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Holographic study of rho meson mass in an external magnetic field

Paving the road towards a magnetically induced superconducting QCD vacuum?

Nele Callebaut*, **David Dudal, Henri Verschelde**
Ghent University, Department of Physics and Astronomy
Krijgslaan 281-S9, 9000 Gent, Belgium
E-mail: ncalleba.callebaut@UGent.be, david.dudal@ugent.be, henri.verschelde@ugent.be

We study the rho meson mass in a uniform background magnetic field eB at zero temperature, in search of indications for the magnetically induced rho meson condensation, as predicted recently by Chernodub. The holographic model used is the Sakai-Sugimoto model with two flavours and a non-zero constituent quark mass. We fix the free holographic parameters by matching them to the phenomenological value for the constituent quark mass and the experimental values for the pion decay constant and the rho meson mass, this in absence of a magnetic field. In a first approximation, the Landau levels are recovered, indeed indicating an instability of the QCD vacuum at a critical magnetic field, $eB_c \sim m_\rho^2$, to a phase where rho mesons are condensed. We improve on this result by also taking into account the holographic analogue of chiral magnetic catalysis, numerically solving the mass eigenvalue equation for the rho meson, which depends on eB both explicitly and implicitly through the changed embedding of the flavour probe branes. This turns out to raise eB_c with a few percents. As a byproduct of our analysis we find that the separation between the chiral symmetry restoration temperature $T_\chi(eB)$ and the deconfinement temperature T_c is 3.2 percent at $eB = 30m_\pi^2 \approx 0.57$ GeV^2.

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1. Introduction

The interest in QCD in the presence of a strong magnetic background has arisen recently since strong magnetic fields are expected to appear in heavy ion collisions [1]. In [2, 3] it was suggested that the cold ($T = 0$) QCD vacuum is unstable towards formation of a condensate of charged rho mesons when a sufficiently strong external magnetic field[1] ($B \sim m_\rho^2$) is applied. The argumentation in [2] is based on the DSGS-Lagrangian [4] describing self-consistent quantum electrodynamics for the rho mesons, whereas in [3] a similar reasoning was built up in a NJL-model.

Inspired by these papers and the already known existence of rho meson condensation at finite isospin chemical potential in holographic QCD [5], we have studied the dependence of the rho meson mass on a background magnetic field in a holographic approach. Holographic QCD-models give a description of hadronic physics through a dual supergravity theory in a higher-dimensional world. The duality is valid at large 't Hooft coupling where QCD itself is unmanageable, thus providing a setting for studying non-perturbative QCD effects. The holographic model which we have used, the Sakai-Sugimoto model [6, 7], manages to reproduce much of the low-energy physics of QCD, in particular confinement and dynamical chiral symmetry breaking at low temperature.

In section 2 we give a short review of this model, including our numerical values for the free parameters obtained by matching to phenomenological and experimental results, and we describe the method to introduce the magnetic field. The effect of the magnetic field on the geometry of the model in the current approximation is summarized in section 3, and then used in section 4 to determine the B-dependence of the rho meson mass and the critical value of the magnetic field at which the charged rho mesons condense. In the last section we calculate the magnetically induced split between deconfinement and chiral symmetry restoration temperature, finding a reasonable agreement with phenomenological and lattice output. In this proceeding, we only summarize our findings, details will appear shortly in [8].

2. Holographic setup

The Sakai-Sugimoto model [6, 7] involves a system of N_f pairs of D8-$\overline{\text{D8}}$ flavour probe branes in the D4-brane background

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$e^\phi = g_s\left(\frac{u}{R}\right)^{3/4} \quad , \quad F_4 = \frac{N_c}{V_4}\varepsilon_4 \quad , \quad f(u) = 1 - \frac{u_K^3}{u^3},$$

where $d\Omega_4^2$, ε_4 and $V_4 = 8\pi^2/3$ are, respectively, the line element, the volume form and the volume of a unit four-sphere, while R is a constant parameter related to the string coupling constant g_s, the number of colours N_c and the string length ℓ_s through $R^3 = \pi g_s N_c \ell_s^3$. This background has a natural cut-off at $u = u_K$ and the QCD-like theory is said to "live" at $u \to \infty$. We set $N_c = 3$, and the number of flavours $N_f = 2$. We stress here that we ignore the back reaction of the flavour branes on the background, meaning we are working in a holographic analogue of the quenched approximation.

[1] For notational convenience, we shall usually write just B, rather than eB.

