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String Gauge Symmetries in the Light-Front Polyakov D1 Brane Action

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We investigate the question of string gauge symmetries for the conformally gauge-fixed light-front Polyakov D1 brane action in the presence of background gauge fields.

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Polyakov action is one of the most widely studied topics in string theories [1] - [16]. Recently, we have studied this action for the D1 brane in the conformal gauge (CG), using the instant-form (IF) of dynamics (on the hyperplanes defined by the world-sheet (WS) time $\sigma^0 = \tau =$ constant) [17] - [21] and the light-front (LF) dynamics (on the hyperplanes of the LF defined by the light-cone (LC) WS time $\sigma^+ = (\tau + \sigma) =$ constant)[17] - [29]. The LF theory is seen to be a constrained system in the sense of Dirac, which is in contrast to the corresponding IF theory, where the theory remains unconstrained in the sense of Dirac. The LF theory is seen to possess a set of twenty six second-class constraints. Further, the conformally gauge-fixed Polyakov $D1$ brane action (CGFPD1BA) describing a gauge-noninvariant (GNI) theory (being a gauge-fixed theory) is seen to describe a gauge-invariant (GI) theory in the presence of an antisymmetric NSNS $2-$form gauge field $B_{\alpha\beta}(\tau,\sigma)$. Recently we have shown that this NSNS $2-$form gauge field behaves like a Wess-Zumino (WZ) field and the term involving this field behaves like a WZ term for the CGFPD1BA[9] - [16]. We have also studied [9, 10] the Hamiltonian and path integral formulations[9] -[14] of the CGFPD1BA with and without a scalar dilaton field in the IF as well as in the LF dynamics. In both the above cases the theory is seen (as expected) to be gauge- noninvariant (GNI)[9] - [21], possessing a set of second-class constraints in each case [9] - [21], owing to the conformal gauge-fixing [1] -[14] of the theory. The CGFPD1BA being GNI does not respect the usual (string) gauge symmetries defined by the WS reparametrization invariance (WSRI) and the Weyl invariance (WI). However, in the presence of a constant 2-form gauge field $B_{\alpha\beta}$ it is seen to describe a gauge-invariant (GI) theory respecting the usual (string) gauge symmetries defined by the WSRI and the WI. The IF and LF Hamiltonian and path integral formulations of this theory in the presence of the constant 2-form gauge field $B_{\alpha\beta}$ have been studied by us in Refs. [9] - [13]. In the present work, we consider the question of the string gauge symmetries associated with the Polyakov D1 brane action in the presence of some other background fields such as the $U(1)$ gauge field $A^\mu(\tau,\sigma)$ and the constant scalar axion field $C(\tau,\sigma)$.

The Polyakov $D1$ brane action in a d-dimensional curved background $h_{\alpha\beta}$ is defined by [1] - [13]:

$$\tilde{S} = \int \tilde{\mathscr{L}} d^2\sigma \tag{1a}$$

$$\tilde{\mathscr{L}} = \left[-\frac{T}{2}\sqrt{-h} h^{\alpha\beta} G_{\alpha\beta} \right] \tag{1b}$$

$$h = \det(h_{\alpha\beta}) \tag{1c}$$

$$G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{1d}$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1,+1,...,+1) \tag{1e}$$

$$\mu,\nu = 0,1,....,(d-1);\ \alpha,\beta = 0,1 \tag{1f}$$

Here $\sigma^\alpha \equiv (\tau,\sigma)$ are the two parameters describing the worldsheet (WS). The overdots and primes denote the derivatives with respect to τ and σ. T is the string tension. $G_{\alpha\beta}$ is the induced metric on the WS and $X^\mu(\tau,\sigma)$ are the maps of the WS into the d-dimensional Minkowski space and describe the strings evolution in space-time [1] - [10]. $h_{\alpha\beta}$ are the auxiliary fields (which turn out to be proportional to the metric tensor $\eta_{\alpha\beta}$ of the two-dimensional surface swept out by the string). One can think of $\tilde{S}$ as the action describing d massless scalar fields X^μ in two dimensions moving on a curved background $h_{\alpha\beta}$. Also because the metric components $h_{\alpha\beta}$ are varied in Eq. (1), the

