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Non-Perturbative Hamiltonian Light-Front Field Theory: Progress and Prospects

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Fundamental theories, such as Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD), offer promise for predicting phenomena on all scales from the microscopic to cosmic scales. New tools that go beyond perturbation theory are required to build bridges from one scale to the next. Recent theoretical and computational progress in quantum many-body theory show how to build such bridges and those developments are applicable to light-front field theory. In particular, by choosing light-front gauge and adopting a basis function representation, one obtains a large, sparse, Hamiltonian matrix for mass eigenstates of gauge theories that is solvable by adapting the *ab initio* no-core methods of nuclear many-body theory. In this way, one obtains the invariant masses and correlated parton amplitudes suitable for accessing all experimental observables. Full covariance is recovered in the continuum limit, the infinite matrix limit. I outline the approach and discuss the computational challenges.

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1. Introduction

Non-perturbative solutions of quantum field theory represent opportunities and challenges that span particle physics and nuclear physics. Fundamental understanding of, among others, the spin structure of the proton, the neutron electromagnetic form factor, and the generalized parton distributions of the baryons should emerge from results derived from a non-perturbative light-front Hamiltonian approach. The light-front Hamiltonian quantized within in a basis function approach offers a promising avenue that capitalizes on theoretical and computational achievements in quantum many-body theory over the past decade.

By way of background, one notes that Hamiltonian light-front field theory in a discretized momentum basis [1] and in transverse lattice approaches [2, 3] have shown significant promise. I outline here a Hamiltonian basis function approach following Ref. [4] that exploits recent advances in solving the non-relativistic strongly interacting nuclear many-body problem [5, 6]. There are many issues faced in common - i.e. how to (1) define the Hamiltonian; (2) renormalize for the available finite spaces while preserving all symmetries; (3) solve for eigenvalues and eigenvectors; (4) evaluate experimental observables; and, (5) take the continuum limit.

I begin with a brief overview of recent advances in solving light nuclei with realistic nucleon-nucleon (NN) and three-nucleon (NNN) interactions using *ab initio* no-core methods. After reviewing some advances with two-dimensional theories, I outline a basis function approach suitable for light front gauge theories including the issues of renormalization/regularization. I present an outline of the approach to cavity-mode QED and sketch its extension to QCD.

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2. No Core Shell Model (NCSM) and No Core Full Configuration (NCFC) methods

To solve for the properties of nuclei, self-bound strongly interacting systems, with realistic Hamiltonians, one faces immense theoretical and computational challenges. Recently, *ab initio* approaches have been developed that preserve all the underlying symmetries and they converge to the exact result. The basis function approach [5, 6] is one of several methods shown to be successful. The primary advantages are its flexibility for choosing the Hamiltonian, the method of renormalization/regularization and the basis space. These advantages support the adoption of the basis function approach in light-front quantum field theory.

Refs. [5, 7, 8, 9] and [6, 10, 11] provide examples of the recent advances in the *ab initio* NCSM and NCFC, respectively. The NCSM adopts a renormalization method that provides an effective interaction dependent on the chosen many-body basis space cutoff (N_{max} below). The NCFC either retains the un-renormalized interaction or adopts a basis-space independent renormalization so that the exact results are obtained either by using a sufficiently large basis space or by extrapolation to the infinite matrix limit. Recent results for the NCSM employ realistic nucleon-nucleon (NN) and three-nucleon (NNN) interactions derived from chiral effective field theory to solve nuclei with Atomic Numbers 10-13 [7]. Recent results for the NCFC feature a realistic NN interaction that is sufficiently soft that binding energies and spectra from a sequence of finite matrix solutions may be extrapolated to the infinite matrix limit [11]. Experimental binding energies, spectra, magnetic moments and Gamow-Teller transition rates are well-reproduced in both the NCSM and NCFC approaches. Convergence of long range operators such as electric quadrupole are more challenging.