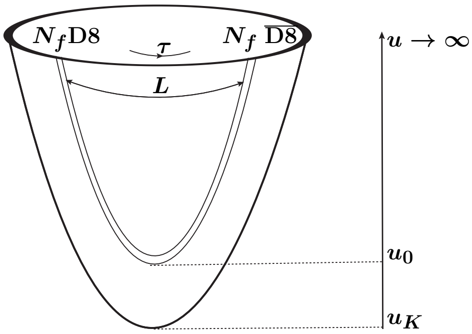

Figure 1: The Sakai-Sugimoto model.

The Sakai-Sugimoto model incorporates dynamical chiral symmetry breaking as a consequence of the U-shaped embedding of the flavour branes: the D8-branes and $\overline{\text{D8}}$-branes merge at a certain value of the extra holographic dimension $u = u_0$. In contradistinction to the original setup of [6, 7], we consider the more general setting with $u_0 > u_K$, this in order to have non-zero constituent quark mass [9]

$$m_q = \frac{1}{2\pi\alpha'} \int_{u_K}^{u_0} \frac{du}{\sqrt{f(u)}}\,, \tag{2.1}$$

with $\alpha' = \ell_s^2$ the string tension. The bare quark masses are always zero, so we are working in the chiral limit.

The gauge field $A_m(x^\mu, u)$ $(m = 0,1,2,3,u)$ living on the D8-branes describes mesons in the boundary field theory. The action for this gauge field is given by the non-Abelian DBI-action[2]

$$S_{DBI} = -T_8 \int d^4x d\tau\ \varepsilon_4 e^{-\phi} \mathrm{STr}\sqrt{-\det\left[g_{mn}^{D8} + (2\pi\alpha')F_{mn}\right]} \tag{2.2}$$

with $T_8 = 1/((2\pi)^8 \ell_s^9)$ the D8-brane tension, STr the symmetrized trace, g_{mn}^{D8} the induced metric on the D8-branes, and $F_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n] = F_{mn}^a t^a$ the field strength with anti-hermitian generators

$$t^a = \frac{i}{2}(\mathbf{1}, \sigma_1, \sigma_2, \sigma_3), \qquad \mathrm{Tr}(t^a t^b) = -\frac{\delta_{ab}}{2}.$$

The parameters R, g_s, ℓ_s, M_K, u_K and $\lambda = g_{YM}^2 N_c$ are related through the following equations:

$$R^3 = \frac{1}{2}\frac{\lambda \ell_s^2}{M_K}, \quad g_s = \frac{1}{2\pi}\frac{g_{YM}^2}{M_K \ell_s}, \quad u_K = \frac{2}{9}\lambda M_K \ell_s^2. \tag{2.3}$$

Without loss of generality one can put $u_K = \frac{1}{M_K}$ [7]. To fix the seven unknown parameters (R, λ, ℓ_s, M_K, u_K, g_s and u_0) we thus need three additional conditions. We intend to choose these in such a way that the major features of $N_f = 2$ QCD are reproduced as well as possible. In particular, we use the phenomenological value for the constituent quark mass ($m_q = 0.310$ GeV, as in [10]), and the experimental values for the pion decay constant ($f_\pi = 0.093$ GeV) and for the rho meson mass

[2] Because the 't Hooft coupling λ is assumedly large, we ignore in a first approximation the Chern-Simons part of the action in the analysis, being a factor $1/\lambda$ smaller than the DBI-action.

in absence of magnetic field ($m_\rho = 0.776$ GeV). The results of our numerical analysis are [8]

$$M_K \approx 0.721 \text{ GeV}, \quad u_0 \approx 1.92 \text{ GeV}^{-1} \text{ and } \kappa = \frac{\lambda N_c}{216\pi^3} \approx 0.006778. \tag{2.4}$$

From this, we do extract a relatively large 't Hooft coupling, $\lambda \approx 15$. Similar values can be found in [7], however these were obtained in the limiting case $u_0 = u_K$, corresponding to zero constituent quark mass. In this setting it would be, as we shall see shortly, impossible to model a magnetic catalysis effect, which is by now generally accepted to occur in QCD.

