S-waves and the extraction of $\beta_s$

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The $CP$ Violating asymmetry in $B_s$ mixing ($\beta_s$) is one of the most promising measurements where physics beyond the Standard Model could be revealed. As such, analyses need to be subjected to great scrutiny. The mode $B_s \rightarrow J/\psi \phi$ has been used, and the mode $B_s \rightarrow \phi \phi$ proposed for future measurements. These modes both have two vector particles in the final state and thus angular analyses must be used to disentangle the contributions from $CP^+$ and $CP^-$ configurations. The angular distributions, however, could be distorted by the presence of S-waves masquerading as low mass $K^+K^-$ pairs, that could result in erroneous values of $\beta_s$. The S-waves could well be the result of a final state formed from an $s$-quark $\bar{s}$-quark pair in a $0^+$ spin-parity state, such as the $f_0(980)$ meson. Data driven and theoretical estimates of the $B_s$ decay rate into the $CP^+$ final state $J/\psi f_0(980)$ are given, when $f_0 \rightarrow \pi^+ \pi^-$. The S-wave contribution in $J/\psi \phi$ should be taken into account when determining $\beta_s$ by including a $K^+K^-$ S-wave amplitude in the fit. This may change the central value of current results and will also increase the statistical uncertainty. Importantly, the $J/\psi f_0(980)$ mode has been suggested as an alternative channel for measuring $\beta_s$. 

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1. Introduction

Measurements of Charge-Parity (CP) violation in the $B_s$ meson system are sensitive to the presence of heavy, as yet undiscovered, particles. The CP violating angle $-2\beta_s$, the so called “phase of $B_s - \bar{B}_s$ mixing” is a particularly important place to look for physics beyond the Standard Model, since the expected asymmetry is very small, $2\beta_s = 0.036 \pm 0.002$ [1], thus allowing the effects of any new physics to be more easily observed. Both CDF [2] and D0 [3] have investigated S-wave component in the region of the $K^+K^−$ masses between 1.01–1.03 GeV [10]. The interval. (Note that these fractions depend on the experiment resolution.)

Since the final state consists of two spin-1 particles, it is not a CP-eigenstate, yet it is well known that CP violation can be measured using angular analyses [4]. Except for one very recent analysis [5], previous determinations have ignored the possibility of an S-wave $K^+K^−$ system in the region of the $\phi$. Not accounting for the S-wave can bias the result, and the resulting quoted error is smaller than if the S-wave components are allowed in the fit.

2. Evidence for S-waves

In fact, there is a great deal of evidence for S-waves in many decays where vectors are dominant. Consider the reaction $D_s^+ \rightarrow K^+K^−\pi^+$. This mode has long been known to have large $\phi\pi^+$ and $K^*K$ components [6]. CLEO has looked explicitly at the low mass $K^+K^−$ region in the $K^+K^−\pi^+$ Dalitz plot [7]. Fig. 1 shows the $K^+K^−$ mass projection in the region near 1 GeV. The signal is extracted by fitting the $D_s$ yield in each bin of $K^+K^−$, so no background remains in the plot. There is an additional component of signal beneath the $\phi$. To estimate the size of this component, the CLEO data are fit to a Breit-Wigner to describe the $\phi$, convoluted with a Gaussian for detector resolution, and in addition a linear component that we take as an S-wave based on Dalitz plot studies. The fraction of S-wave depends on the mass interval considered. For $\pm 10$ MeV around the $\phi$ mass there is a 6.3% S-wave contribution, which rises to 8.9% for a $\pm 15$ MeV interval. (Note that these fractions depend on the experimental resolution.)

If the S-wave here is the $0^+ f_0(980)$ state, then we should see an $f_0$ signal peak in the $D_s^+ \rightarrow \pi^+\pi^+\pi^−$ final state, since the $f_0(980)$ decays into $\pi^+\pi^−$ as well as $K^+K^−$. A BaBar Dalitz plot analysis shows a large $f_0$ signal [8].

S-wave effects have also been observed in semileptonic charm decays. FOCUS observed an interfering S-wave amplitude in the $D^+ \rightarrow K^+\pi^−\mu^+\nu$ channel with a rate fraction of $(2.7\pm0.4)\%$ with respect to the P-wave $K^+$ in the $K^−\pi^+$ invariant mass region between 0.8–1.0 GeV [9]. BaBar measured an S-wave fraction in $D_s^+ \rightarrow K^+K^−\epsilon^+\nu$ decays of $(0.22^{+0.12}_{−0.08})\%$ for $K^+K^−$ invariant masses between 1.01–1.03 GeV [10]. The $K^+K^−$ invariant mass spectrum when weighted by the $K^+$ decay angle shows a clear distortion due to the interference (Fig. 2).