2-dimensional gravitational field $h_{\alpha\beta}$ is treated not as a given background field, but rather as an adjustable quantity coupled to the scalar fields [9, 10]. The action $\tilde{S}$ has the well-known three local gauge symmetries given by the 2-dimensional WSRI and WI[1] - [13] as follows:

$$X^\mu \longrightarrow \tilde{X}^\mu = [X^\mu + \delta X^\mu] \tag{2a}$$

$$\delta X^\mu = [\zeta^\alpha(\partial_\alpha X^\mu)] \tag{2b}$$

$$h^{\alpha\beta} \longrightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] \tag{2c}$$

$$\delta h^{\alpha\beta} = \left[\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma}\right] \tag{2d}$$

$$h_{\alpha\beta} \longrightarrow [\Omega] h_{\alpha\beta} \tag{2e}$$

Where the WSRI is defined for the two parameters $\zeta^\alpha \equiv \zeta^\alpha(\tau,\sigma)$; and the WI and is specified by a function $\Omega \equiv \Omega(\tau,\sigma)$ [1] - [14]. In the following we, however, work in the so-called orthonormal gauge where one sets $\Omega = 1$ [1] - [14]. Also for the CGFPD1BA one makes use of the fact that the 2-dimensional metric $h_{\alpha\beta}$ is also specified by three independent functions as it is a symmetric 2×2 metric. one can therefore use these gauge symmetries of the theory to choose $h_{\alpha\beta}$ to be of a particular form [1] - [14]:

$$h_{\alpha\beta} := \eta_{\alpha\beta}; \quad h^{\alpha\beta} := \eta^{\alpha\beta} \tag{3}$$

For the IF dynamics we take [1, 2]:

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \tag{4a}$$

$$h^{\alpha\beta} = \eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \tag{4b}$$

with

$$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta})} = \quad +1 \tag{5}$$

In LF formulation we use the Light-Cone (LC) variables defined by [1, 2, 3, 7]:

$$\sigma^\pm := (\tau \pm \sigma) \quad \text{and} \quad X^\pm := (X^0 \pm X^1)/\sqrt{2} \tag{6}$$

In this case we take

$$h_{\alpha\beta} := \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix} \tag{7a}$$

$$h^{\alpha\beta} := \eta^{\alpha\beta} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \tag{7b}$$

with

$$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta)}} = \quad +1/2 \tag{8}$$

Now the action $\tilde{S}$ in the CG (in the IF and LF) finally reads [1, 2, 3, 7]:

$$S^N = \int \mathscr{L}^N d^2\sigma \tag{9a}$$
$$\mathscr{L}^N = [(-T/2)][\partial^\beta X^\mu \partial_\beta X_\mu] \tag{9b}$$
$$\beta = 0,1 \quad and \quad \mu = 0,1,i \; ; \quad i = 2,3,...,25 \;\; (IF) \tag{9c}$$
$$\beta = +,- \quad and \quad \mu = +,-,i \; ; \quad i = 2,3,...,25 \;\; (LC/FF) \tag{9d}$$

The action S^N is the CGFPD1BA. This action is seen to lack the local gauge symmetries. This is in contrast to the fact that the original action $\tilde{S}$ had the local gauge symmetries and was therefore GI. The theory defined by the action S^N, on the other hand describe GNI. This is not surprising at all because the theory defined by S^N is after all (conformally) gauge-fixed theory and consequently not expected to be GI anyway. In fact, the IF theory defined by S^N is seen to be unconstrained [9] - [21] whereas the LF theory is seen to possess a set of 26 second-class constraints [9] - [14]. In both the cases it does not respect the usual local string gauge symmetries defined by WSRI and WI.

