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Relativistic three-body model for final state interaction in $D^+ \to K^-\pi^+\pi^+$ decay

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A challenge in mesonic three-body decays of heavy mesons is to quantify the contribution of rescattering between the final mesons. D decays have the unique feature that make them a key to light meson spectroscopy, in particular to access the $K\pi$ S-wave phase-shifts. We built a relativistic three-body model for the final state interaction in $D^+ \to K^-\pi^+\pi^+$ decay based on the ladder approximation of the Bethe-Salpeter equation projected on the light-front. The decay amplitude is separated in a smooth term, given by the direct partonic decay amplitude, and a three-body fully interacting contribution, that is factorized in the standard two-meson resonant amplitude times a reduced complex amplitude that carries the effect of the three-body rescattering mechanism. The off-shell reduced amplitude is a solution of an inhomogeneous Faddeev type three-dimensional integral equation, that includes only isospin 1/2 $K^-\pi^+$ interaction in the S-wave channel. The elastic $K^-\pi^+$ scattering amplitude is parameterized according to the LASS data[1]. The integral equation is solved numerically and preliminary results are presented and compared to the experimental data from the E791 Collaboration[2, 3] and FOCUS Collaboration[4, 5].

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1. Introduction

The non-leptonic decays of charmed mesons offer a good opportunity to study the mechanism of heavy quark decay and subsequent hadronization. Due to the experimental efforts of the past years (see e.g. [2, 4, 5, 6]) there are data available to access the hadronic interaction in the final state. The first analysis of $D^+ \to K^-\pi^+\pi^+$ decay was performed by the E791 collaboration and revealed that approximately 50% of this decay proceed through a low-mass scalar resonance with isospin 1/2: the $K_0^*(800)$, also called the κ[2]. The κ meson was the second elusive scalar to be firmly detected in D^+ decays since the scalar-isoscalar $f_0(600)$, or σ has been detected by the same collaboration. Other recent experimental data analysis from the FOCUS[4, 5] and CLEO[6] collaborations based on large data samples had conclusions similar from that obtained by pioneering E791 experiment[2]. Some previous theoretical treatments and analysis[7, 8, 9, 10] of the $K^-\pi^+$ data showed the presence of the κ pole in scattering amplitude. Other theoretical approaches[11, 12] determine the κ and σ poles using Roy's equation [13]. Also models based on chiral perturbation theory [14, 15] extracted κ values from $K\pi$ data.

Although, the κ pole of the $K\pi$ amplitude is suggested by different models, a description of the reaction $D^+ \to K^-\pi^+\pi^+$ is still not available. In addition, this decay has a large branching ratio fraction, easy to be reconstructed from the data, with a very low background and with an S-wave contribution amounting to 80% of the total decay width. The E791 Collaboration[3] used MIPWA (Model Independent Partial Wave Analysis) to obtain the decay amplitude in the lowest partial waves, with no assumptions on the analytic form, which are represented by a generic complex function to be determined directly from data. A comparison between the S-wave $K^-\pi^+$ phases obtained by FOCUS from a MIPWA of $D^+ \to K^+\pi^-\pi^-$ decay and by LASS [1], showed a sizeable energy dependent shift between these two results for the phases. A possible origin of this additional energy dependent phase is a three-body final state interaction.

Our goal is to construct a relativistic three-body model of the final state interaction in $D^+ \to K^-\pi^+\pi^+$ decay by projecting onto the light-front the ladder approximation of the four-dimensional Bethe-Salpeter (BS) equation. Previously, we have formulated the covariant BS equation for the $3 \to 3$ $K^-\pi^+\pi^+$ full transition off-mass-shell amplitude without isospin degrees of freedom[16]. In the model, the decay amplitude is separated in a smooth term, given by the direct partonic decay amplitude, and a three-body fully interacting contribution, that is factorized in the standard two-meson resonant amplitude depending only on the square mass of the pair times a reduced complex amplitude that carries the effect of the three-body rescattering mechanism. In the present calculation, the off-shell reduced amplitude is a solution of an three-dimensional inhomogeneous Faddeev type integral equation in light-front momentum, that has the S-wave isospin 1/2 $K^-\pi^+$ transition matrix as the input. The parameters of the S-wave isospin 1/2 $K^-\pi^+$ T-matrix are fitted to the experimental data from LASS experiment[1]. The integral equation is solved numerically by a standard discretization method and the results for the complex $D^+ \to K^-\pi^+\pi^+$ decay amplitude are compared to experimental data analysis from [2] and [4].

