K_{SL}^0 K^+ K^-$. In particular, a strong effect is seen for the $m_{K^+ K^-}^2$ projection. When a $K_{SL}^0 K^+ K^-$ decay is tagged with a CP+ eigenstate, the $K_{SL}^0 K^+ K^-$ is in a CP+ state and often decays via the narrow CP+ $K_{SL}^0 \phi$ resonance (Figure 6). When a $K_{SL}^0 K^+ K^-$ decay is tagged with a CP− eigenstate, the $K_{SL}^0 K^+ K^-$ is in a CP− state so the $K_{SL}^0 \phi$ resonance is not present (Figure 8).

A statistically-sensitive binning of the Dalitz plane is to use regions of similar $\Delta \delta_P$[10]. Example binnings for $K_{SL}^0 \pi^+ \pi^-$ and $K_{SL}^0 K^+ K^-$ are shown in Figures 9 and 10 respectively. The branching

5The log likelihood term used in the $K_{SL}^0 \pi^+ \pi^-$ analysis is given in Ref [9].
ratio of $D^0 \rightarrow K_S^0 K^+ K^-$ is approximately five times smaller than that of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$, so in order to achieve reasonable statistical precision a coarser binning is required for $D^0 \rightarrow K_S^0 K^+ K^-$. 

The results on the $(c_i, s_i)$ plane using these binnings are shown in Figures 11 and 12. The results are in reasonable agreement with the quantities predicted by the corresponding decay models. However, to emphasise, these measurements are completely independent of the model.

4.1.1 Impact on $\gamma$ measurement

A toy Monte Carlo study is performed in order to evaluate the impact on $\gamma$ obtained by using the CLEO-c results in $B^\pm \rightarrow DK^\pm$ measurements at LHCb. A large number of $B^\pm \rightarrow (K_S^0 h^+) D K^\pm$ decays are simulated using the CLEO-c values of $c_i$ and $s_i$. The $c_i$ and $s_i$ are smeared using their reported uncertainties, taking correlations into account. Central and smeared
values of $N_i$ are determined using the $(c, s)_i$ and preset values of $r_B$, $\delta_B$ and $\gamma$. The parameters $r_B$, $\delta_B$ and $\gamma$ are then floated in the fit and extracted by fitting the smeared $N_i$ to the central values. The RMS of their distributions is taken in order to determine the CLEO-c contribution to the error on $\gamma$ from the uncertainty on the measured values of $c_i$ and $s_i$. For $K^0_S\pi^+\pi^-$, this contribution $\sim 1.7^\circ$; for $K^0_SK^-K^-$, the contribution $\sim 3.7^\circ$. These additional errors are small compared to the expected LHCb statistical precision.

4.2 Two-body $D$ decays: $D \rightarrow K^{\pm}\pi^\mp$

The Atwood-Dunietz-Soni (ADS) method [11] is used to determine $\gamma$ from $B^\pm \rightarrow DK^\pm$ decays in which the $D$ decays to $K^{\pm}\pi^\mp$. This method also benefits from external input of the $D$ decay parameters measured in charm experiments. For decays of the form $B^\pm \rightarrow (K^{\pm}\pi^\mp)_D K^\pm$, there are four permutations of the charge of the $B$ and the charge of the $K$ produced by the $D$ decay. The decay rates of these permutations are:

$$
\Gamma(B^+ \rightarrow (K^+\pi^-)_D K^+) \approx 1 + (r_B^{K^+\pi^-} D^+)^2 + 2r_B^{K^+\pi^-} \cdot \cos(\delta_B + \gamma - \delta_{K^+\pi^-}), \quad (4.9)
$$
$$
\Gamma(B^+ \rightarrow (K^-\pi^+)_D K^+) \approx r_B^2 + (r_D^{K^-\pi^+})^2 + 2r_B^{K^-\pi^+} \cdot \cos(\delta_B + \gamma + \delta_{K^-\pi^+}), \quad (4.10)
$$
$$
\Gamma(B^- \rightarrow (K^+\pi^-)_D K^-) \approx r_B^2 + (r_D^{K^+\pi^-})^2 + 2r_B^{K^+\pi^-} \cdot \cos(\delta_B - \gamma + \delta_{K^+\pi^-}), \quad (4.11)
$$
$$
\Gamma(B^- \rightarrow (K^-\pi^+)_D K^-) \approx 1 + (r_B^{K^-\pi^+} D^-)^2 + 2r_B^{K^-\pi^+} \cdot \cos(\delta_B - \gamma - \delta_{K^-\pi^+}) \quad (4.12)
$$

where $r_B$ and $\delta_B$ were introduced in Section 1 and $r_D^{K^\pm\pi^\mp}$ and $\delta_{K^\pm\pi^\mp}$ are respectively the amplitude ratio and strong phase difference between the Cabibbo-favoured and suppressed $D^0$ decays:

$$
K^\mp_{D^0} e^{i\delta_{K^\mp}^{D^0}} \equiv \frac{\mathcal{A}_D(D^0 \rightarrow K^+\pi^-)}{\mathcal{A}_D(D^0 \rightarrow K^-\pi^+)} \quad (4.13)
$$

The world average of $\delta_{K^\mp}^{D^0}$ is $(26.4^{+9.6}_{-9.9})^\circ$ [12]. This result has inputs from B-factory mixing measurements and the CLEO-c analysis of 281 pb$^{-1}$ of $\psi(3770) \rightarrow D^0\overline{D}^0$ data [4]. The CLEO-c analysis reconstructed $D^0 \rightarrow K^{\pm}\pi^\mp$ events tagged with flavour and CP eigenstates.