It is important to note two recent analytical and technical advances. First, non-perturbative renormalization has been developed to accompany these basis-space methods and their success is impressive. Several schemes have emerged and current research focuses on understanding of the scheme-dependence of convergence rates (different observables converge at different rates) [10]. Second, large scale calculations are performed on leadership-class parallel computers to solve for the low-lying eigenstates and eigenvectors and to evaluate a suite of experimental observables. Low-lying solutions for matrices of basis-space dimension 10-billion on 200,000 cores with a 5-hour run is the current record. However, one expects these limits to continue growing as the techniques are evolving rapidly [9] and the computers are growing dramatically. Matrices with dimensions in the several tens of billions will soon be solvable with strong interaction Hamiltonians.

In a NCSM or NCFC application, one adopts a 3-D harmonic oscillator for all the particles in the nucleus (with harmonic oscillator energy $\hbar\Omega$), treats the neutrons and protons independently, and generates a many-fermion basis space that includes the lowest oscillator configurations as well as all those generated by allowing up to N_{max} oscillator quanta of excitations. The single-particle states specify the orbital angular momentum projection and the basis is referred to as the *m*-scheme basis. For the NCSM one also selects a renormalization scheme linked to the basis truncation while in the NCFC the renormalization is either absent or of a type that retains the infinite matrix problem. In the NCFC case [6], one extrapolates to the continuum limit as I now illustrate.

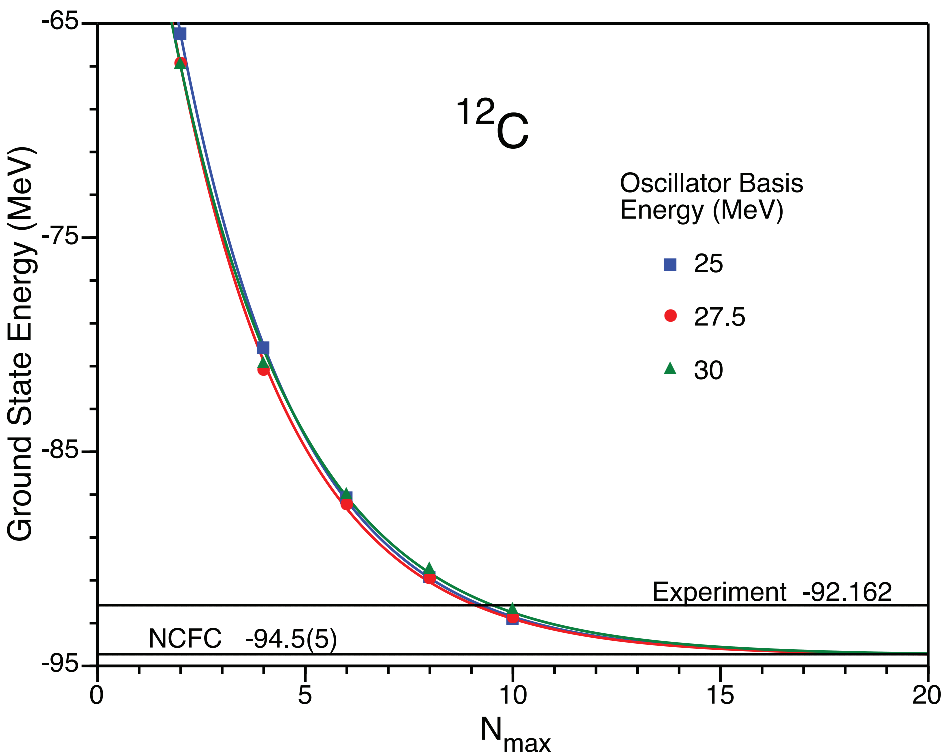

Figure 1: Calculated ground state (gs) energy of ^{12}C for $N_{max} = 2-10$ (symbols) at selected values of $\hbar\Omega$. For each $\hbar\Omega$, the results are fit to an exponential plus a constant, the asymptote, constrained to be the same for all $\hbar\Omega$[6]. Horizontal lines indicate the experimental gs and the NCFC result (uncertainty = 0.5 MeV).