2. Covariant four-dimensional three-body model for heavy-meson decay

Following our model proposed in ref. [16], the partonic amplitude for the decay of the D meson into the $K\pi\pi$ channel with off-shell momenta q_i^μ, and masses m_i $(i = \pi, K, \pi')$ is expressed by a function $D(q_\pi, q_{\pi'})$. This amplitude should be convoluted with the $3 \to 3$ off-shell transition matrix, which take into account the three-meson interacting final state, as shown in figure 1 including the three-body connected ladder series, where the $2 \to 2$ scattering process is summed up in the $K\pi$ transition matrix.

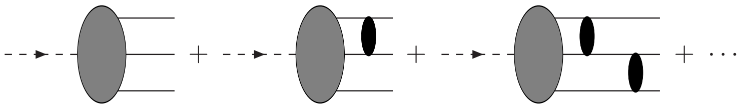

Figure 1: Diagrammatic representation of the heavy meson decay process into $K\pi\pi$, starting from the partonic amplitude (gray) and adding the hadronic multiple scattering in the ladder approximation. The input $K\pi$ scattering amplitude (black) is required fully off-mass-shell.

The full decay amplitude shown in figure 1 is given by:

$$\mathscr{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \int \frac{d^4q_\pi d^4q_{\pi'}}{(2\pi)^8} T(k_\pi, k_{\pi'}; q_\pi, q_{\pi'}) S_\pi(q_\pi) S_\pi(q_{\pi'}) S_K(K - q_{\pi'} - q_\pi) D(q_\pi, q_{\pi'}), \quad (2.1)$$

where the momentum of the pions from the decay of the D are k_π and $k_{\pi'}$. The matrix element of the $3 \to 3$ transition matrix is $T(k_\pi, k_{\pi'}; q_\pi, q_{\pi'})$. The Feynman propagator for the meson is $S_i(q_i) = \imath(q_i^2 - m_i^2 + \imath\varepsilon)^{-1}$ in the approximation where self-energies are disregarded.

The $3 \to 3$ transition matrix is obtained from the following assumptions: *i)* the $K\pi\pi$ Bethe-Salpeter equation is solved in the ladder approximation, and *ii)* the effective S-wave interaction between the kaon and pion is local on the fields with the $K\pi$ scattering amplitude, $\tau_i(M^2_{K\pi})$, parameterized to reproduce the $K\pi$ S-wave phase-shift from the LASS experiment [1]. After that, the full $3 \to 3$ ladder scattering series is summed up by solving the integral equations for the Faddeev decomposition of the scattering matrix.

Owing the S-wave contact interaction for the $K\pi$ system, the decay amplitude can be decomposed as:

$$\mathscr{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) \; + \; \tau(M^2_{K\pi})\xi(k_{\pi'}) + \tau(M^2_{K\pi'})\xi(k_\pi) \; , \quad (2.2)$$

where the first term, $D(k_\pi, k_{\pi'})$, corresponds to a smooth background given by the partonic decay amplitude. It is represented by the gray blob with three legs at leftmost corner of figure 1. The second and third terms in the rhs of Eq. (2.2) carry the full effect of the final state interaction through the two-meson resonant amplitude, τ, times a spectator amplitude, ξ, that contains the three-body re-scattering contributions. The contributions to the decay from $\tau(M^2_{K\pi})\xi(k_{\pi'}) + \tau(M^2_{K\pi'})\xi(k_\pi)$ correspond to the sum of the second, third and higher order diagrams of figure 1. They represent the full hadronic re-scattering series of the $K\pi\pi$ system in the ladder approximation.

The re-summation of the scattering series by the reduced amplitude $\xi(k)$ can be done by an integral equation shown diagrammatically in figure 2,

$$\xi(k) = \xi_0(k) + \int \frac{d^4q}{(2\pi)^4} \tau((K-q)^2) S_K(K - k - q) S_\pi(q) \xi(q), \quad (2.3)$$

where

$$\xi_0(k) = \int \frac{d^4q}{(2\pi)^4} S_\pi(q) S_K(K-k-q) D(k,q), \tag{2.4}$$

is the driving term (source term) in the above integral equation. This term multiplied by $\tau(M^2_{K\pi'})$ is represented by the second diagram in figure 1. The other term in eq. (2.3) comes from three-body connected diagrams. The lowest order rescattering term is the connected amplitude given by the third diagram in figure 1.