The analogous channel to $J/\psi\phi$ in $B^0$ decay $J/\psi\bar{K}^0$ is well known to have an S-wave $K\pi$ component in the $K^*$ mass region. This interference, in fact, has been used by BaBar to measure $\cos(2\beta)$ and thus remove an ambiguity in the value of $\beta$ from the $\sin(2\beta)$ measurement. The S-wave component in the region of the $K^*$ is measured as $(7.3\pm1.8)\%$ of the P-wave for $0.8 < m(K\pi) < 1.0$ GeV [11]. BaBar uses this interference to remove ambiguities in the measurement of $\cos(2\beta)$, where $\beta$ is the CP violating angle measured in $B^0 \rightarrow J/\psi K_S$ decays, for example. Visual
**Figure 1:** Dalitz plot projections for $K^+K^-$ invariant mass in $D_s^+ \rightarrow K^+K^-\pi^+$ from the CLEO Collaboration [7]. The signal is extracted individually in each mass bin, thus there is no background. The data are fit with a Breit-Wigner signal function for the $\phi$ convoluted with a Gaussian for detector resolution and linear representation of an S-wave component (dashed line). The solid curve shows the sum. (Only the data is ascribed to CLEO, the fits have been added.)

**Figure 2:** $K^+K^-$ invariant mass distribution weighted by the measured value of the cosine of the decay angle from BaBar [10] in the channel $D_s^+ \rightarrow K^+K^-e^+\nu$. 
evidence of the S-wave can be seen in Fig. 3 where there is an obvious asymmetry of the decay angle distribution of the kaon in the $K^{*0}$ rest-frame.

Perhaps it may be hoped that the S-wave $K^+ K^-$ under the $\phi$ in $J/\psi \phi$ is smaller due to the relatively narrow width ($\Gamma$) of the $\phi$ (4.3 MeV) compared to the $K^*$ (51 MeV), but even so, the question is how much does the presence of the S-wave amplitude affect the extraction of $\beta_s$? Similar considerations apply to the measurement of $CP$ violation in the process $B_s \rightarrow \phi \phi$. Here the problem is exacerbated by the presence of two $\phi$’s in the final state. The decay diagrams for both of these processes are shown in Fig. 4. In both cases the $s\bar{s}$ forms a $\phi$. Other manifestations of $s\bar{s}$ quarks are the $\eta, \eta'$ and $f_0(980)$ mesons. The first two are pseudoscalars, while the last is a scalar.

![Figure 3: The cosine of the decay angle of the kaon from the $K^{*}$ decay in $B^{0} \rightarrow J/\psi K^{*0}$ decay. The histogram is Monte Carlo without S-P wave interference, and the points the BaBar data [11].](image)

The formalism for the time dependent $B_s$ and $\bar{B}_s$ decay rates as a function of the decay angular distributions is given in Ref. [12]. Addition of the S-wave amplitudes was done by Xie et al. [13]. The number of terms to consider increases from 6 to 10. Another approach has been given by Azfar et al. [14]. Adding the S-wave terms in the fit can only increase the experimental error. The size of the effect depends on many factors including the magnitude and phase of the S-wave amplitude, $\beta_s$, values of the strong phases, detector acceptances, biases, etc..

While S-waves are a nuisance in analyzing the $J/\psi \phi$ final state, they can also be used beneficially. When the $f_0$ materializes as a $\pi^+ \pi^-$ there cannot be an iso-vector $\rho$ state as $s\bar{s}$ pair is isoscalar. Near the $\phi$ mass the $f_0(980)$ can materialize as a $\pi^+ \pi^-$ pair in the $O^+$ state and this $J/\psi f_0$ state is useful for $\beta_s$ measurements [15]. The final state is a $CP^+$ eigenstate, thus no angu-
lar analysis is necessary! Note, that the modes $J/\psi \eta$ and $J/\psi \eta'$ can also be used, but they involve photons in the decay and thus have lower efficiency at hadron colliders.