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I show in Fig. 1 results for the ground state (gs) of ^{12}C as a function of N_{max} obtained with a realistic NN interaction, JISP16 [8]. The smooth curves portray fits that achieve asymptotic independence of N_{max} and $\hbar\Omega$. The NCFC gs energy (the common asymptote) of -94.5 MeV indicates $\sim 3\%$ overbinding. The assessed uncertainty in the NCFC result is 0.5 MeV indicated in parenthesis in the figure. The largest calculations correspond to $N_{max} = 10$, with a matrix dimension near 8 billion. $N_{max} = 12$ produces a matrix dimension near 81 billion which we hope to solve in the future.

3. Light-front Hamiltonian field theory

It has long been known that light-front Hamiltonian quantum field theory has similarities with non-relativistic quantum many-body theory and this has prompted applications with established non-relativistic many-body methods (see Ref. [1] for a review). These applications include theories in 1+1, 2+1 and 3+1 dimensions. Several of my efforts in 1+1 dimensions, in collaboration with others, have focused on developing an understanding of how one detects and characterizes transitional phenomena in the Hamiltonian approach. To this end, I list the following developments:

1. identification and characterization of the quantum kink solutions in the broken symmetry phase of two dimensional ϕ^4 including the extraction of the vacuum energy and kink mass that compare well with classical and semi - classical results [12];

2. detailed investigation of the strong coupling region of the topological sector of the two-dimensional ϕ^4 theory demonstrating that low-lying states with periodic boundary conditions above the transition coupling are dominantly kink-antikink coherent states [13];

3. switching to anti-periodic boundary conditions in the strong coupling region of the topological sector of the two-dimensional ϕ^4 theory and demonstrating that low-lying states above the critical coupling are dominantly kink-antikink-kink states as well as presenting evidence for the onset of kink condensation[14]. Fig. 2 presents the detailed transition of the lowest 5 mass eigenstates in the broken phase from kink to kink-antikink-kink structure over a narrow range in the coupling. Increasing the resolution K shrinks the range in coupling over which the transitions occur.

More recently, full-fledged applications to gauge theories in 3+1 dimensions have appeared and there are several talks at this conference showing initial results for QED (e.g. talks by Honkanen, by Hiller and by Chabysheva) plus roadmaps for addressing QCD. A brief summary of some of the major developments in 3+1 dimensional Hamiltonian light front field theory includes the solutions of

1. light-front QED wave equations for the electron plus electron-photon system [15]

2. simplified gauge theories with a transverse lattice [2, 3, 16]

3. Hamiltonian QED for the electron plus electron-photon system in a trap with a basis function approach [4, 20] that I discuss in the next section.

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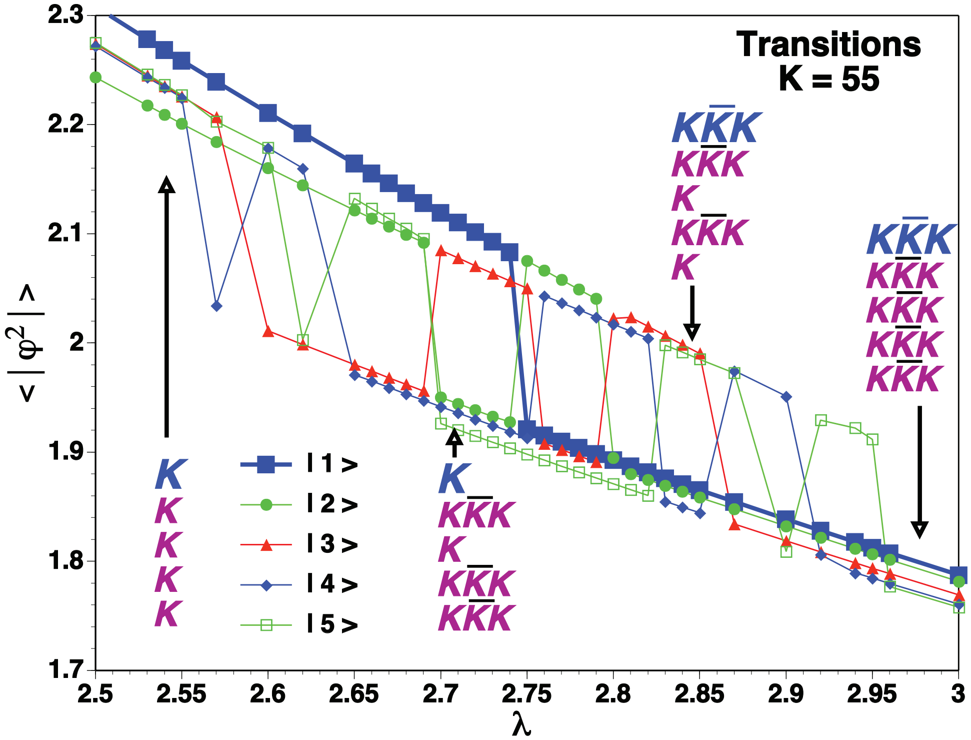