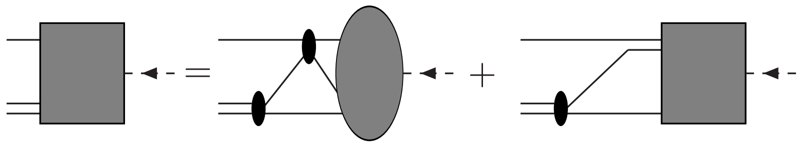

Figure 2: Diagrammatic representation of the integral equation for the three-body function $\tau(M^2_{K\pi'})\xi(k_\pi)$ (gray box). The driving term contains the partonic amplitude convoluted with the two-body scattering amplitude (black).

The contribution to the three-body rescattering process given by the reduced amplitude solution of eq. (2.3) is built by mixing resonances of the two possible $K\pi$ pairs. The reduced amplitude from the model is a function only of the momentum of the spectator pion. The model separates the background of the decay amplitude in two parts: one that corresponds to a smooth function of the momentum of the pions $(D(k_\pi, k_{\pi'}))$ and the other giving by $\xi(k)$ times the pair amplitude, which is a fully three-body interacting term modulated by the $K\pi$ resonant amplitude.

3. $K\pi$ S-wave I=1/2 transition matrix and a dynamical AdS/QCD Model

In our model the input for the calculation of the $3 \to 3$ T-matrix is the S-wave I=1/2 $K\pi$ amplitude. Besides the $K_0^*(1430)$ resonance of the parametrization of the LASS data [1] given in ref. [17], we introduced the resonances $K_0^*(1630)$ (in PDG there is no assignment of spin to $K(1630)$) and $K_0^*(1950)$ in the S-wave $K\pi$ scattering amplitude.

The suggestion to include the higher radial excitations of K_0^* comes from a recent proposal to interpret the scalar meson family (f_0) as radial excitations of the σ meson[18]. This interpretation comes within a Dynamical AdS/QCD model [19] in which the dilaton-gravity background is a solution of 5D Einstein equations. Confinement is dynamically encoded in the model and leads to a Regge behavior for light- scalar, - pseudoscalar mesons as well higher spin mesons ($S \geq 1$). These models implement Maldacena's Conjecture [20] which introduced new perspectives to the treatment of the strong-interaction physics by using gauge/string dualities. New insights and analytical tools to study hadron properties in the non-perturbative regime of the strong force follows the conjecture. The dynamical AdS/QCD model applied to the strange sector gives a mass spectrum ($M^2 \times n$) for the Kappa family with a slope of ~ 0.6 GeV^2 [21]. Therefore, the resonances $K_0^*(1430)$, $K_0^*(1630)$ and $K_0^*(1950)$ are suggested to be radial excitations of $K_0^*(800)$, which couple to the S-wave $K\pi$ I=1/2.

In this way, we include the $K_0^*(1630)$ and $K_0^*(1950)$ resonances in our relativistic model of the S-wave $K\pi$ amplitude extending the parametrization given in ref. [17]:

$$\tau\left(M^2_{K\pi}\right) = 4\pi\,\frac{M_{K\pi}}{k}\left(S_{K\pi}-1\right)\ , \tag{3.1}$$

with the S-matrix given by:

$$S_{K\pi} = \frac{k\cot\delta + \imath k}{k\cot\delta - \imath k}\prod_{r=1}^{3}\frac{M_r^2 - M_{K\pi}^2 + \imath z_r\bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - \imath z_r\Gamma_r}\ , \tag{3.2}$$

where $z_r = kM_r^2/(k_r M_{K\pi})$. The relative c.m. momentum of the $K\pi$ pair is

$$k = \left[\left(\frac{M^2_{K\pi} + m_\pi^2 - m_K^2}{2M_{K\pi}}\right)^2 - m_\pi^2\right]^{\frac{1}{2}}\ . \tag{3.3}$$

The parameters M_r, Γ_r, $\bar{\Gamma}_r$ and k_r refer to each resonance. The relative momentum k_r is obtained at the resonance position. The parameter $-\Gamma_r < \bar{\Gamma}_r < \Gamma_r$ is introduced to allow inelasticity in the $K\pi$ scattering. The background s-matrix is parameterized by the effective range expansion $k\cot\delta = -a^{-1} + \frac{1}{2}r_0 k^2$ with $a = -1.6$ GeV^{-1} and $r_0 = 3.32$ GeV^{-1}. The resonance parameters $(M_r, \Gamma_r, \bar{\Gamma}_r)$ in GeV for $K_0^*(1430)$, $K_0^*(1630)$ and $K_0^*(1950)$ are (1.48, 0.25, 0.25), (1.67, 0.1, 0.1) and (1.9, 0.2, 0.14), respectively.