To turn on an electromagnetic background field A_μ^{em} in the boundary field theory we use the standard technique [7] of putting

$$A_\mu(u \to \infty) = eQ_{em}A_\mu^{em} = e\begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} A_\mu^{em} = e\left(\frac{1}{6}\mathbf{1}_2 + \frac{1}{2}\sigma_3\right) A_\mu^{em} \tag{2.5}$$

which, for the case of an external magnetic field in the x_3-direction in the boundary field theory ($F_{12}^{em} = \partial_1 A_2^{em} = B$), amounts to setting

$$A_\mu(u \to \infty) = eQ_{em}x_1 B\delta_{\mu 2} = \frac{x_1 eB\delta_{\mu 2}}{3}\left(\frac{i\mathbf{1}_2}{2}\right) + x_1 eB\delta_{\mu 2}\left(\frac{i\sigma_3}{2}\right), \tag{2.6}$$

or, adapting the notation $\overline{A}_\mu = A_\mu(u \to \infty)$:

$$\overline{A}_2^3 = x_1 eB \quad \text{and} \quad \overline{A}_2^0 = \overline{A}_2^3/3. \tag{2.7}$$

Because we want to look at the effect of the electromagnetic field on the rho mesons, which transform in the adjoint of $U(N_f)_L \times U(N_f)_R$ and hence only couple to the isospin component $\overline{A}_\mu^3$ of the electromagnetic field, we may ignore $\overline{A}_\mu^0$.

We work in the $A_u = 0$ gauge and fix the residual gauge symmetry by working in a particular gauge where the total gauge field is given by [7]

$$A_\mu(x^\mu, u) = \overline{A}_\mu + \sum_{n\geq 1} V_{\mu,n}(x^\mu)\psi_n(u), \tag{2.8}$$

with $V_{\mu,n}(x^\mu)$ a tower of vector mesons with masses m_n, and $\{\psi_n(u)\}_{n\geq 1}$ a complete set of functions of u, satisfying the eigenvalue equation

$$u^{1/2}\gamma_B^{-1/2}(u)\partial_u\left[u^{5/2}\gamma_B^{-1/2}(u)\partial_u\psi_n(u)\right] = -R^3 m_n^2\psi_n(u), \tag{2.9}$$

and the normalization condition

$$\frac{1}{2g_s}(2\pi\alpha')2R^{3/2}T_8V_4\int_{u_0}^{\infty} du\, u^{-1/2}\gamma_B^{1/2}R^3\psi_m(u)\psi_n(u) = \frac{1}{4}\delta_{mn}, \tag{2.10}$$

with

$$\gamma_B(u) = \frac{u^8 A(u)}{u^8 A(u) f(u) - u_0^8 A_0 f_0}, \quad A(u) = 1 + B^2\left(\frac{R}{u}\right)^3, \quad A_0 = 1 + B^2\left(\frac{R}{u_0}\right)^3. \tag{2.11}$$

3. Influence of the magnetic field on the embedding of the flavour probe branes: magnetic catalysis of chiral symmetry breaking

As already discussed in [11] for the Sakai-Sugimoto model, an external magnetic field promotes chiral symmetry breaking: when holding the asymptotic separation L between D8- and $\overline{\text{D8}}$-branes fixed, u_0, which is in a one-to-one correspondence with L [6, 7, 9], grows with B. This means that the probe branes get more and more bent towards each other, driving them further and further away from the chirally invariant situation (straight branes). This feature corresponds to a holographic modeling of the magnetic catalysis of chiral symmetry breaking [12]. For our value of $L = 1.57$ GeV^{-1} (corresponding to $u_0(B=0) = 1.92$ GeV^{-1}) kept fixed, the numerically obtained B-dependence of u_0 and consequently of the constituent quark mass m_q, via the relation (2.1), is plotted in Figure 3 [8]. Since m_q rises with B, we expect that taking this magnetic chiral catalysis into account will translate into the rho meson mass m_ρ also rising with B, at least when ignoring the lowest Landau level shift (see next section).

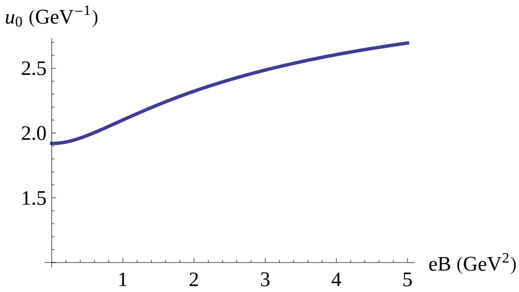

Figure 2: u_0 as a function of the magnetic field.