Figure 2: Expectation value of the square of the scalar field as a function of the coupling constant λ at light-front harmonic resolution K=55 for the lowest five excitations of two dimensional ϕ^4 in the broken phase [14]. The pattern of transitions correspond to 5 states falling with increasing λ and crossing the 5 lowest states, thus replacing them and becoming the new 5 lowest states. At selected values of λ, the character of the lowest states is indicated on the figure with the top level of each column signifying the nature of the lowest state. Successive excited states are signified by the labels proceeding down the column. The letter "K" represents "kink" while "$K\bar{K}K$" represents "kink-antikink-kink".

These successes open pathways for ambitious research programs to evaluate non-perturbative amplitudes and to address the multitude of experimental phenomena that are conveniently evaluated in a light-front quantized approach. As one important example, consider the deeply virtual Compton scattering (DVCS) process which provides the opportunity to study the 3-dimensional coordinate space structure of the hadrons. Recent efforts with model 3+1 dimensional light-front amplitudes [18] have shown that the Fourier spectra of DVCS should reveal telltale diffractive patterns indicating detailed properties of the coordinate space structure.

4. Cavity mode light-front QED and QCD

Together with co-authors, I have introduced the "Basis Light Front Quantized (BLFQ)" approach [4] which adopts a light-front basis space consisting of the 2-D harmonic oscillator for the transverse modes (radial coordinate ρ and polar angle ϕ) and a discretized momentum space basis

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for the longitudinal modes with either periodic or anti-periodic boundary conditions. The 2-D oscillator states are chosen to retain rotational symmetry about x^- direction and they are characterized by their principal quantum number n and orbital quantum number m. Adoption of this basis is also consistent with recent developments in AdS/CFT correspondence with QCD [19]. Note, however, that the choice of basis functions is arbitrary except for the standard conditions of orthonormality and completeness.

In our initial applications, we focus on QED and consider systems in a transverse scalar harmonic trap [20]. This setup will be useful for addressing a range of strong field QED problems such as electron-positron pair production in relativistic heavy-ion collisions and with ultra-strong pulsed lasers planned for the future. We adopt the sector dependent non-perturbative renormalization scheme [21].

The chosen basis allows the imposition of symmetry constraints that reduce the Hamiltonian matrix dimension considerably. For example, we impose the constraint of a fixed total magnetic projection (J_z) and fixed total longitudinal momentum in dimensionless units (K) consistent with longitudinal boost invariance. We also impose a cutoff in the Fock space basis controlling the number of fermion and boson degrees of freedom and we impose a limit on the maximum total 2-D oscillator quanta (N_{max}) in the basis. We then investigate how the non-perturbative results depend on the cutoffs and seek to obtain results in the continuum limit where the cutoffs are removed.

The total light-front Hamiltonian is $H = H_0 + V$ (KH gives the invariant mass-squared) where the unperturbed Hamiltonian H_0 for this system is defined by the sum of the occupied modes with the scale set by the combined constant $\Lambda^2 = 2M_0\Omega$ with Ω representing the harmonic oscillator frequency:

$$H_0 = 2M_0 P_c^- = \frac{\Lambda^2}{K}\sum_i \frac{2n_i + |m_i| + 1 + \bar{m}_i^2/\Lambda^2}{x_i}, \tag{4.1}$$

where $\bar{m}_i$ is the mass of the parton i and x_i is its light-front momentum fraction. We keep the photon mass set to zero and the electron mass $\bar{m}_e$ is set at the physical mass 0.511 MeV in our non-renormalized calculations. We also set $M_0 = \bar{m}_e$. The interaction vertices are taken directly from light-front quantized QED in the light-front gauge and are used to generate the interaction matrix elements in the basis. Considerable analytical and numerical efforts are required to achieve an efficient evaluation scheme for these matrix elements.