4. Light-Front Three-Body Model

The Bethe-Salpeter formalism is four-dimensional and explicitly Lorentz covariant. Three-dimensional reductions result in equations of the quasi-potential type. In Light-Front Dynamics (LFD) the state vector describing the system is expanded in Fock components with increasing number of particles. The Fock-space state vector is defined on a hypersurface of the four-dimensional space-time defined by the light-front $x^+ = t + z = cte$. The time evolution is governed by the light-front (LF) hamiltonian which is the generator of the LF-time boosts. This form of dynamics is suited for relativistic systems, since the interaction of a probe (an electron, for instance) with the constituents of the system is separated from its interaction with the vacuum fluctuations - the LF vaccum is trivial apart the contribution of zero modes [22, 23]. The perturbative amplitudes for the physical process obtained within LF quantization are equivalent to the usual equal time formalism in the infinite momentum frame. From a qualitative point of view, all the physical processes become as slow as possible because of time dilation in this reference frame. LF quantization and the associated Fock-space decomposition greatly simplifies the description of the system which is equivalent to a snapshot not spoiled by vacuum fluctuations. It is thus very natural that in the description of high energy experiments like deep inelastic scattering the probabilities associated with the LF wave functions are measured.

The projection of the four-dimensional BS equation 2.3 to the light-front takes the advantage of the quasi-potential formalism developed in ref.[24] and the result is written as:

$$\xi_{\frac{3}{2}}(y,k_\perp) = \frac{5}{3}\xi_0(y,k_\perp) + \frac{i}{3}\int_0^{1-y}\frac{dx}{x(1-y-x)}\int\frac{d^2q_\perp}{(2\pi)^3}\frac{\tau_{\frac{1}{2}}(M^2_{K\pi})\xi_{\frac{3}{2}}(x,q_\perp)}{M_D^2 - M^2_{0,K\pi\pi} + i\varepsilon}\ . \tag{4.1}$$

A similar form, but considering the homogeneous equation, has been used to study relativistic three-boson bound states [25, 26]). The driving term becomes

$$\xi_0(y,\vec{k}_\perp) = -\int \frac{d^4q}{(2\pi)^3}\frac{1}{q^2-m_\pi^2+i\varepsilon}\,\frac{1}{(K-k-q)^2-m_K^2+i\varepsilon}$$
$$= \frac{i}{2}\int_0^1 dx\int\frac{d^2q_\perp}{(2\pi)^3}\left[\frac{1}{M_{k\pi}^2-M_{0,K\pi}^2+i\varepsilon}-\frac{1}{\mu^2-M_{0,K\pi}^2+i\varepsilon}\right]+i\lambda(\mu^2) \quad (4.2)$$

where $\lambda(\mu^2)$ is the renormalization parameter and

$$M_{K\pi}^2=(M_D^2-\frac{k_\perp^2+m_\pi^2}{y})-k_\perp^2,\;\; M_{0,K\pi}^2=\frac{q_\perp^2+m_\pi^2}{x}+\frac{q_\perp^2+m_K^2}{1-x}. \quad (4.3)$$

The physical amplitude for the S-wave $D^+\to K^-\pi^+\pi^+$ decay parameterized according to [4] as $A_0(S_A,S_B)=a_0(S_A)e^{i\Phi_0(S_A)}+a_0(S_B)e^{i\Phi_0(S_B)}$ and our theoretical framework gives:

$$A_0(M_{K\pi'}^2)=a_0(M_{K\pi'}^2)e^{i\Phi_0(M_{K\pi'}^2)}\propto 1+\tau_{\frac{1}{2}}\left(M_{K\pi'}^2\right)\xi_{\frac{3}{2}}\left(y,k_{\perp\pi}\right), \quad (4.4)$$

where $k_{\perp\pi}^2=\left(M_D^2+m_\pi^2-M_{K\pi'}^2\right)^2/(2M_D)^2-m_\pi^2$ and $y=\sqrt{k_{\perp\pi}^2+m_\pi^2}/M_D$.