We now consider this CGFPD1BA in the presence of a constant background antisymmetric 2-form gauge field $B_{\alpha\beta}$ studied earlier by Schmidhuber, de Alwis and Sato, Tseytlin and Abou Zeid and Hull and others defined by [1] -[10]:

$$S^I = \int \mathscr{L}^I \; d^2\sigma \tag{10a}$$
$$\mathscr{L}^I = [\mathscr{L}^C + \mathscr{L}^B] \tag{10b}$$
$$\mathscr{L}^C = [\lambda \mathscr{L}^N] = \left[-\frac{T}{2}\right][\lambda \partial^\beta X^\mu \partial_\beta X_\mu] \tag{10c}$$
$$\mathscr{L}^B = \left[-\frac{T}{2}\right][\Lambda \varepsilon^{\alpha\beta} B_{\alpha\beta}] \tag{10d}$$
$$\lambda = \sqrt{(1+\Lambda^2)}; \;\; \Lambda = constant \tag{10e}$$
$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{10f}$$
$$B_{\alpha\beta} := \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu} \tag{10g}$$
$$B_{\alpha\beta} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \tag{10h}$$
$$B = B_{01} = -B_{10} \; (IF) \tag{10i}$$
$$B = B_{+-} = -B_{-+} \; (LC/FF) \tag{10j}$$
$$\alpha,\beta = 0,1 \quad and \quad \mu = 0,1,i; \quad i = 2,3,....,25 \;\; (IF) \tag{10k}$$
$$\alpha,\beta = +,- \quad and \quad \mu = +,-,i; \quad i = 2,3,....,25 \;\; (LC/FF) \tag{10l}$$

In IF, the above action is seen to possess only one first-class constraint and to possess three local gauge symmetries given by the two-dimensional WSRI and the WI:

$$
\begin{aligned}
X^{\mu} &\longrightarrow \tilde{X}^{\mu} = [X^{\mu} + \delta X^{\mu}] && \text{(11a)}\\
\delta X^{\mu} &= [\zeta^{\alpha}(\partial_{\alpha} X^{\mu})] && \text{(11b)}\\
h^{\alpha\beta} &\longrightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] && \text{(11c)}\\
\delta h^{\alpha\beta} &= \left[\zeta^{\gamma}\partial_{\gamma} h^{\alpha\beta} - \partial_{\gamma}\zeta^{\alpha} h^{\gamma\beta} - \partial_{\gamma}\zeta^{\beta} h^{\alpha\gamma}\right] && \text{(11d)}\\
B_{\alpha\beta} &\longrightarrow \widetilde{B}_{\alpha\beta} = [B_{\alpha\beta} + \delta B_{\alpha\beta}] && \text{(11e)}\\
\delta B_{\alpha\beta} &= [\zeta^{\alpha}\partial_{\alpha} B_{\alpha\beta}] && \text{(11f)}\\
h_{\alpha\beta} &\longrightarrow [\Omega] h_{\alpha\beta} && \text{(11g)}
\end{aligned}
$$

It is important to recollect here that the 2-form gauge field $B_{\alpha\beta}$ is a scalar field in the target-space whereas it is a constant anti-symmetric tensor field in the world-sheet space. The Hamiltonian and path integral formulations of this theory under the gauge $B \approx 0$ have been studied by us in Ref. [9].

We now consider the string gauge symmetries of the gauge-fixed Polyakov D1 brane action describing a gauge-noninvariant theory in the presence of a $U(1)$ gauge field $A_{\alpha}(\equiv A_{\alpha}(\tau,\sigma))$ and a constant scalar axion field $C(\equiv C(\tau,\sigma))$ and show that the gauge-fixed Polyakov D1 brane action describing a gauge-noninvariant theory (being a gauge-fixed theory) is seen to describe a GI theory when considered in the presence of above background fields. We also show that the $U(1)$ gauge field $A_{\alpha}(\tau,\sigma)$ and the constant scalar axion field $C(\tau,\sigma)$ are both seen to behave like the Wess-Zumino (WZ) fields and the term involving these fields is seen to behave like a WZ term for the CGFPD1BA in the presence of an axion field and an $U(1)$ gauge field. Here the field A_{α} is a scalar field in the target space and a vector field in the WS space and the axion field C is a constant scalar field in both the target space as well as in the WS space. We find that the resulting theory obtained in the above manner describes a GI system respecting the usual string gauge symmetries defined by the WSRI and the WI. It is seen that the axion field C and the $U(1)$ gauge field A_{α}, in the resulting theory behave like the WZ fields and the term involving these fields behaves like a WZ term for the CGFPD1BA. The situation in the present case is seen to be exactly analogous to a theory where one considers the CGFPD1BA in the presence of a 2-form gauge field $B_{\alpha B}$ as studied by us in Refs. [9] - [13], where the field $B_{\alpha\beta}$ behaves like a WZ field and the term involving this field behaves like a WZ term for the CGFPD1BA [11, 12, 13]. The CGFPD1BA in the presence of a constant background scalar axion field C and an $U(1)$ gauge field A_α is defined by [1] - [14]:

$$S^I = \int \mathscr{L}^I \; d^2\sigma ; \qquad \mathscr{L}^I = [\mathscr{L}^C + \mathscr{L}^A] \tag{12a}$$

$$\mathscr{L}^C = [\lambda \mathscr{L}^N] = \left[-\frac{T}{2}\right] [\lambda \partial^\beta X^\mu \partial_\beta X_\mu] \tag{12b}$$

$$\mathscr{L}^A = \left[-\frac{T}{2}\right] [-\Lambda C \varepsilon^{\alpha\beta} F_{\alpha\beta}] \tag{12c}$$

$$\lambda = \sqrt{(1+\Lambda^2)}; \quad \Lambda = constant; \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{12d}$$

$$F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \; ; \quad f = F_{01} = -F_{01}(IF); f = F_{+-} = -F_{-+}(FF) \tag{12e}$$

$$\alpha, \beta = 0,1 \quad and \quad \mu = 0,1,i; \quad i = 2,3,....,25 \;\; (IF) \tag{12f}$$

$$\alpha, \beta = +,- \quad and \quad \mu = +,-,i; \quad i = 2,3,....,25 \;\; (LC/FF) \tag{12g}$$

Now the matrix of the Poisson brackets of the constraints Ψ_i is seen to be singular implying that the constraints Ψ_i form a set of first-class constraints and that the theory described by S_1 is a GI theory [9]-[21]. It is indeed seen to posses three local gauge symmetries given by the two dimensional WSRI and the WI defined by [1]-[14]:

$$X^\mu \longrightarrow \tilde{X}^\mu = [X^\mu + \delta X^\mu] \tag{13a}$$

$$\delta X^\mu = [\zeta^\alpha (\partial_\alpha X^\mu)] \tag{13b}$$

$$h^{\alpha\beta} \longrightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] \tag{13c}$$

$$\delta h^{\alpha\beta} = \left[\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma}\right] \tag{13d}$$

$$A_\beta \longrightarrow \tilde{A}_\beta = [A_\beta + \delta A_\beta] \tag{13e}$$

$$\delta A_\beta = [\zeta^\alpha \partial_\alpha A_\beta] \tag{13f}$$

$$C \longrightarrow \tilde{C}_\alpha = [C + \delta C] \tag{13g}$$

$$\delta C = [\zeta^\alpha \partial_\alpha C] \tag{13h}$$

$$h_{\alpha\beta} \longrightarrow [\Omega h_{\alpha\beta}] \tag{13i}$$

The above theory is thus seen to be GI possessing the three local gauge symmetries defined by the two-dimensional WSRI and the WI in both the IF and LF dynamics.

In conclusion, the Polyakov D1 brane action in a d-dimensional curved background $h_{\alpha\beta}$ defined by $\tilde{S}$ is GI and it possesses the well-known three local string gauge symmetries. However, under conformal gauge-fixing, the CGFPD1BA is no longer GI as expected and it also does not possess the local string gauge symmetries being a gauge-fixed theory. However, this GNI theory when considered in the presence of a constant background scalar axiom field C and an $U(1)$ gauge field A_α it is seen to become a GI theory possessing the three local string gauge symmetries. The scalar axion field C and the $U(1)$ gauge field A_α are seen to behave like the WZ fields and the term involving these fields is seen to behave like a WZ term for the CGFPD1BA, which in the absence of this term is seen to posses a set of second-class constraints and consequently describes a GNI theory which does not respect the local string gauge symmetries. The situation in the present case is analogous to a theory where one considers the CGFPD1BA in the presence of a constant 2-form gauge field $B_{\alpha\beta}$ which behaves like a WZ field and the term involving this field behaves like a WZ term for the CGFPD1BA [11, 12, 13].

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