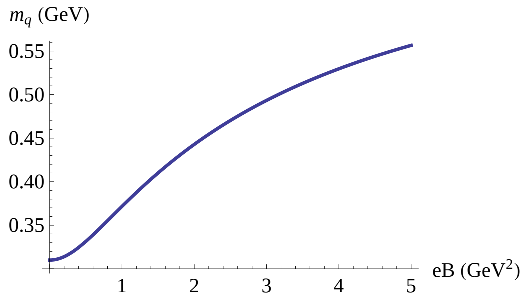

Figure 3: The constituent quark mass as a function of the magnetic field.

4. Equations of motion for the mesons in the magnetic field

Plugging the gauge field ansatz (2.8) into the action (2.2) and expanding it to order $V_{\mu,n}^2$, we derive the equations of motion for the vector mesons $V_{\mu,n}$, and thus in particular the rho mesons ρ_μ (we concentrate on the lightest vector mesons, because they will probably be the first to condense at strong B). We find [8]

$$m_\rho^2 \rho_\mu^- - D_\nu^2 \rho_\mu^- + 2iG_{\mu\nu}\rho^{\nu -} = 0, \quad \text{with } D_\mu = \partial_\mu - ieA_\mu^{em}, \quad G_{\mu\nu} = \partial_\mu A_\nu^{em} - \partial_\nu A_\mu^{em}, \tag{4.1}$$

for the negatively charged rho meson $\rho_\mu^- = \rho_\mu^1 - i\rho_\mu^2$, and the complex conjugate of this equation for the positively charged rho meson $\rho_\mu^+ = \rho_\mu^1 + i\rho_\mu^2$. Inserting the background gauge field ansatz $\overline{A}_2^3 = x_1 eB$ into the above equation of motion, and Fourier transforming ρ_μ^-, we obtain the Landau energy levels [13]

$$E^2(\rho_1^- \mp i\rho_2^-) = \left(eB(2N+1) + p_3^2 + m_\rho^2 \mp 2eB\right)(\rho_1^- \mp i\rho_2^-) \tag{4.2}$$

with p_3 the momentum of the meson in the x_3-direction. The combinations $(\rho_1^- - i\rho_2^-)$ and $(\rho_1^+ + i\rho_2^+)$ have spin $s_3 = 1$ parallel to the external magnetic field. In the lowest energy state ($N = 0$,

$p_3 = 0$) their effective mass,

$$m_{eff}^2 = m_\rho^2 - eB, \tag{4.3}$$

can become zero if the magnetic field is strong enough.

This suggests that the QCD vacuum in a strong magnetic field $B = F_{12}$ is unstable towards condensation of the fields

$$\rho = \rho_1^- - i\rho_2^- \quad \text{and} \quad \rho^\dagger = \rho_1^+ + i\rho_2^+$$

at a critical value of the magnetic field

$$eB_c = m_\rho^2. \tag{4.4}$$

Here, in our first approximation, the B-dependence of m_ρ^2 itself was ignored (as in [2]). At $B \approx B_c$ the decay modes of rho mesons into pions, which normally would guarantee a short lifetime of the rho mesons, are no longer kinematically allowed because the charged pions' masses increase due to the magnetic field (see discussion in [2]).

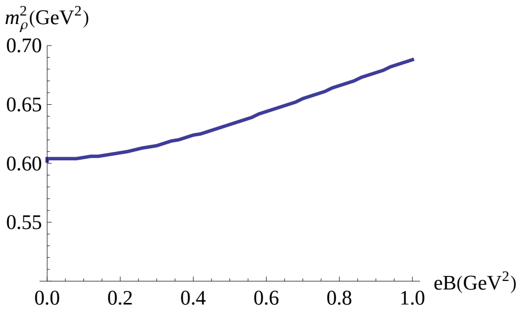

Figure 4: Mass of the rho meson squared as a function of the magnetic field.

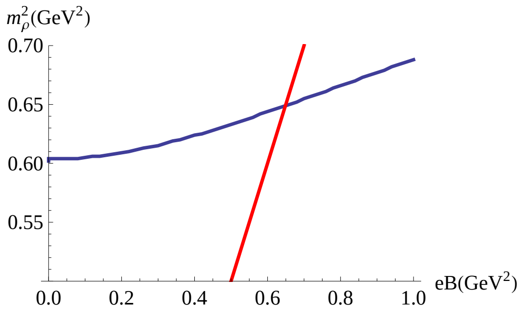

Figure 5: eB_c is determined as the value where the effective rho meson mass $m_{eff}^2 = m_\rho^2 - eB$ becomes zero, i.e. at the crossing of the 2 curves.