Predictions of the ratio

$$R_{f_0/\phi} \equiv \frac{\Gamma(B_s^0 \to J/\psi f_0, f_0 \to \pi^+ \pi^-)}{\Gamma(B_s^0 \to J/\psi \phi, \phi \to K^+ K^-)}$$

have been given based on $D_s$ decay data, and purely from theory. Stone and Zhang using $D_s^+ \to f_0 \pi^+$ decays where $f_0$ was detected in both $K^+ K^-$ and $\pi^+ \pi^-$ modes predicted $R_{f_0/\phi} \approx 20\%$ [15]. CLEO made an estimate of $R_{f_0/\phi} = (42 \pm 11)\%$ based on the ratio of the branching fractions for $D_s^+ \to f_0 e^+ \nu$ to $D_s^+ \to \phi e^+ \nu$ at $q^2 = 0$ where the mass difference between the $D_s$ and the final state hadron is largest, which best approximates $B_s \to J/\psi$ decays [16].

Theoretical predictions for $R_{f_0/\phi}$ are difficult, however there are a few heroic attempts. Colangelo, De Fazio and Wang use Light Cone Sum Rules to make two predictions [17]. For the first they use their calculations of $\mathcal{B}(B_s \to J/\psi f_0)$ which are $(3.1 \pm 2.4) \times 10^{-4}$ at leading order (LO) and $(5.3 \pm 3.9) \times 10^{-4}$ at non-leading order (NLO), combined with the measured $\mathcal{B}(B_s \to J/\psi \phi) = (1.3 \pm 0.4) \times 10^{-3}$ to predict $R_{f_0/\phi}$=24\% (LO) and $R_{f_0/\phi}$=41\% (NLO). Secondly, they use the form-factor calculation of Ball and Zwicky [18] to predict $R_{f_0/\phi}^L$=13\% (LO) and $R_{f_0/\phi}^L$=22\% (NLO), where $R^L$ refers to only longitudinal $\phi$ production, so since transverse $\phi$ production is about 46\% this lowers their $R_{f_0/\phi}$ predictions for the second case by almost a factor of two [6]. The experimental predictions above for $R_{f_0/\phi}$ based on $D_s$ decays are also enhanced by normalizing to $\phi$ final states that are mostly longitudinal. Thus they should be lowered.

In a later paper using QCD factorization Colangelo, De Fazio and Wang [19] predict $\mathcal{B}(B_s \to J/\psi f_0) = (4.7 \pm 1.9) \times 10^{-4}$ using CDSS form-factors [20], and $(2.0 \pm 0.8) \times 10^{-4}$ using Ball and Zwicky [18]. These predictions are somewhat smaller than those given above, but still have $R_{f_0/\phi}$ as 36\% or 20\%. Within the framework of QCD factorization O. Leitner et al. [22] give a range of predictions for $R_{f_0/\phi}$ that are in the 30-50\% range. Thus predictions for $R_{f_0/\phi}$ have a rather wide range from 7-50\%.

The only reported experimental search for $J/\psi f_0$: $f_0 \to \pi^+ \pi^-$ was done by BELLE using 23.6 fb$^{-1}$ of data taken on the $Y(5S)$ resonance, about 1/5 of their total accumulated data sample. They find $R_{f_0/\phi} < 27.5\%$ at 90\% c.l. [21]. Their data however show a hint of signal with a central value about half of the upper limit. It will be quite interesting to see which experiment finds the signal first. CDF has recently put the S-wave amplitudes in their fits for $\beta_s$ in the $J/\psi \phi$ channel. They find that the fitted fraction of $K^+ K^-$ S-wave in the signal region is < 6.7\% at 95\% c.l. [5]. They do not however, report any result for a direct search using the $f_0 \to \pi^+ \pi^-$ channel.

In conclusion, S-waves are ubiquitous, they appear whenever looked for and must be taken into account in $B_s \to J/\psi \phi$ measurements of amplitudes, phases, and $CP$ violation. Kudos to CDF for including S-waves in their most recent fits. In addition it appears to be wise to add S-wave amplitudes in the analysis of $B \to K^* \mu^+ \mu^-$ and surely in $B_s \to \phi \phi$. Furthermore, especially since angular analysis is not required, $B_s \to J/\psi f_0$ may be a useful mode to add to the statistical precision on the measurement of $-2\beta_s$ and will provide a useful systematic check [23].

Acknowledgments

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References


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