We can extend this approach to QCD by implementing the SU(3) color degree of freedom for each parton - 3 colors for each fermion and 8 for each boson. We have investigated two methods for implementing the global color singlet constraint. In the first case, we follow Ref. [22] by constraining all color components to have zero color projection and adding a Lagrange multiplier term to the Hamiltonian to select global color singlet eigenstates. In the second case, we restrict the basis space to global color singlets [4, 23]. The second method produces a factor of 30-40 lower many-parton basis space dimension at the cost of increased computation time for matrix elements. Either implementation provides an exact treatment of the global color symmetry constraint but the use of the second method provides overall more efficient use of computational resources.

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5. Conclusion

The short history of light-front Hamiltonian field theory features many advances that pave the way for non-perturbative solutions of gauge theories. The goal is to evaluate the light-front amplitudes for strongly interacting composite systems and predict experimental observables. High precision tests of the Standard Model may be envisioned as well as applications to theories beyond the Standard Model.

Following successful methods of *ab initio* nuclear many-body theory, we have introduced a basis light-front quantization (BLFQ) approach to Hamiltonian quantum field theory and illustrated some of its key features with a cavity mode treatment of QED. We have developed methods for treating color in order to extend the light-front BLFQ appraoch to QCD. The computational requirements of this approach are substantial, and we foresee extensive use of leadership-class computers to obtain practical results.

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References

[1] H. C. Pauli and S. J. Brodsky, *Solving Field Theory In One Space One Time Dimension*, *Phys. Rev. D* **32**, (1985)1993; S. J. Brodsky, H. C. Pauli and S. S. Pinsky, *Quantum Chromodynamics and Other Field Theories on the Light Cone*, *Phys. Reports* **301** (1998) 299 [*hep-ph/9705477*].

[2] D. Chakrabarti, A. Harindranath and J. P. Vary, *A Study of q-qbar States in Transverse Lattice QCD Using Alternative Fermion Formulations*, *Phys. Rev. D* **69** , (2004) 034502 [*hep-ph/0309317*].

[3] D. Grunewald, E. M. Ilgenfritz, E. V. Prokhvatilov and H. J. Pirner, *Formulating Light Cone QCD on the Lattice*, *Phys. Rev. D* **77** (2008) 014512.

[4] J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang, *Hamiltonian light-front field theory in a basis function approach*, *Phys. Rev. C* **81** (2010) 035205 [*nucl-th/0905.1411*]

[5] P. Navrátil, J. P. Vary and B. R. Barrett, *Properties of 12-C in the ab-initio Nuclear Shell Model*, *Phys. Rev. Lett.* **84** (2000) 5728; *Large-basis ab-initio No-core Shell Model and its application to 12-C*, *Phys. Rev. C* **62** (2000) 054311.

[6] P. Maris, J. P. Vary and A. M. Shirokov, *Ab Initio no-core full configuration calculations of light nuclei*, *Phys. Rev. C.* **79** (2009) 014308 [*nucl-th/0808.3420*].

[7] P. Navratil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand and A. Nogga, *Structure of A = 10-13 nuclei with two- plus three-nucleon interactions from chiral effective field theory*, *Phys. Rev. Lett.* **99** (2007)042501 [*nucl-th/0701038*].

[8] A. M. Shirokov, J. P. Vary, A. I. Mazur and T. A. Weber, *Realistic Nuclear Hamiltonian: Ab exitu approach*, *Phys. Letts. B* **644** (2007) 33 [*nucl-th/0512105*].

[9] P. Maris, M. Sosonkina, J. P. Vary, E. G. Ng and C. Yang, *Scaling of ab-initio nuclear physics calculations on multicore computer architectures*, International Conference on Computer Science, ICCS 2010, *Procedia Computer Science* **1** (2010) 97.