4.3 Multibody $D$ decays: $D \rightarrow K^{\pm}\pi^\mp\pi^\mp\pi^\mp, D \rightarrow K^{\pm}\pi^\mp\pi^\mp\pi^0$

When the $D^0$ decays to three or more bodies in a non-CP eigenstate, the ADS method can be used but must be extended to take into account intermediate resonances. The $D^0$ decay amplitude varies across phase space; it is useful to quantify the extent to which the amplitudes of the intermediate resonances interfere. For a multibody final state $F$, the coherence factor $R_F$ [13] is defined as

$$
R_F e^{-i\delta_F^F} \equiv \frac{\int \mathcal{A}_F(x) \mathcal{A}_F^*(x) dx}{A_F A_F^*} \quad (4.14)
$$

where $x$ denotes the multidimensional phase space variables, $\delta_F^F$ is the average across phase space of the strong phase difference of the $D^0$ decay, $\mathcal{A}_F(x)$ is the $D^0$ decay amplitude, and $A_F \equiv \int |A_F(x)|^2 dx$. The value of $R_F$ lies between 0 and 1. If $R_F \sim 0$, the intermediate resonances are out of phase and no single resonance dominates; conversely, if $R_F \sim 1$, the resonances are in phase (i.e coherent) and a single resonance dominates.

Quantum-correlated CLEO-c data have been used to measure the coherence factors for $D^0 \rightarrow K^{\pm}\pi^\mp\pi^\mp\pi^0$ and $D^0 \rightarrow K^{\pm}\pi^\mp\pi^0\pi^0$, denoted $R_{K3\pi}$ and $R_{K3\pi\pi}$ respectively [14]. The yields of $K^{\pm}\pi^\mp\pi^\mp\pi^0$...
Double-tag yield & Sensitive to 
\( K^+\pi^-\pi^+\pi^- \) vs \( K^+\pi^-\pi^+\pi^- \) & \( R_{K3}\pi^2 \) 
\( K^+\pi^-\pi^+\pi^- \) vs \( K^+\pi^-\pi^+\pi^- \) & \( R_{K3}\pi\cdot\cos(\delta_{K3}\pi - \delta_D\pi) \) 
\( K^+\pi^-\pi^+\pi^- \) vs \( K^+\pi^-\pi^+\pi^0 \) & \( R_{K3}\pi\cdot\cos(\delta_{K3}\pi - \delta_D\pi^0) \) 
\( K^+\pi^-\pi^+\pi^- \) vs CP eigenstate & \( R_{K3}\pi\cdot\cos(\delta_{K3}\pi) \)

Table 1: Dependence of yields of \( K^+\pi^-\pi^+\pi^- \) and \( K^+\pi^-\pi^+\pi^0 \) on coherence factors \( R_F \) and average strong phases \( \delta_D \).

and \( K^+\pi^-\pi^+\pi^0 \) are sensitive to the coherence factors and average strong phases as outlined in Table 1. Two-dimensional likelihood scans for the coherence factor and average strong phase of \( K^+\pi^-\pi^+\pi^- \) and \( K^+\pi^-\pi^+\pi^0 \) are shown in Figures 13 and 14 respectively. High coherence is observed for \( D^0 \to K^+\pi^-\pi^+\pi^- \) and low coherence is seen for \( D^0 \to K^+\pi^-\pi^+\pi^0 \).

4.3.1 Impact on \( \gamma \) measurement

Measurements of the coherence factors, along with the \( D \to K^+\pi^+ \) strong phase, provide invaluable additional input for the determination of \( \gamma \) at LHCb. Using the CLEO-c results in the \( B^\pm \to DK^\pm \) analysis at LHCb is estimated to provide additional sensitivity equivalent to doubling the \( B\bar{B} \) dataset for 2 fb\(^{-1} \) [15].

5. Future

Several analyses at CLEO-c will be updated in the future. The measurements of double-tagged \( K^+\pi^\pm \) decays, used as input to the ADS analysis, will be updated using the full 818 pb\(^{-1} \) \( \psi(3770) \) dataset. The analysis of \( D^0 \to K_S^0K^-K^- \) will be finalised and the analysis of \( D^0 \to K_S^0\pi^+\pi^- \) will be updated with different Dalitz plane binnings (determined using updated decay models from BaBar [7] and Belle [8]). In addition there will be a coherence factor analysis of the \( D^0 \to K_S^0K^+\pi^- \)
decay. Similar analyses can be performed on data taken at BESIII, exploiting the higher sample sizes which will become available at this experiment.

LHCb and other $b$-physics experiments will be able to take advantage of the input from CLEO-c and BESIII when making $B^\pm \rightarrow D K^\pm$ measurements to determine $\gamma$. Excellent precision on $\gamma$ is expected to be achieved when combining these measurements \cite{15}.

6. Conclusions

Results from analyses of $D^0$-hadronic decays play an important role in the precise measurement of $\gamma$ using $B^\pm \rightarrow D K^\pm$ decays at LHCb and other $b$-physics experiments.

Quantum correlations at CLEO-c (and in the future BESIII) enable precise measurement of the $D^0$ decay parameters. The CLEO-c collaboration has performed analyses of $D^0$ decays to $K_S^0 \pi^+ \pi^-$, $K^\pm \pi^\mp \pi^0$, and $K^\pm \pi^\mp \pi^0$ and the results have been summarised in these proceedings. Preliminary results from an analysis of $D^0 \rightarrow K_S^0 K^+ K^-$ have also been shown.

References

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