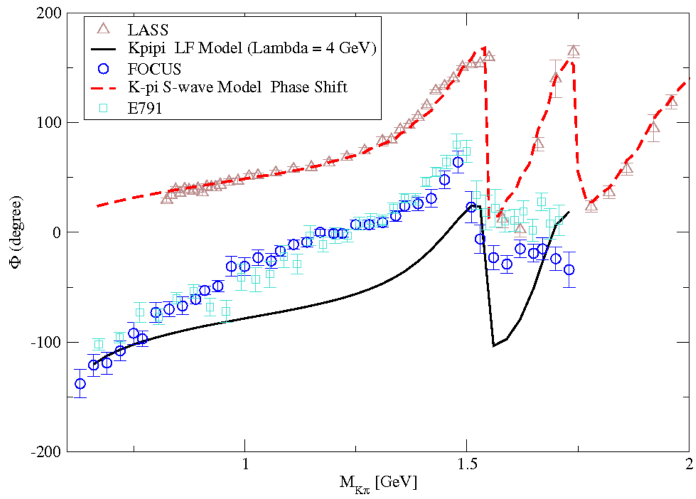

Figure 3: Comparison of the three- and two-body models S-wave phase-shifts with the experimental data as function of the $K\pi$ mass. The $D^+\to K^-\pi^+\pi^+$ phase is defined according to (4.4). Experimental data from E791 and FOCUS collaborations, and for the $K\pi$ I=1/2 S-wave phase-shift from LASS.

5. Numerical Results and Conclusion

Our aim is to solve numerically light-front eq. (4.1) for the reduced amplitude. For that preliminary calculation, we introduce a cut-off, $\Lambda=4$ GeV, of the transverse momentum in eq.

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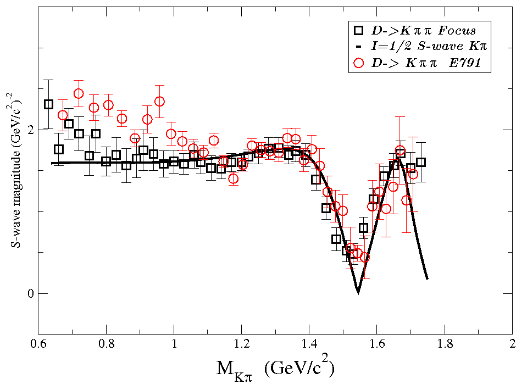

Figure 4: Magnitude of the $D^+ \to K^-\pi^+\pi^+$ S-wave amplitude (4.4)as a function of the $K\pi$ mass. Experimental data from E791 and FOCUS collaborations.

(4.1), to calculate the spectator function $\xi(k)$. Also, we have considered a finite value of 0.1 GeV for ε in the meson propagators. The spectator function was considered for virtual pion momentum fully off-energy-shell. The partonic decay amplitude is assumed momentum independent with, i.e., $D(q_\pi, q_{\pi'}) = 1$ and $\lambda(0) = 0.12 + \imath 0.06$.

Our preliminary results for the s-wave phase for the $D \to K\pi\pi$ decay are shown in figure 3 and for the complex reduced S-wave amplitude in figure 4. We show the results for the three-resonance model (3.2) for the S-wave $K\pi$ I=1/2 phase-shift which present a reasonable compared agreement with the LASS data.

The three-body calculation shows a considerable effect on the phase near the $K\pi$ threshold with a shift of about -150^o. These results are similar to the Focus data from the $D \to K\pi\pi$ decay. But we stress that the phase from the computation of the inhomogeneous term comes from $\lambda(0)$. Given that, the phase for $M_{K\pi} < 1.45$ GeV follows the trend of the data but the details are still missing. The modulus of the S-wave $D \to K\pi\pi$ decay amplitude $|\frac{1}{2} + \tau(M^2_{K\pi})\xi(k_{\pi'})|$ essentially follows the data as well the modulus of the $K\pi$ amplitude. However, our toy model results are still far from satisfactory. We have to consider other effects that could enhance the final state interaction. Moreover, the analytical continuation of the S-wave $K\pi$ scattering amplitude to the unphysical region has to be studied in more detail, as we have naively used the on-mass-shell formula given by eq. (3.3) for $M^2_{K\pi} < (m_K + m_\pi)^2$. Also, the momentum cut-off dependence has to be explored and also the momentum dependence of the partonic amplitude may be important. Therefore, we believe that our model still need to be further improved to allow more definite conclusion about the effect of hadronic three-body final state interaction. However, with the present work we give the first steps to systematically address the issue of final state interaction in the $D \to K\pi\pi$ decay.

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