Next, we can calculate the B-dependence of m_ρ^2 in (4.2) itself, the lowest eigenvalue of the equation (2.9), which depends on B both explicitly and implicitly through the changed embedding of the probe branes, represented by the value of $u_0(B)$. This reflects the effect of chiral magnetic catalysis. The result of our numerical study [8] is depicted in Figure 4, showing that m_ρ^2 is an increasing function of B, as expected. The value of the magnetic field at the onset of the condensation becomes slightly larger:

$$eB_c = 1.08 m_\rho^2. \tag{4.5}$$

We clearly observe that the effect is of the order of a few percent, which is consistent with the rough estimate provided in [2].

5. Split between deconfinement and chiral symmetry restoration temperature

In the Sakai-Sugimoto model the chiral symmetry restoration temperature T_χ is known to increase when a background magnetic field is present [11]. Because of the way the magnetic field is introduced in the Sakai-Sugimoto model, the deconfinement temperature T_c is independent[3] of

[3] T_c is determined solely from the background geometry, which is totally B-independent in the current approach.

B, with $T_c \approx 115$ MeV for our numerical values of the holographic parameters [8], so there will arise a split between the deconfinement transition and chiral symmetry restoration. We numerically calculate this split, assuming $T_\chi(B=0) = T_c$, and find it to be 1.2% at $B = 15m_\pi^2 \approx 0.28$ GeV^2 and 3.2% at $B = 30m_\pi^2 \approx 0.57$ GeV^2, see Figure 6. The percent estimation of the magnitude of the split is in quantitative agreement with [14, 15], and in qualitative agreement with [10, 16]. Notice however that none of these papers work in the chiral limit, and that all of these also witnessed an increasing T_c in terms of B. Results in the chiral limit can be extracted from the work [17] wherein, interestingly, T_c seems to depend only marginally on B, at least up to $eB = 20m_\pi^2$, the maximum value considered there. It is perhaps interesting to notice that for ever-growing values

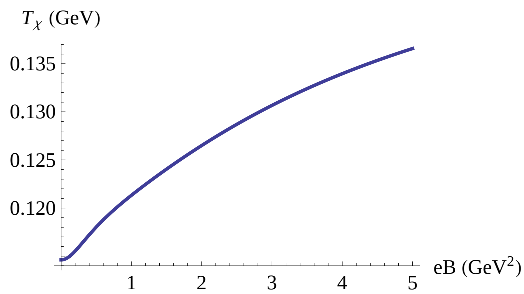

Figure 6: The chiral symmetry restoration temperature as a function of the magnetic field. The deconfinement temperature remains at $T_c \approx 0.115$ GeV.

of B, a saturation (horizontal asymptote) is seen in all our quantities ($u_0(B)$, $m_q(B)$ and $T_\chi(B)$), in agreement with what was found in [11]. It is unclear whether this behaviour really corresponds to QCD. From QCD-related models like [10, 14, 16, 18] and lattice simulations like [19], one rather expects that e.g. $m_q(B)$ (or more precisely, the chiral condensate) and $T_\chi(B)$ will keep growing for large B. It looks tempting to associate such a discrepancy to the probe approximation. At large values of B, perhaps the quark back reaction might become more prominent, leading to other results than found for e.g. $m_q(B)$ at large B. Likewise, also the deconfinement temperature might become B-dependent beyond the probe approximation, in harmony with works like [10, 14, 16, 18]. It thus seems interesting to study the back reaction of the flavour branes on the background geometry, with or without magnetic field present[4].

So far, we did not study the actual condensation of the rho mesons, a work which we relegate to the future. Let us however already refer to [21] for related work using a different holographic setup. It would also be interesting to include the pions in a future analysis. Since the rotational symmetry is broken by $\vec{B} = B\vec{e}_3$, one might expect a mixing between the longitudinal vector mesons and pions in some manner. It is apparent that such happens when taking into account the to the chiral anomaly related Chern-Simons piece of the action [8], but as said before, this effect is suppressed by an additional inverse 't Hooft coupling. For sure, getting a clear holographic understanding of the occurrence of a magnetically induced superconducting QCD vacuum at zero temperature, as touched upon in [2, 3], will need further efforts.

[4]In absence of the probe branes, a constant magnetic field B does not change the background geometry, see [20] and references therein.

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