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[10] S. K. Bogner, R. J. Furnstahl, P. Maris, R. J. Perry, A. Schwenk and J. P. Vary, *Convergence in the no-core shell model with low-momentum two-nucleon interactions*, *Nucl. Phys. A* **801**, (2008) 21[*nucl-th/0708.3754*].

[11] P. Maris, A. M. Shirokov and J. P. Vary, *Ab initio nuclear structure simulations: the speculative* ^{14}F *nucleus*, *Phys. Rev. C* **81** (2010) 021301(R).

[12] D. Chakrabarti, A. Harindranath, L. Martinovic and J. P. Vary, *Kinks in discrete light cone quantization*, *Phys. Lett. B* **582**, (2004) 196 [arXiv:hep-th/0309263].

[13] D. Chakrabarti, A. Harindranath, L. Martinovic, G. B. Pivovarov and J. P. Vary, *Ab initio results for the broken phase of scalar light front field theory*, *Phys. Lett. B* **617**, (2005) 92 [arXiv:hep-th/0310290].

[14] D. Chakrabarti, A. Harindranath and J. P. Vary, *A transition in the spectrum of the topological sector of phi**4(2) theory at strong coupling*, *Phys. Rev.* D **71**, (2005) 125012 [arXiv:hep-th/0504094].

[15] S. J. Brodsky, V. A. Franke, J. R. Hiller, G. McCartor, S. A. Paston and E. V. Prokhvatilov, *A nonperturbative calculation of the electron's magnetic moment*, *Nucl. Phys. B* **703** (2004) 333; S. S. Chabysheva and J. R. Hiller, *"A nonperturbative calculation of the electron's magnetic moment with truncation extended to two photons*, *Phys. Rev. D* **81** (2010) 074030; S. S. Chabysheva and J. R. Hiller, *On the nonperturbative solution of Pauli–Villars-regulated light-front QED: A comparison of the sector-dependent and standard parameterizations*, arXiv:0911.3686 [hep-ph].

[16] S. Dalley and B. van de Sande, *Transverse lattice calculation of the pion light-cone wavefunctions*, *Phys. Rev. D* **67** (2003) 114507, [arXiv:hep-ph/0212086].

[17] H. Honkanen, P. Maris, J. P. Vary and S. J. Brodsky, *Electron in a transverse harmonic cavity*, arXiv:1008.0068 [hep-ph], see also the paper by H. Honkanen in these proceedings.

[18] S. J. Brodsky, D. Chakrabarti, A. Harindranath, A. Mukherjee and J. P. Vary, *Hadron optics: Diffraction patterns in deeply virtual Compton scattering*, *Phys. Lett. B* **641**, (2006) 440 [arXiv:hep-ph/0604262]; S. J. Brodsky, D. Chakrabarti, A. Harindranath, A. Mukherjee and J. P. Vary, *Hadron optics in three-dimensional invariant coordinate space from deeply virtual Compton scattering*, *Phys. Rev. D* **75**, 014003 (2007) [arXiv:hep-ph/0611159].

[19] S. J. Brodsky and G. F. de Teramond, *Light-Front hadron dynamics and AdS/CFT correspondence*, *Phys. Lett. B* **582** (2004) 211 [*hep-th/0310227*]; A. Karch, E. Katz, D. T. Son and M. A. Stephanov, *Linear confinement and AdS/QCD*, *Phys. Rev. D* **74**, (2006)015005.

[20] H. Honkanen, P. Maris, J. P. Vary and S. J. Brodsky, *Electron in a transverse harmonic cavity*, arXiv:1008.0068 [hep-ph].

[21] V. A. Karmanov, J.-F. Mathiot and A. V. Smirnov, *Systematic renormalization scheme in light-front dynamics with Fock space truncation*, *Phys. Rev. D* **77** (2008) 085028 [*hep-th/0801.4507*]

[22] R. J. Lloyd and J. P. Vary, *All-charm Tetraquarks*, *Phys. Rev. D* **70** (2004) 014009 [*hep-ph/0311179*].

[23] Jun Li, *Light front Hamiltonian and its application in QCD* Ph. D. thesis, Iowa State University (2009), unpublished.

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